Optimal Tone Reproduction Curves for Color Printing

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Abstract

This paper describes a method for making multi-color tone reproduction curves. Curves are modeled using Bernstein polynomials, and their parameters are adjusted using numerical optimization to minimize the color errors of an arbitrary sample set. With a well-chosen sample set, these curves are superior to those obtained by other methods.

Background

In his seminal paper, "Monochrome reproduction in photoengraving" (Murray, 1936), Alexander Murray states, "A purely photographic process of multi-color plate-making could not be realized unless it gave a practically perfect monochrome reproduction of the density scale on each plate." This is the rationale for TVI-based tone reproduction curves, widely used to calibrate printing processes, and to specify those processes in print standards (ISO, 2013).

In 2003, we (the authors) used TVI-based tone curves to calibrate a halftone digital proof to an offset press sheet. The resulting color match was visually unacceptable. Initially, we thought we'd made an error, so we rechecked our work. The tonality (TVI values) of the calibrated proof was correct. Further measurements indicated that the tonality of CMYK neutrals corresponded with our visual assessment.

We recalled the RIT "TRAND" technique, a graphical method used to calibrate drum scanners (Elyjiw & Archer, 1972). Alternate tone curves made with this technique greatly improved the proof to press sheet match. Murray's premise was

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not sufficient for a good color match. Tone curves built to match the pure process colors might not result in correct gray balance and/or tonality of the neutral scale.

We felt this was an important discovery, and did further testing, measuring many pre-press proofs and press sheets to increase our understanding. In 2004, we presented a TAGA paper (Birkett & Spontelli, Improving Print Standards by Specifying Isometric Tone Reproduction, 2004). That was followed in 2005 by

another TAGA paper (Birkett & Spontelli, A Regression-Based Model of Colorimetric Tone Reproduction for Use in Print Standards, 2005). Based on that work, we proposed a new metric to replace TVI – colorimetric tone value or CTV (Birkett & Spontelli, Colorimetric Tone Value (CTV), A Proposed Single- Value Measure for Presswork, 2005). CTV eventually became the basis for spot color tone value or SCTV (ISO, 2017).

We were actively involved in print standards during this time period. We hoped our insights would lead to improved standards for commercial offset printing, which were sorely needed. The Idealliance Print Properties & Colorimetric Council (PPC) developed the "G7 methodology" (ANSI/CGATS/IDEAlliance, 2013) similar to the RIT "TRAND" technique. At an ISO TC 130 meeting in 2006, the U.S. delegation proposed to incorporate aspects of the G7 method into international print standards and standard data sets. Their proposal was met with strong opposition from some European delegates (ISO TC 130 WG 3, 2006). This began a schism that continues to this day (ISO TC 130, 2014).

Although our own standards proposals were never considered (Birkett & Spontelli, Position Paper On Proposed Revisions to ISO 12647-2:2004 A Move to Colorimetry and Matching a Reference Printing Standard, 2007), we continued working on pressroom calibration. Our work suggested a better solution was possible (Birkett & Spontelli, TC 130 Generic Matching Tool, 2006). In 2009, we developed the method described in this paper. Since then, we've used it extensively with great success. We call it the OPTIMAL method.

Problem Definition

Starting in the late 1990's, CtP (computer to plate) and digital proofs began replacing film and analog proofs. By 2006, the commercial printing industry was in a state of color anarchy. Then, FOGRA and Idealliance released reference data sets based on the ISO 12647-2:2004 standard. *Today, it is well- established practice to use color-managed ink-jet proofs as the critical color reference. Now, printers need a reliable way to match those proofs on press.*

FOGRA and CGATS published their first reference data sets in the early 1990s (International Color Consortium (ICC), 2015) (International Color Consortium

(ICC), 2006). The materials and procedures used to produce these data sets were documented and eventually incorporated into early print specifications and standards (ANSI 1995, ISO 1996). This was before the introduction of CtP technology. Printing plates were made from film, which was calibrated to be linear. The tonality of the printing process (TVI) was measured as a guide for pressmen. But tone reproduction was controlled at the color scanner, not in the plate room.

Although TVI measures were included in early print specifications and standards, they were only meant for reference. Solid ink densities were considered the primary target. If the ink trap and TVI values were off, it was up to the pressman to make any necessary changes, usually to the ink, fountain, or some mechanical adjustment. The use of tone curves to calibrate a printing process followed the adoption of CtP.

The TVI method is the obvious solution when full-range tone curves of the reference printing are available. The method is quite simple. Make a test print with correct ink densities, measure the TVI of each process color ramp, then build tone curves to match the reference. The gray balance may not be quite right, but the pressman will adjust that to match the proofs.

The G7 method actually began as a way to control the gray balance of reference print runs. It arose from a long and troubled effort to make a standard data set (TR 004) for commercial offset printing (Warter, 2004). The GRACoL2006 data set was derived from FOGRA data, modified to have G7 gray balance and tonality. Since then, Idealliance has promoted the G7 method as a way to calibrate a printing process. It is widely accepted among US printers.

Although both methods seemed reasonable at the time they were developed, the problem they solve has changed. Today, the goal is to match presswork to digital proofs. These proofs are color-managed to reference data sets. This is what McDowell (the "father" of international print standards) envisioned in 1999 (McDowell, 1999), and later (ANSI CGATS, 2013).

McDowell believed that color management would also be used in the pressroom. Many prepress workflow systems now have device-link profile capabilities for this purpose. Device-link profiles support multi-dimensional CLUTs, which are the ultimate color transform mechanism. Despite the theoretical superiority of device-link profiles, tone curves are generally preferred for their simplicity.

Because they are just one-dimensional functions, tone curves cannot produce a perfect match for all colors! They are a compromise – a change that helps one color may harm other colors. That is why we use optimization to compute tone curves, and why the choice of samples is important. We want to optimize the colors that are most likely to occur in real jobs.

Be aware that reference data sets are not direct measurements of an especially good press run. They are composite data, computer processed to remove flaws, and to attain the desired gamut, tonality, and gray balance. Consequently, tone curves made by older methods may not produce the expected results. With our optimization method, the origin of the reference data set doesn't matter. The technique is completely agnostic, and works equally well with data sets from any source (CGATS, FOGRA, IFRA, JPMA, etc.)

Stepwise Calculation vs. Optimization

In 1920, Loyd Jones published his paper, "On the Theory of Tone Reproduction, with a Graphic Method for the Solution of Problems" (Jones, 1920). This paper introduced the "Jones diagram", which was used to solve tone reproduction problems before computers, and is still used for illustration. A typical Jones diagram is below (Figure 1.)

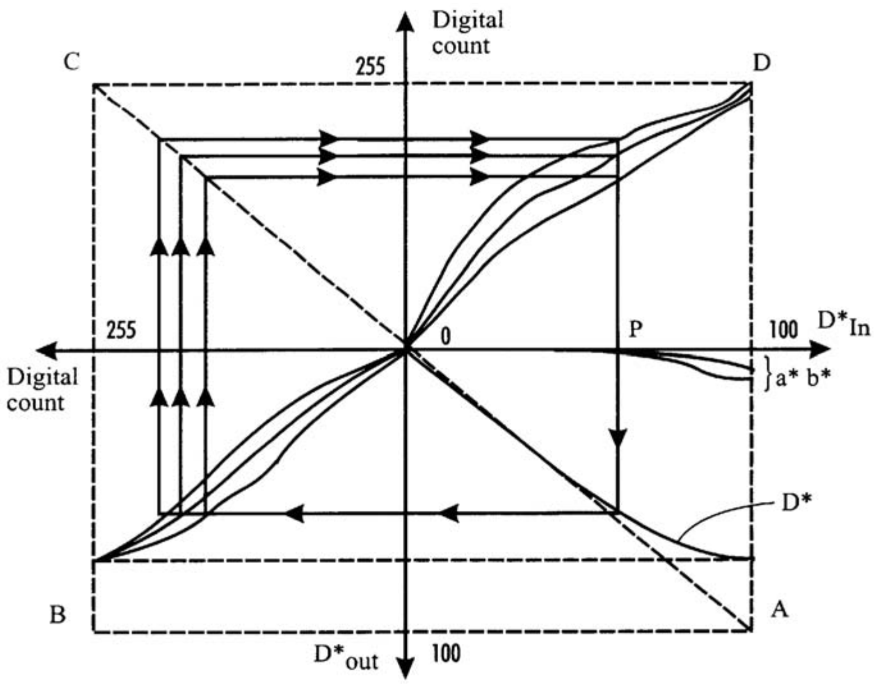

Figure 1. *Typical Jones diagram (Degani, T., & Henderson, 2002)*

A Jones diagram may be used to calculate TVI-based tone curves. For each process color, two functions are combined – the tone value function for the reference printing and the tone value function for the test printing. Typically, a set of input values (e.g. 0%, 5%, 10%, ... 100%) is transformed to the required output values. This set of paired values is then entered into the platemaking software as a tone curve.

The problem with this approach is that it doesn't consider the colors that will most likely appear in images. The same is true for the near-neutral method. This is a consequence of trying to calculate the curves exactly. Printing is a complex multi-dimensional process. Calibrating it with tone curves is an over-determined problem. In general, there is no set of tone curves that will correctly reproduce all colors. The solution for this type of problem is optimization.

To understand optimization, first consider a simple B&W printing process. We print a test chart containing a range of inputs (x-values), and measure the outputs (y-values) (Figure 2.)

X	Y
0.000	0.050
0.100	0.057
0.200	0.062
0.300	0.133
0.400	0.185
0.500	0.298
0.600	0.401
0.700	0.530
0.800	0.678
0.900	0.861
1.000	1.048

Figure 2. *Table of inputs (x-values) and outputs (y-values)*

Here is the same data plotted as a graph (Figure 3.) Notice that the progression of the points is not smooth. The y-values have a random component, which is typical of pressroom measurements.

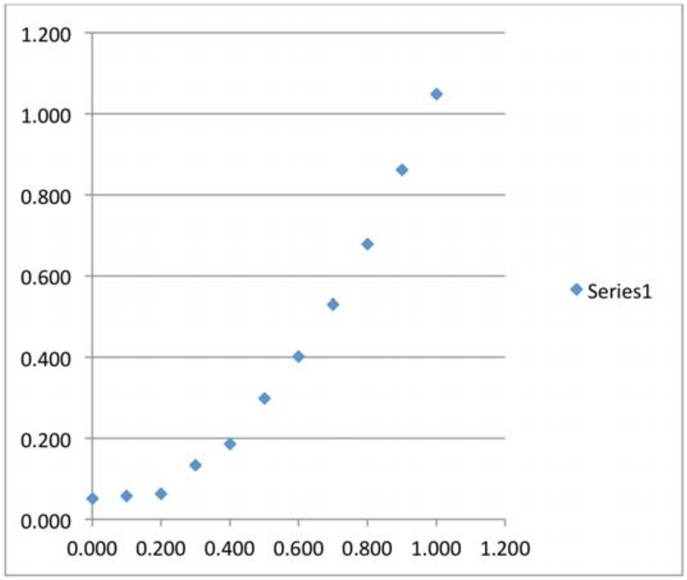

Figure 3. *Plot of X-Y Data*

To compute tone curves with this data, it will be necessary to interpolate values between those actually measured. One way to do this is by connecting the points with line segments (piecewise linear interpolation) (Figure 4.) Jones did this with a pencil and paper (Jones, 1920).

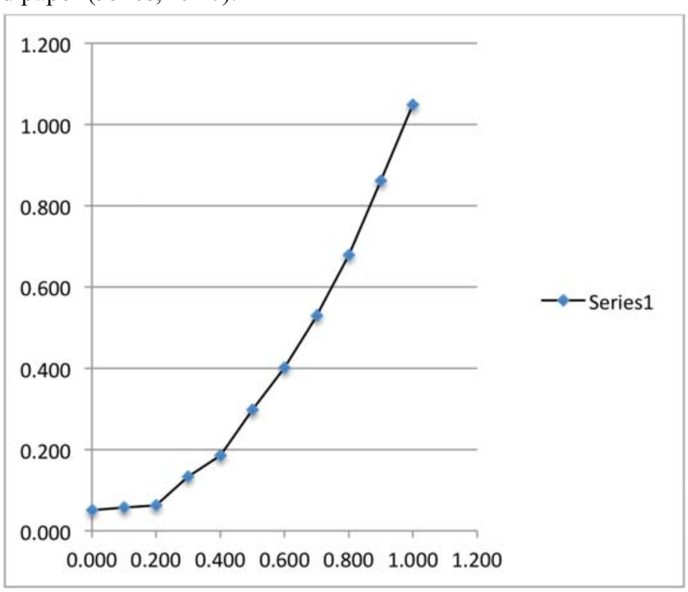

Figure 4. *Piecewise Linear Interpolation*

There is another way to use this data. In Excel, you can add a "trendline", which is a mathematical function "fitted" to the data (Figure 5.) The trendline does not intersect the actual measurements. It represents the "best fit" of the function to the data. This technique is commonly known as regression.

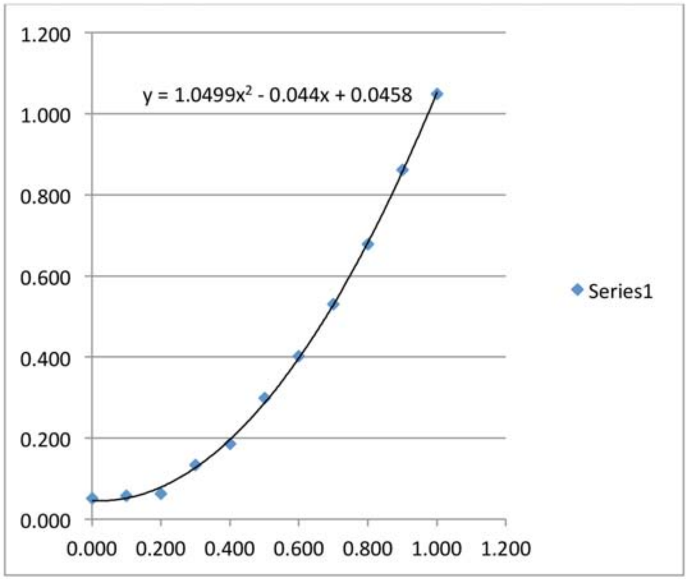

Figure 5. *Trendline fitted to X-Y data*

From our sample data, Excel computed the polynomial function $y = 1.0499x^2 - 0.044x + 0.0458$. This function was obtained using the least squares method, a basic statistical technique. None of the data points exactly intersect the trendline function, but the y-value errors are minimized. If the measurement variation has a Gaussian distribution, a least squares solution is a very good estimate of the actual underlying function.

We modeled the function as a polynomial, $y = ax^2 + bx + c$, then determined the coefficients a, b, and c using the least squares method. The coefficients were adjusted to minimize the y-value errors. If the function were part of a larger system, we could choose some other measurements to optimize. This approach can be extended to complex systems having multi-dimensional inputs and outputs. *The OPTIMAL method optimizes tone curves for each ink channel simultaneously to minimize the overall error for the chosen sample set.*

Figure 6 is a flowchart of the optimization process. A test chart is printed and measured with a spectrophotometer. The curves are initially linear (identity). The chart device values (usually CMYK) are transformed by the curves, then converted to L*a*b* values using the reference profile/data set. These values are then compared to the measured values to determine the L*a*b* error for each sample.

The curve parameters are then adjusted to reduce the error, using some clever math. This process is repeated until the error cannot be reduced any further. The resulting curves are then inverted, and output for the prepress workflow system.

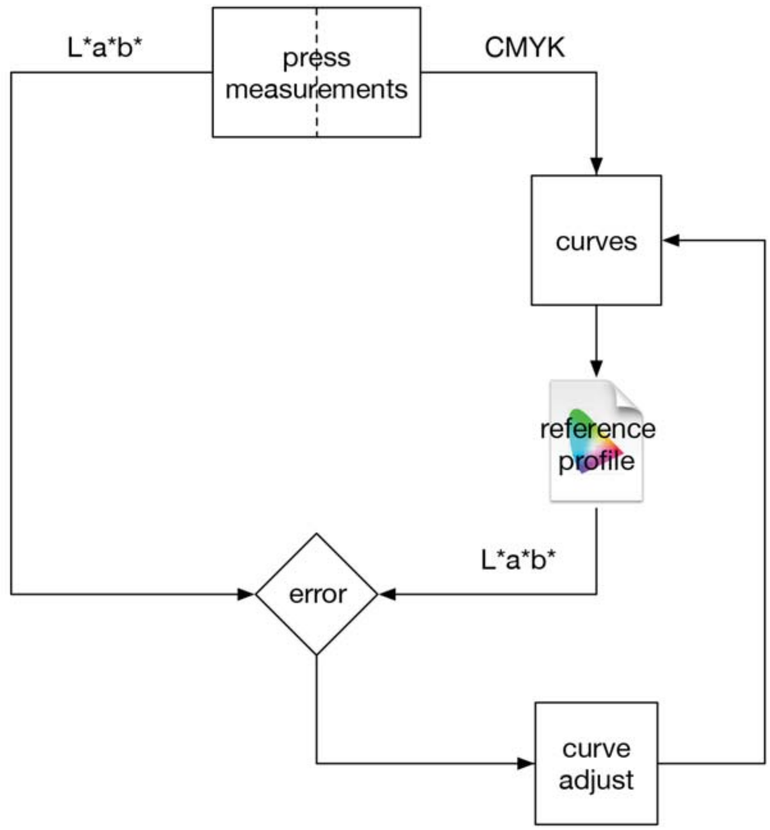

Figure 6. *Flowchart for the OPTIMAL method*

Modeling Tone Curves

In our flowchart (Figure 6) the block marked "curves" represents a set of parametric curves, one curve for each color (CMYK). Each curve has an input and output for the device values. In addition, there is a set of control inputs, which are the parameters defining the shape of the curve. These parameters are adjusted during optimization to minimize the color errors of the sample set.

The simplest form of parametric curve is a set of equally spaced x-y points. The x-values are inputs and the y-values are outputs. For example, here is a set of five points, [0, 0], [0.25, 0.3], [0.5, 0.6], [0.75, 0.9], [1, 1]. Outputs for the five input values are exactly specified. For other input values, we interpolate between the points surrounding that input value. Interpolation can be done with straight lines (linear) or cubic functions (cubic spline). Our method works with either of these curve types.

However, our preferred parametric curve is the Bernstein polynomial, which is a linear combination of Bernstein basis functions (Joy, 2000). These polynomials have some useful properties that make them ideal for modeling tone curves. Like tone curves, Bernstein polynomials are defined over the domain 0 to 1. There are n + 1 basis functions in a set of degree n. A Bernstein polynomial is a linear combination of these functions. The polynomial coefficients are the curve parameters that we optimize.

The following illustrations show the Bernstein basis functions of degree 4 (Figure 7) and degree 6 (Figure 8.) Notice there are five functions of degree 4 and seven functions of degree 6. The degree may be any positive integer, but we typically use values between 3 and 6.

The gradation controls of drum scanners (Molla, 1988) were analogous to a Bernstein polynomial of degree 4 (Figure 7). The highlight and shadow controls set the coefficients of the endpoint basis functions, $bf0$ and $bf4$. The mid-tone control set the coefficient of the middle basis function, $bf2$. The highlight contrast and shadow contrast controls set the coefficients of the remaining basis functions, $bf1$ and $bf3$.

Although drum scanners are no longer used, this analogy shows the suitability of Bernstein polynomials for modeling tone curves. Five-point gradation control was hard-wired into drum scanners, but the complexity of a Bernstein tone curve is easily adjusted by the degree of the basis functions (Figure 8).

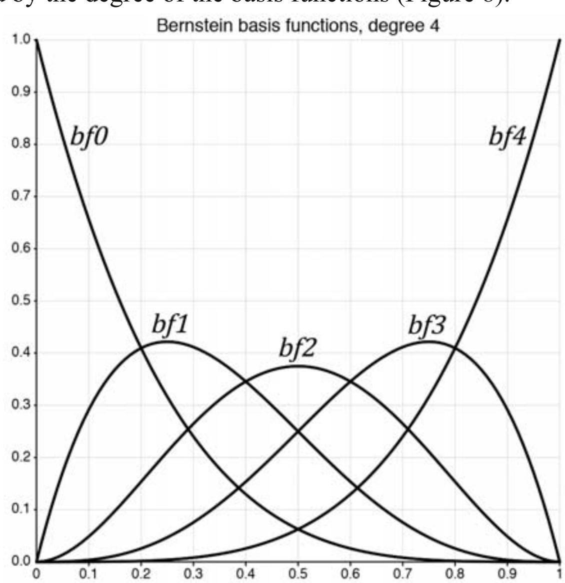

Figure 7. *Bernstein basis functions, degree 4*

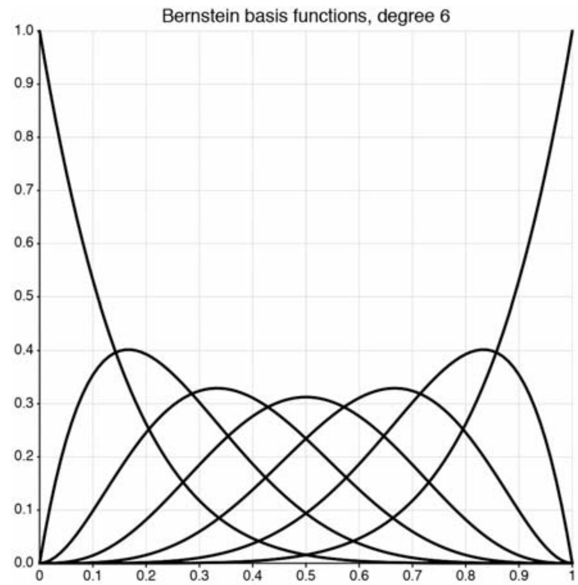

Figure 8. *Bernstein basis functions, degree 6*

Consider a Bernstein polynomial with the parameters [0, 0.5, 0.2, 1.5]. There are four parameters. Therefore, the basis functions ($bf0$, $bf1$, $bf2$, $bf3$) are of degree 3. The Bernstein polynomial is,

$$f(x) = 0 \times bf0(x) + 0.5 \times bf1(x) + 0.2 \times bf2(x) + 1.5 \times bf3(x)$$

Below is a plot of that function (Figure 9.)

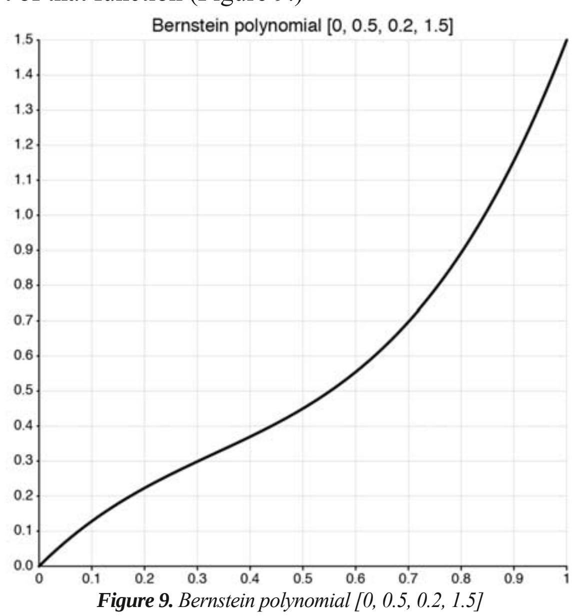

Figure 9. *Bernstein polynomial [0, 0.5, 0.2, 1.5]*

Notice that each of the basis functions is greatest at points that roughly divide the range into equal segments. The first coefficient of the polynomial (0 in this example) sets the function value at x = 0. Likewise, the last coefficient (1.5 in this example) sets the function value at x = 1. These coefficients also affect the intermediate values, but in a rapidly diminishing way as you move away from the endpoints. The second and third coefficients affect the output most strongly at $x \approx 1/3$ and $x \approx 2/3$, respectively. The Bernstein polynomial parameters can be viewed as largely controlling their respective region of the curve, and gradually relinquishing control to the adjacent parameters.

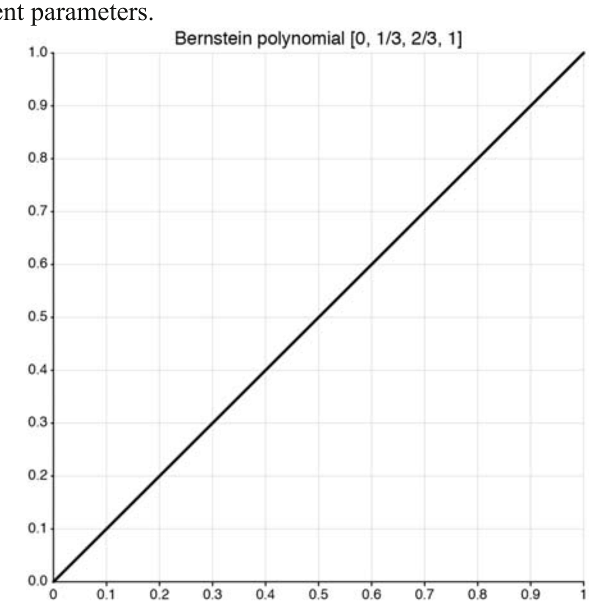

Figure 10. *Bernstein Polynomial [0, 1/3, 2/3, 1]*

Another Bernstein polynomial, [0, 1/3, 2/3, 1], is shown in Figure 10. Notice that this is a perfectly straight line passing through points [0, 0] and [1, 1]. This is the identity function (often called a “linear” curve). In fact, *for any Bernstein polynomial of degree n, the polynomial [0, 1/n, 2/n, 3/n ... (n - 1)/n, 1] is the identity function*. This is a very useful property, because it allows us to make a linear curve of any degree. We initialize each curve to be the identity function before optimizing.

The degree of a Bernstein polynomial controls the possible complexity of the curve. For example, if the degree is 1, only linear curves are possible. If the degree is 3, a cubic spline is possible. There is an optimum value for the degree; we generally want to limit the degree to the minimum needed for a good result. This, of course, depends on the process, and how well it has been linearized. We have found that a degree between 3 and 6 works well for offset printing. The degree is also limited by the sample set, which will be discussed later.

Optimization Techniques

We optimize the tone curves by adjusting their parameters to minimize color differences between the press and the reference profile/data set. There are many standard optimization methods, which are chosen according to the nature of the problem (Björck, 1996) (Madsen, Nielsen, & Tingleff, 2004) (Nocedal & Wright, 2006). Our problem involves a non-linear process (printing), and we are working with non-linear measurements (L*a*b*). This type of problem is usually solved with a "non-linear least squares" method.

We use the **Gauss-Newton** method, which is named after Karl Gauss and Sir Isaac Newton, famous mathematicians. This is a well-established, basic technique. It works very well when the solution is clearly defined, which is the case for our problem. The technique is usually described using matrix math. The following **normal equation** relates curve parameter changes ($\Delta\beta$) to the resulting L*a*b* differences (Δy),

$$(J^T J)\Delta\beta = J^T\Delta y$$

The **Jacobian matrix** (J) contains partial derivative values, which are the ratio of a change in the L*a*b* values of each sample, to a very small change in each of the Bernstein curve coefficients (β). The Jacobian matrix can be quite large. For instance, if we had 300 samples and 20 (4 x 5) curve parameters, the Jacobian would be 900 rows x 20 columns, or 18,000 values.

Once the Jacobian matrix (J) is computed, the curve parameter changes ($\Delta\beta$) are determined by solving the normal equation. These changes are applied, and the procedure repeated until the reduction of error values (Δy) is very small. Within five iterations, the optimization is essentially complete. Needless to say, this involves a lot of computing. Fortunately, modern computers are powerful. The above example takes about one second on a 2015 laptop computer.

We've also used the **Levenberg-Marquardt** method (Levenberg, 1944) (Marquardt, 1963), which is a variation of Gauss-Newton that is less aggressive in finding the solution, and therefore, more reliable when the solution is not so well behaved. We use an open-source software implementation called **levmar** (Lourakis, 2011), which has the capability to constrain the optimization process. We will discuss constraints in the next section.

Optimization Goals and Constraints

It should be clear by now that our goal is to create curves that minimize color errors between press measurements, and the corresponding reference values. We employ the Gauss-Newton "optimization engine" to accomplish this. (With the least squares method, error values may be positive or negative, but their square will always be positive, and that is what is minimized).

We calculate the error for each sample by comparing the measured colors values to the reference values. Here is an example of color samples and their error values (Figure 11.)

sample	device values				ref L*a*b* values			press L*a*b* values			dEab
1	0.000	1.000	1.000	0.000	49.8	70.3	51.4	51.5	69.6	49.6	2.60
2	1.000	0.000	1.000	0.000	53.0	-69.4	29.0	53.3	-68.9	32.6	3.56
3	1.000	1.000	0.000	0.000	26.8	20.5	-45.6	27.0	21.3	-44.9	1.11
4	0.000	0.000	0.000	0.000	100.0	0.0	0.0	100.2	-0.0	-0.0	0.23
5	0.000	1.000	1.000	0.000	49.8	10.3	51.4	51.9	69.4	49.5	2.99
6	0.000	1.000	1.000	0.200	43.1	58.4	42.5	40.4	52.7	36.5	8.69
7	0.000	1.000	1.000	0.400	36.4	47.2	34.3	31.2	40.0	26.1	12.11
8	1.000	0.400	0.000	0.600	27.5	-9.7	-25.9	21.8	-4.6	-19.9	9.68
9	0.000	1.000	1.000	1.000	12.3	8.3	8.5	14.5	8.5	5.4	3.81
10	0.898	0.851	0.851	1.000	10.6	-0.4	1.6	11.8	0.4	1.1	1.49

Figure 11. *Color Samples and* ΔE^*_{ab} *Values*

For each measured sample, the CMYK device values (0 - 1) are on the left. Next are the reference L*a*b* values, then, the actual measured L*a*b* values from the press sheet. The ΔE^*_{ab} error values are on the far right. The reference color values are computed using the curves and the reference profile/data set (Figure 6.) The measured (press) color values remain fixed.

Other color difference metrics could be used, e.g. ΔE^*_{94}, ΔE^*_{00}, CMC, or DIN99. However, ΔE^*_{ab} is very efficient because it allows us to optimize ΔL*, Δa*, and Δb* instead of ΔE^*_{ab} (and obtain the same result). The other metrics will reduce the error values of colored samples, and thereby diminish their influence on the optimization outcome.

When discussing color, it is natural to assume errors are based on colorimetry. But other metrics could be used, such as density, tone value, or even spectral values.

It is easy to weight the individual samples during optimization. We can use weights to make the optimization goal very specific with regard to the samples. For instance, we could increase the weight of a sample having the color of an important client's logo. Another example, which we will discuss in the next section, is to weight the samples according their black value. The matrix formula for weighted least squares is,

$$(J^T W J)\Delta\beta = J^T W \Delta y$$

Compare this to the **normal equation** in the previous section. The weights matrix (W) is a simple diagonal matrix containing the weight values.

It is common practice to "pin" the endpoints of tone curves, so that inputs of 0 or 1 produce the same outputs. With our method, this is optional. Initially, the tone curves are identity functions. We can choose to include either endpoint in the optimization, or not. If we include an endpoint, it will be adjusted to whatever value is optimal. This may or may not produce a good result. For instance, the solid black value might map to 0.9. This would cause solid black elements, such as type, to be screened. That would not be good for offset printing with coarse screen rulings, but it might be fine for inkjet printing with stochastic screening. If the solid endpoints map to values other than 1, this indicates the solid ink levels (densities or ink limits) are not set correctly for the profile/data set we're trying to match.

With optimization, various constraints are possible (Nocedal & Wright, 2006). The **levmar** software (Lourakis, 2011) mentioned previously supports linear equation, box, and inequality constraints. Constraints are useful in preventing unwanted outcomes or forcing wanted outcomes. For instance, it is possible for a Bernstein curve to be non-monotonic, but that can be prevented with an inequality constraint. We could also force a particular color to be mapped in a certain way.

The media white point of the print may not exactly match that of the reference profile/data set. This is commonplace. This mismatch is usually handled by using media relative colorimetry. For both sets of measurements, the color is adjusted so that paper white has an L*a*b* value of 100, 0, 0. This is normally done by multiplying the XYZ values by appropriate constants, before converting to L*a*b*. However, it is also possible use absolute colorimetry. Depending on which measurements have the whiter media, this will result in either highlight clipping or background toning. To prevent that, we can pin the 0 endpoint, as described above. That will cause the tone curves to bend smoothly from media relative at 0, towards absolute colorimetry elsewhere.

Whenever both ends (0 and 1) of a curve set are pinned, the gamut of the printing process is mapped exactly to the gamut of the reference profile/data set. With the OPTIMAL method, it is best to scale the reference gamut to the size of the printing process before the curves are optimized. This is done with linear scaling of the XYZ values, to minimize the errors of the solid inks and their overprints. This form of gamut scaling is equivalent to adding (or subtracting) a very small amount of white light to the reference XYZ values. If the ink densities have been set carefully, there's no need for gamut scaling. But sometimes, the supplied press sheets aren't ideal for the intended reference.

Sample Sets

So far, we have explored optimization techniques, and the various ways they can be configured to solve our problem. We've mentioned the importance of sample sets, and the fact that our method can use any sample set. Now, let's look at sample sets in more detail.

Suppose we have a CMYK process we wish to characterize. If we divide each ink color into ten equal segments, and count the number of samples at each vertex, we would have 11 x 11 x 11 x 11 = 14,641 samples. These samples are equally distributed in CMYK color space. Now, consider the samples we would use to compute TVI curves. There are 4 x 11 = 44 samples, located along four edges of the CMYK color space. To compute G7 curves with the same level of detail, we would use 11 gray samples, and 11 black samples, for a total of 22 samples. The TVI and G7 sample sets are very tiny snapshots of the printing process.

You could argue that these small sample sets contain the crucial information that defines the behavior of the entire printing process, but it is easy to find examples that prove otherwise. *The truth is that these small sample sets were chosen to support the stepwise calculation of curves, as explained earlier. If we enlarge our sample set to be more representative of the colors actually printed, we can no longer compute tone curves that way.* The problem becomes over- determined, and there is no exact solution for all those samples. The best we can do is a compromise that minimizes the overall errors, and that compromise is found using optimization techniques.

What is a well-chosen sample set? Consider the range of colors that might be found in a photograph. A photo begins as an RGB image, and is converted to CMYK using an ICC profile. Hopefully the graphic designer will know to use a profile that represents the printing process. The converted CMYK image will use the black generation built into that profile. So, if the RGB photo contained 11 x 11 x 11 = 1,331 colors, the converted CMYK image would also contain that same number of colors. Although there are 14,641 possible CMYK colors, (using an 11 step tone scale), an RGB image converted to CMYK only contains 1,331 colors.

Of course, many different profiles could be made from the same data set. But most likely, the graphic designer will use the profile suggested by their printer(s), created by Adobe, Idealliance, ECI, or a similar source. For non- photo elements, it is certainly possible that a graphic designer might create their own CMYK combinations, but they are more likely to pick a spot color, which would eventually be converted to CMYK using color management. Therefore, it is reasonable to use a sample set derived from the reference ICC profile, combined with any other colors that are important to clients. Keep in mind this is just a recommendation. You can use any sample set with our method, even from prior TVI or G7 calibrations.

Black generation is a bigger issue than you might think. To illustrate, suppose you have an RGB image of a 21 step gray scale. That image is converted to CMYK using the GRACoL2013_CRPC6.icc profile from Idealliance, and perceptual rendering intent. Here is a table showing the L*a*b* and CMYK values of the (gray) steps (Figure 12.)

L*a*b*				CMYK			
100	0	0	–	0.000	0.000	0.000	0.000
95	0	0	–	0.059	0.039	0.041	0.000
90	0	0	–	0.112	0.081	0.081	0.000
85	0	0	–	0.164	0.121	0.120	0.000
80	0	0	–	0.219	0.164	0.162	0.001
75	0	0	–	0.274	0.207	0.205	0.008
70	0	0	–	0.323	0.248	0.245	0.029
65	0	0	–	0.370	0.288	0.285	0.062
60	0	0	–	0.414	0.327	0.323	0.106
55	0	0	–	0.457	0.368	0.364	0.157
50	0	0	–	0.501	0.412	0.406	0.213
45	0	0	–	0.549	0.458	0.450	0.272
40	0	0	–	0.595	0.504	0.490	0.341
35	0	0	–	0.634	0.545	0.526	0.421
30	0	0	–	0.668	0.581	0.557	0.508
25	0	0	–	0.698	0.615	0.584	0.594
20	0	0	–	0.727	0.646	0.607	0.681
15	0	0	–	0.754	0.674	0.622	0.771
10	0	0	–	0.784	0.705	0.627	0.864
5	0	0	–	0.827	0.739	0.612	0.960
0	0	0	–	0.845	0.751	0.603	1.000

Figure 11. *Color Samples and ΔE^*_{ab} Values*

If you're using this profile to convert your RGB files, and you're concerned about gray balance, these are CMYK samples you should use to build your tone curves.

For comparison, the table below (Figure 13) is the gray sample set from the G7 P2P25 target. Notice the CMYK values are similar from 0% to about 25% cyan. Then, black generation of the ICC profile kicks in, with the black ink carrying the shadows, and the CMY inks limited by UCR. If you build tone curves using the P2P25 sample set, you are using dark samples that will never appear in a photo, and ignoring the effects of black generation. If the solid CMY color of the presswork is not neutral, using the P2P25 samples in the shadow region will cause problems (Birkett & Spontelli, TC 130 Generic Matching Tool, 2006).

CMYK			
0.000	0.000	0.000	0.000
0.020	0.012	0.012	0.000
0.039	0.028	0.028	0.000
0.059	0.043	0.043	0.000
0.078	0.055	0.055	0.000
0.102	0.074	0.074	0.000
0.149	0.110	0.110	0.000
0.200	0.149	0.149	0.000
0.251	0.188	0.188	0.000
0.302	0.231	0.231	0.000
0.349	0.271	0.271	0.000
0.400	0.314	0.314	0.000
0.451	0.357	0.357	0.000
0.498	0.400	0.400	0.000
0.549	0.451	0.451	0.000
0.600	0.502	0.502	0.000
0.651	0.553	0.553	0.000
0.698	0.604	0.604	0.000
0.749	0.659	0.659	0.000
0.800	0.718	0.718	0.000
0.851	0.780	0.780	0.000
0.898	0.843	0.843	0.000
0.949	0.922	0.922	0.000
0.980	0.969	0.969	0.000
1.000	1.000	1.000	0.000

Figure 13. *Sample Set from the G7 P2P25 Target*

While gray balance is important, we should not ignore the reproduction of colors. We can include a full assortment of colored samples, also generated using the reference profile. We may decide to include a black ramp, for the sake of B&W halftones; and if there are important logo colors, add them too. However, keep in mind that optimization is a compromise, and including many unlikely colors may harm those that matter.

There will be situations where the sample set is not in our control, e.g. an IT874 chart from a previous press run. We can use the data as-is, knowing that many of the samples are unwanted. A better way to handle this is to use weighted optimization (see the previous section). Each sample is assigned a weight, calculated from the difference in the black ink value after a round trip through the reference profile.

Samples with a large black difference get a small weight, or are just omitted (the round-trip selection criterion).

We claim that any data set can be used. But there is a limitation that comes from the matrix math used to solve the normal equations. Generally, the number of samples is much larger than the number of Bernstein parameters (an over- determined system). But if the number of samples is less than the number of Bernstein parameters (an under-determined system), optimization will fail. It is also possible that a poorly chosen sample set may lead to a bad solution, such as non-monotonic curves. If that happens, lowering the degree of the curve set will improve the results.

Linearity

In our 2005 TAGA paper (Birkett & Spontelli, A Regression-Based Model of Colorimetric Tone Reproduction for Use in Print Standards, 2005), we observed that the tonality of offset printing could be accurately modeled with Hermite spline curves. Hermite spline curves are equivalent to Bernstein polynomials of degree 3. Our observations were based on printing produced with linearized CtP plates.

There is a current trend towards using process-free plates that cannot be measured. We have encountered rough (non-smooth) tone curves with this type of plate. We have also seen rough tone curves from digital presses. In both cases, there is no way to make the printing “smooth” (linearized) prior to making tone curves.

In these instances, we recommend building high-resolution curves (many tone steps) using TVI or CTV measurements of the process ramps. These high- resolution curves are then “trimmed” using our optimization method to introduce proper gray balance and tone. This technique is implemented in one of our software examples.

We suggest that CTV is the preferred metric for this purpose. TVI is based on density, which requires spectral measurements. Most reference data sets are not spectral (an exception is FOGRA 51). While it is possible to use individual XYZ values in place of density, CTV is a better all-around metric (ISO, 2017). An ISO Technical Specification (ISO, 2009) states, “One very critical issue is that the reference characterization data being matched and the measurements of the printing sample both need to be based on the same measurement parameter.” CTV delivers this harmony of measurement.

Software Implementation

We developed a toolkit for solving color reproduction problems (Birkett W. B., 2016). The toolkit is written in the **Perl** language (The Perl Foundation, 2018) using object-oriented modules. Some of the modules are interfaces to powerful software libraries, such as **BLAS** (Netlib, 2018), **LAPACK** (Netlib, 2018), and

levmar (Lourakis, 2011). Because Perl is a scripting language, it is quite easy to test an idea, or solve a one-off problem. A script may be run with a single keystroke at any time. The modules are used to create objects having

predefined methods, which are called to perform commonly needed tasks. These methods are numerous and highly developed, so it is possible to build powerful scripts with just a few lines of code. For example, here is a program that computes round trip values (lines starting with an '#' are comments),

```
use ICC::Profile;

# make GRACoL2013 profile object
$profile = ICC::Profile->new('GRACoL2013_CRPC6.icc');

# get the A2B1 and B2A1 tag objects
($a2b, $b2a) = $profile->tag(qw(A2B1 B2A1));

# compute round trip for CMYK (50, 40, 40, 0) sample
@rt = $b2a->transform($a2b->transform(0.5, 0.4, 0.4, 0));

# print the result
print "round trip = @rt\n";
```

This is the output,

```
round trip = 0.425150960726281 0.334916615796323
0.336943580093474 0.111595483409626
```

This example shows the use of profile objects and methods, which make it easy to perform a normally difficult task.

We developed the optimal method using this toolkit. There are eleven example programs for creating tone curves by various methods, including the TVI and G7 (ANSI/CGATS/IDEAlliance, 2013) methods. These programs are fully commented, and illustrate every technique and option we've mentioned. A great deal of practical information is contained in these programs, which supplement the paper. The tool kit may be obtained online (CPAN, 2018). Contact the authors for the example programs.

Conclusions

The optimal method offers many advantages:

- Targets the proofing reference profile/data set
  - (e.g. GRACoL2013, FOGRA51)
- Flexible
  - Any printing process
  - Any number of inks
  - Any gray balance
  - Any sample set
  - Any error metric
  - Adjustable curve complexity
- Robust
  - Reliable (no surprises or failures, with well-chosen sample set)
  - Smooths process and measurement variations
  - Small impact from bad or missing sample measurements
  - Outliers may be identified and removed
- Powerful
  - Samples may contain any mixture of inks
  - Accounts for the effects of UCR/GCR
  - Accounts for ink trapping variations (including black ink)
  - Makes curves for multiple references with a single test run
- Easy to learn and use
- Uses standard mathematical techniques
- Open source software implementation available
- Free (no patents, royalties, training fees, or certification fees)

We believe the efforts to define reference printing in the early years of color management were on the right track (ANSI CGATS, 1995). Printing processes are best characterized by measuring a large number of test runs, which are then mathematically refined into reference data sets and profiles (McDowell, 1999). Photographers and graphic designers produce image files and proofs using these reference profiles. Today, this is normal practice throughout the world, although with different reference profiles, regionally.

A printer faces the problem of matching these proofs on press. The OPTIMAL method solves this problem DIRECTLY for all reference data sets/profiles. It does not impose a preconceived gray balance or tonality. It works with any printing process, with any set of colorants, conventional or digital.

Printers who currently use the TVI method will see benefits from the OPTIMAL method. The ISO 12647-2 standard specifies the substrate, colorant, screening, ink set, and print sequence for each printing condition (ISO, 2013). A change in any of these parameters will affect color. Furthermore, the colors of the solid ink overprints are not toleranced, because they cannot be controlled (ISO 2013, NOTE 4). Curves built with the OPTIMAL method correct for these process variations by using a sample set of likely CMYK colors, rather than individual process ramps.

Printers who currently use the G7 method will also see benefits from the OPTIMAL method. The solid CMY overprint, which the G7 method presumes to be neutral, may have a significant color cast. This will distort tone curves in the shadow region (Birkett & Spontelli, TC 130 Generic Matching Tool, 2006). Furthermore, dark colors and neutrals contain significant black ink (because of UCR/GCR) that is not considered at all by the G7 method. Curves built with the OPTIMAL method overcome these limitations by using a sample set of likely

CMYK colors, rather than a ramp of gray CMY colors. The optimal method produces a good result, even when the CMY ramp veers away from neutral.

Printers with international work will benefit from the ease of making curves for any printing process. For instance, a printer could make curves for both FOGRA 51 and GRACoL2013 by optimizing the same test data with each of those reference profiles.

The OPTIMAL method is especially useful when the printing process differs from the reference profile. For example, flexography from files built with GRACoL profiles. Theoretically, these instances are the realm of device link profiles. But when pure process colors must map to pure process colors, OPTIMAL curves are a good solution. Other applications include digital presses and wide format printers, which often have a larger gamut than the reference profile. OPTIMAL curves will utilize the larger gamut while preserving the general appearance of the (standard) proofs.

Another good application is N-color printing, such as extended gamut printing with touch plates. One of the main difficulties in this realm is the size of sample sets needed for color management. The OPTIMAL method does not need a large sample set, just the colors that are likely to be printed. This might include a group of simulated spot colors. N-color printing is an evolving field, and standard reference profiles do not yet exist. But when they do, the OPTIMAL method will calibrate these extended gamut processes as easily as CMYK.

Optimization techniques are used daily by scientists and engineers in many diverse fields. When a problem has different, competing solutions, optimization is likely a better approach. This paper could be a case study for that idea. Perhaps it will spark other innovations.

We still hope to have some positive impact on print standards. It is our belief that print standards should benefit the industry – printers, their clients, and suppliers of equipment and materials. A standard should resolve the ambiguities and incompatibilities that waste our time and money. Standards should not be used to enrich trade organizations and consultants. There are plenty of examples where standards were developed through cooperation and sound technical thinking, without lasting discord.

This was the spirit of our print standards work from 2004-2009. In a sense, the OPTIMAL method is "the road not taken" in 2006 (Birkett & Spontelli, TC 130 Generic Matching Tool, 2006). We hope you find it useful.

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