An Evaluation of Color Aberration caused by the Optical Dot Gain

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ABSTRACT

One of the issues relating to the four-color process prints is the optical dot gain caused by light penetration into the paper substrate. Neugebauer's Equation is well known for describing the color reproduction by halftone dots, but it is a little complicated.

By assuming that the optical densities may be added to superimposed solid portions, Neugebauer's Equation can be resolved into factors which correspond to primary, secondary and tertiary colors of the respective inks and can be transformed into Pollak's Equation shown below.

$$R^4_{pollak} = Rp\lambda(1 - c + cRsc\lambda)(1 - m + mRsm\lambda)(1 - y + yRsy\lambda)(1 - k + kRsk\lambda)$$

R^4_{pollak} : Reflectance, c,m y,k : dot values,

$Rp\lambda$: Reflection value of white portion,

$R\ si\lambda$: Reflectance value of solid portion for i ink

When discussing the color reproduction, these factors affected by the optical dot gain should be examined. In this paper, definition of the optical dot gain is the difference between the measured reflectance values and the values calculated by Murray-Davies's Equation, and show that the optical dot gain can be estimated by the quadratic equation of the dot values.

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When considering the real color reproduction of four-color prints, the effects of the optical dot gain should be considered. Then using Pollak's Equation as a tool, it should be modified by the introduction of the correction terms relating to the optical dot gain.

The correction terms called Nonaka's Model are expressed by the quadratic equation of dot area **a** and coefficient $\mathbf{h}_\lambda$. By introducing the correction term based on Nonaka's Model into Pollak's Equation, the color reproduction of the superimposed four-color halftone dots has become possible to evaluate satisfactorily, in the form of spectral reflectance. G.L. Rogers studied optical dot gain in a halftone theoretically. In Paragraph 3.4, the comparison between Nonaka's Model and G.L.Rogers' Model regarding prediction methods of optical dot gain was performed.

The color aberration predicted from the optical dot gain can be evaluated by the color difference. The results are shown in Chapter 3. The average color difference ΔE between the measured values and the predicted values calculated by Pollak's Equation in which the correction terms are excluded are approximately 17.7.

ΔE =5.0 for **Nonaka's Model 1** in which the reflectance is estimated by using 164 coefficients values of $\mathbf{h}_\lambda$ measured at 10 nm intervals.

ΔE =8.4 for **Nonaka's Model 2** in which the correction terms for optical dot gain is estimated from the absorption curve of the respective inks.

These results show the possibility that the introduction of correction terms expressed by **Nonaka's Model** can estimate the color aberration caused by the optical dot gain in a halftone print.

GLOSSARY OF SYMBOLS

a Dot value: it changes between 0 to 1.0.

C Cyan ink, and **c** means the dot value **a** of Cyan ink printed on the paper.

M Magenta ink, and **m** means the dot value **a** of Magenta ink printed on the paper.

Y Yellow ink, and **y** means the dot value **a** of Yellow ink printed on the paper.

K Black ink, and **k** means the dot value **a** of Black ink printed on the paper

i Symbol of ink: it means C, M, Y and K.

λ Symbol of wavelength used for measurement.

$G_{i\lambda}$ (**a**) Optical dot gain function named Nonaka's Model including dot value **a**, variety of inks **i** and measurement wavelength λ as variables.
The curve of $G_{i\lambda}$ (**a**) is shown by the quadratic equation.
It represents the following four symbols : $G_{c\lambda}$ (**a**), $G_{m\lambda}$ (**a**), $G_{y\lambda}$ (**a**) and $G_{k\lambda}$ (**a**).

$\mathbf{h}_{i\lambda}$ Coefficient of quadratic equation which estimates the optical dot gain of halftone dots of **i** ink. Wavelength used for measurement is λ.
Measurement of $\mathbf{h}_{i\lambda}$ was performed from 380 nm to 780 nm at 10nm intervals for respective inks.
It represents the following four symbols : $\mathbf{h}_{c\lambda}$, $\mathbf{h}_{m\lambda}$, $\mathbf{h}_{y\lambda}$ and $\mathbf{h}_{k\lambda}$

$I_i(\lambda)$ Absorption values of **i** ink for wavelength λ.
It represents the following four symbols : $I_c(\lambda)$, $I_m(\lambda)$, $I_y(\lambda)$ and $I_k(\lambda)$.

$\mathbf{h}_{if}$ Coefficient of quadratic equation which estimates the optical dot gain of the halftone dots. This value is approximated by the following equation including $I_i(\lambda)$.

$$\mathbf{h}_{if} = \alpha_i \left[\beta_i + I_i(\lambda)\right]$$

where α_i and β_i are constant values related to the **i** ink.

R^4_{Pollak} Spectral reflectance value of 4-color prints expressed by Pollak's Equation.

$R_{p\lambda}$ Spectral reflectance value of the bare paper

$R_{si\lambda}$ Spectral reflectance value of the solid region where **i** ink is used.
It represents the following four symbols: $R_{sc\lambda}$, $R_{sm\lambda}$, $R_{sy\lambda}$ and $R_{sk\lambda}$.

$R_{average}(a, \lambda)$ Spectral reflectance averaged over a region as a function of the dot value, Equation (12)

$R^4_{spectral}$ Spectral reflectance value of 4-color prints expressed by the modified Pollak's Equation, in which reflectance terms include correction terms derived from spectral optical dot gain characteristics.

R^4_{ink} Spectral reflectance value of 4-color prints expressed by the modified Pollak's Equation in which reflection terms include correction terms derived from the absorption characteristics of the respective inks.

R^4_{rogers} Spectral reflectance value of 4-color prints derived from Rogers's Equation in which the coefficients of correction terms are expressed by $\{1-(R_{si\lambda})^{1/2}\}^2$.

T_0 Transmittance of inked regions

$R_{i\text{-}inked}$ Average reflectance of inked regions where i ink is used.

$R_{i\text{-}noink}$ Average reflectance of non-inked regions where i ink is used.

$R_{p\lambda}$ Spectral reflectance of the bare paper

X, Y, Z Tri-stimulus values of color patch

$S(\lambda)$ Spectral power distributions of the illuminant

1. INTRODUCTION

One factor affecting color reproduction is the optical dot gain caused by light penetration into the paper substrate. For better color reproduction, it is crucial to estimate accurately the effects of the optical dot gain and to perform the appropriate corrections. The effects of the optical dot gain are usually examined by using the Yule-Nielsen's Equation [3][4]

This paper will report on the studies carried out on the characteristics of optical dot gain as a function of spectral reflectance, the introduction of the correction terms for optical dot gain into Pollak's Equation and, on the color difference between the measured values and the predicted values calculated by Pollak's Equation and modified Pollak's Equation.

Optical dot gain is defined as the difference between the reflectance values calculated by using the Murray-Davies' Equation and the measured reflectance values. The optical dot gain is obtained by measuring the reflectance of the printed color vignette specified by the ISO 12641 Input Target standard.

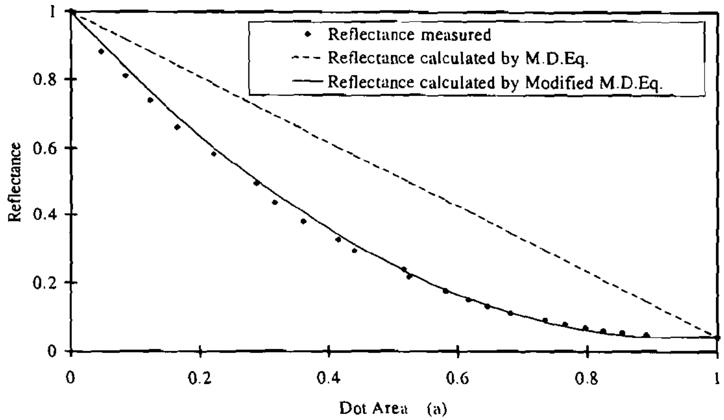


Fig.1 Comparison between the measured reflectance and the reflectance calculated by Murray-Davies's Equation. The difference of both reflectance curves is defined the optical dot gain.
The shape of the curve is a function of the ink and measurement wavelength
Ink: Magenta, Wave length: 550 nm

The relationship between both the measured value of the fractional dot area at various parts of the vignette and the measured optical densities through R, G and B filters at the same locations were investigated in detail. The result showed was the dot gain could be expressed with the quadratic equation of dot values shown by Equation (1). [1]

$$G_{i\lambda}(\mathbf{a}) = \mathbf{h}_{i\lambda} \times \mathbf{a}\,(1-\mathbf{a}) \qquad (1)$$

This paper will report the investigated results when using the single spectrum as a measurement illuminant. The optical dot gain measurement was performed on various combinations of inks, dot values and measurement wavelength, and obtained 164 values of $\mathbf{h}_{i\lambda}$. The measurement results showed that the quadratic equation could closely estimate the optical dot gain in the case of single spectrum illuminant. The expression of dot gain shown by Equation (1) has been named as "**Nonaka's Model**" and used as an analytical method rather than Yule-Nielsen's Model in this paper.

When depicting the $\mathbf{h}_{i\lambda}$ values of spectral reflectance of solid portions for Cyan (C), Magenta (M), Yellow (Y) or Black (K) inks, a close correlation between the spectral curves of $\mathbf{h}_{i\lambda}$ and the spectral absorption curves of respective inks can be observed precisely.

This result follows the coefficient of Equation (1) shown below, as a function of the spectral absorption characteristics of the inks $I_i(\lambda)$.

$$\mathbf{h}_{if} = \alpha_i \left[\beta_i + I_i(\lambda) \right] \qquad (2)$$

On the basis of the analytical results for the optical dot gain, two types of correction terms, as described above, to reflectance terms in the Pollak's Equation were introduced, and the effects of optical dot gain for color reproduction were investigated.

2. POLLAK'S EQUATION MODIFIED BY NONAKA'S MODEL

The original Pollak's Equation is derived from the ideal reflectance model excluding the optical dot gain. In order to evaluate the color aberration caused by the optical dot gain, the Pollak's Equation was modified by inserting the correction terms based on the **Nonaka's Model.** Pollak's Equation is expressed by using spectral reflectance as shown in Equation (3).

$$R^4_{pollal} = R_{p\lambda}\,(1-c+cR_{sc\lambda})(1-m+mR_{sm\lambda})(1-y+yR_{sy\lambda})(1-k+kR_{sk\lambda}) \qquad (3)$$

Where $(1-c+cR_{sc\lambda})$ shows the reflectance of the Cyan portion and **c** is the dot value of Cyan ink. Others are reflectance values of Magenta Yellow and Black inks.

(1) Insertion of spectral correction terms

We can obtain the correction terms from the spectral measurement of $\mathbf{h}_{i\lambda}$values for Cyan, Magenta, Yellow and Black inks. For example, the reflectance term of $(1\text{-}c+cR_{sc\lambda})$ for Cyan in Equation (3) is replaced by the following equation:

$$\{1\text{-}c+c\,R_{sc\lambda}\text{-}\,G_{c\,\lambda}(\mathbf{a})\}=\{1\text{-}c+c\,R_{sc\lambda}\text{-}\,\mathbf{h}_{c\lambda}\,c\,(1\text{-}\,c)\}$$

It is necessary, here to examine how to deal with the case of the Black ink. The value of $\mathbf{h}_{k\lambda}$ is almost constant over the visible range. When the Black ink area is superimposed on the Cyan ink area, the area of Cyan ink can be considered as the statistic addition of Cyan and Black area and its value may be expressed by $1\text{-}\,(1\text{-}c)(1\text{-}k)$.

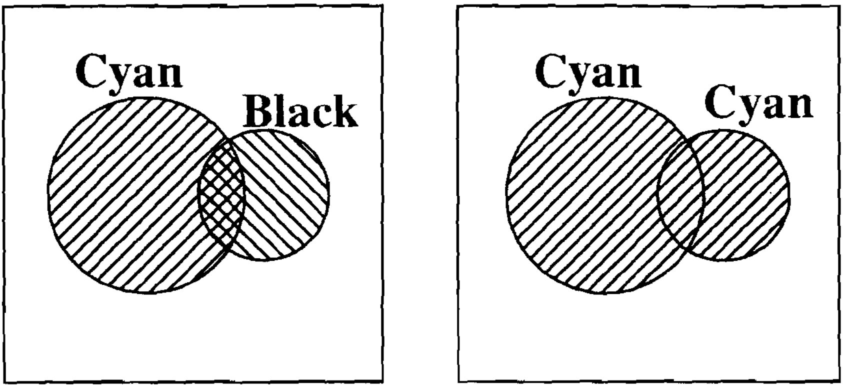


Fig.2 (a) The superimposed Cyan and Black halftone dots.
(b) The case of (a) can be considered as an addition of the Cyan area.

The corrected reflectance $R^4_{spectral}$ as the function of the wavelength can be shown by the modified Pollak's Equation (4).

$$R^4_{spectral} = R_{p\lambda}\{1\text{-}c+cR_{sc\lambda}\text{-}\mathbf{h}_{c\lambda}\,c_l\,(1\text{-}\,c_l)\}\{1\text{-}m+m\,R_{sm\lambda}\text{-}\mathbf{h}_{m\lambda}\,m_l\,(1\text{-}\,m_l)\}$$
$$\{1\text{-}y+y\,R_{sy\lambda}\text{-}\mathbf{h}_{y\lambda}\,y_l\,(1\text{-}\,y_l)\}\,\{1\text{-}k+k\,R_{sk\lambda}\} \qquad (4)$$

where $c_l= 1\text{-}(1\text{-}c)(1\text{-}k)$, $m_l= 1\text{-}(1\text{-}m)(1\text{-}k)$ and $y_l=1\text{-}(1\text{-}y)(1\text{-}k)$.

(2) Insertion of the spectral absorption characteristic of the inks as the correction terms

The correction terms of the second model are given by the spectral absorption characteristics of the inks. The correction terms expressed by Equation (2) are introduced in R^4_{ink} as follows:

$$R^4_{ink} = R_{p\lambda}\{1-c+c\,R_{sc\lambda}-\mathbf{h}_{cf}\,c_l\,(1-c_l)\}\{1-m+mR_{sm\lambda}-\mathbf{h}_{mf}\,m_l\,(1-m_l)\}$$

$$\{1-y+y\,R_{sy\lambda}-\mathbf{h}_{yf}\,y_l\,(1-y_l)\}\,\{1-k+kR_{sk\lambda}\} \quad (5)$$

where $\{1-c+c\,R_{sc\lambda}-\mathbf{h}_{cf}\,c_l\,(1-c_l)\}$is the reflectance of Cyan portion including the correction term.

A comparison was performed between three types of reflectance expressions – R^4_{pollak} (Pollak's Equation), $R^4_{spectral}$ and R^4_{ink}, by using the color difference ΔE on the CIELAB color space between the measured values and the predicted values when considering optical dot gain.

The average error ΔE between the measured values and the predicted values are 17.7 for approximation calculated by Pollak's Equation. ΔE = 5.0 for the first reflectance model of $R^4_{spectral}$ using equation (4) and 164 values of $\mathbf{h}_{i\lambda}$, and ΔE =8.4 for the second reflectance model of R^4_{ink} using the absorption characteristics of the respective inks.

These results show that the introduction of the correction terms to the original Pollak's Equation make it possible to estimate the color aberration by optical dot gain in the case of halftone color dots printed on the paper.

3. EXPERIMENTS

3.1 Preparation of Printing Sample

To evaluate the accuracy of Nonaka's Model and to evaluate the effects of the correction terms inserted in Pollak's Equation for color reproduction, various single color and four-color process print samples were made.

The conditions were as follows:

Image: ISO 12641 Input Target and ISO 12640 SCID
Screen Ruling: 175 line/inch, Square dot
Screen Angle : Cyan: 15°, Magenta: 45°, Yellow: 90° and Black: 75°
Paper: Type-1 (Gloss coated, wood-free) "Tokubishi-Art"(Mitsubishi Paper Mills Co. Ltd.), 135g/m^2
Printer: SG-747 Proof printer (DS)
Pre-sensitized Plates: "FPP-B" (Fuji Photo Film Co. Ltd.)
Printing Ink : "TK For 4 CIL" (Tokyo Ink MFG. Co. Ltd.)
Printing Sequence: K→C→M→Y
Solid Density: Cyan: 1.55, Magenta: 1.55, Yellow: 1.35, Black: 1.90

3.2 Metrology

The spectral reflectance of the proof prints was measured with a C2000 Color Analyzer (Hitachi Co. Ltd.). This Analyzer possesses a range of 380nm ~ 780nm, 10nm interval, Illuminant C, 0°/45° geometry and 2 degree observer. By using this device the mean reflectance values of the halftone prints were measured.

The inked dot values were measured by using a CCD camera with an incorporated microscope. The halftone images of the inked dots were magnified to x100 with the microscope and fed to the CCD camera. The frame position was then manually set to the proper location viewing on the display.

The color dots were converted to gray images and then converted to the binary images using a threshold that was set by studying the brightness value distribution. When taking the statistics of brightness distribution, the histogram of gray levels have two peaks for Cyan, Magenta and Black prints and the minimum value located between both peak values of the brightness histogram is seen as the threshold value.

Because the contrast of the Yellow print was low, the distribution of the brightness values has only a single peak, color separation was used to convert the binary image. By counting the number of pixels included in the inked area, the fractional inked area was derived.

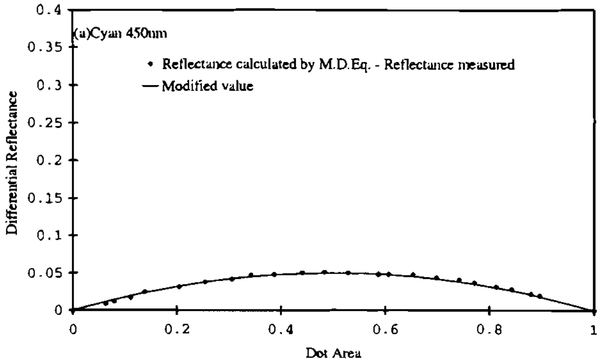
Because the halftone dot images and binary images were capable of being display side by side simultaneously on the monitor, it was ensured that the shape of both images corresponded accurately.

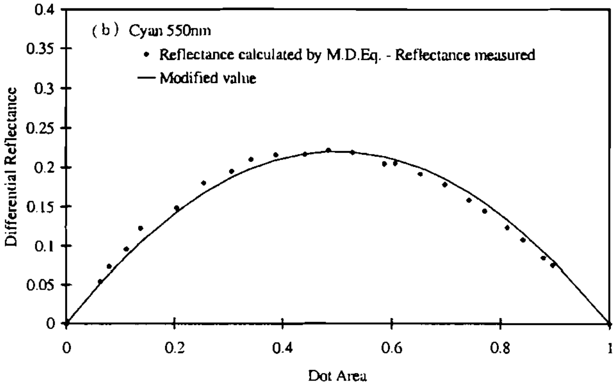
4. RESULTS

4.1 Approximation of the Optical Dot Gain using Nonaka's Model

The optical dot gain is defined by the differential reflectance (D. R.) between the values calculated by Murray-Davies's Equation and the measured reflectance values. The measured optical reflectance value is usually smaller than the value calculated by the Murray-Davies's Equation and the differential reflectance can be expressed by the following equation.

$$\text{D.R.} = (1 - \mathbf{a} + \mathbf{a}\, R_s) - \text{Measured Reflectance} \qquad (6)$$


(a)Cyan 450nm
Reflectance calculated by M.D.Eq. - Reflectance measured
Modified value
Differential Reflectance
Dot Area
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.2
0.4
0.6
0.8
1


(b) Cyan 550nm
Reflectance calculated by M.D.Eq. - Reflectance measured
Modified value
Differential Reflectance
Dot Area
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.2
0.4
0.6
0.8
1

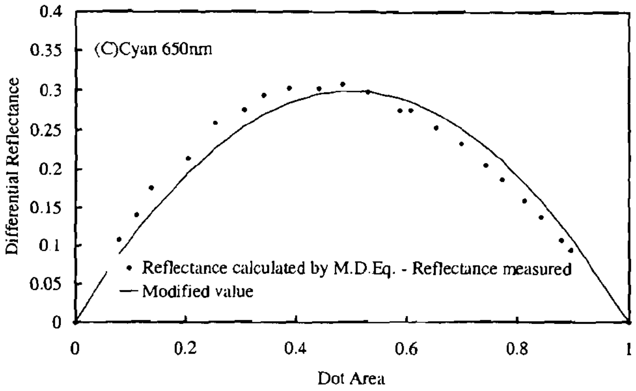


Fig. 3: The relationship between the measured optical dot gain and the estimated curve by the quadratic equation.
(a) Cyan, λ = 450 nm, (b) Cyan, λ= 550 nm, (c) Cyan, λ= 650 nm

The authors have reported already that the D. R. could be estimated by Equation (7) as follows:

$$\text{D. R.} = G_{i\lambda}(\mathbf{a}) = \mathbf{h}_{i\lambda} \times \mathbf{a}\,(1-\mathbf{a}) \qquad (7)$$

Fig.3 shows the relationship between the measured optical dot gain and the estimated curve by the quadratic Equation (7). Wavelengths are 450nm, 550nm and 650nm and the ink used is Cyan.

Fig.4 shows the relationship between $h_{i\lambda}$ values and the measurement wavelengths for various inks. The shape of $h_{i\lambda}$curves are similar to the spectral absorption of the inks

The measurement of $h_{i\lambda}$values were performed from 380nm to 780nm at 10nm intervals. From Fig.4, the following results were extrapolated:

a. $\mathbf{h}_{k\lambda}$ values of K in the visible range have no ripples and values are between 1.0 and 1.2.

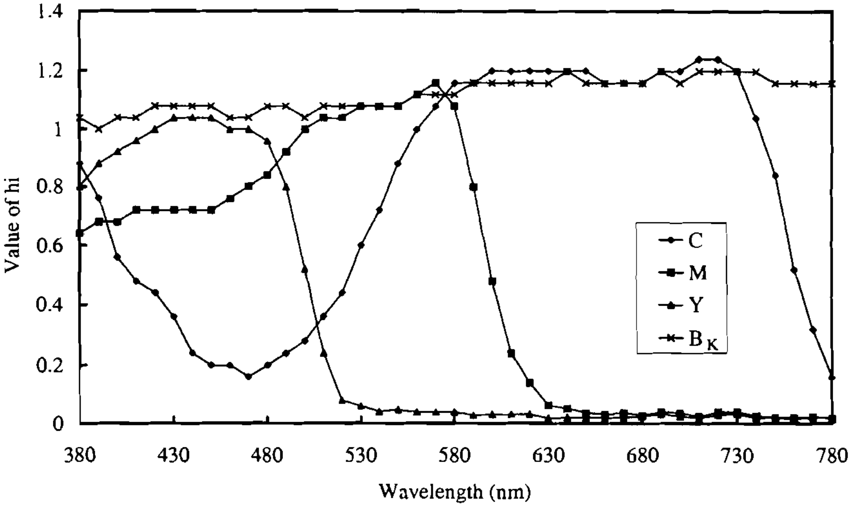


Fig. 4. The relationship between $\mathbf{h}_{i\lambda}$ values and measurement wavelength

b. $\mathbf{h}_{i\lambda}$ values of each ink in the absorption wavelength range are almost equal to that of K.

c. The shape of $\mathbf{h}_{i\lambda}$ for each ink has a strong correlation with the spectral reflectance of the ink printed on the paper.

This shows that the optical dot gain is caused by the penetration of light into inner substrate and the absorption of light at the inner side of the inked area.

4.2 Comparison between $\mathbf{h}_{i\lambda}$ and $\mathbf{h}_{if}$

From the measured values of coefficients $\mathbf{h}_{i\lambda}$ as a function of the wavelength, it may be said that there is some relationship between these values and the spectral absorption characteristics of inks. The coefficient in Equation (8) was estimated. Values of α_i and β_i for the various inks are shown in Table 1.

$$\mathbf{h}_{i\lambda} = \alpha_i\,[\beta_i + I_i(\lambda)] \qquad (8)$$

Table 1 Constant values

Ink (**i**)	coefficients	α_i	β_i
C	$\mathbf{h}_{cf}$	1.65	0.76
M	$\mathbf{h}_{mf}$	1.44	0.78
Y	$\mathbf{h}_{yf}$	1.37	0.81
K	$\mathbf{h}_{kf}$	1.30	0.89

Fig. 5 shows the comparison of $\mathbf{h}_{i\lambda}$ and $\mathbf{h}_{if}$ for the various inks

4.3 Prediction of reproduced color using Nonaka's Model

To investigate the effects of Nonaka's Model as an approximation method for the optical dot gain, the reproduced colorimetric values using color differences between the measured values and the predicted values,derived from Pollak's Equation and the two types of the modified Pollak's Equations were evaluated.

(1) Pollak's Equation

$$R^4_{pollak} = R_{p\lambda}\,(1-c+cR_{sc\lambda})(1-m+mR_{sm\lambda})(1-y+yR_{sy\lambda})(1-k+kR_{sk\lambda}) \qquad (3)$$

(2) Modified Pollak's Equation using the spectral reflectance

(Nonaka's Model 1)

$$R^4_{spectral} = R_{p\lambda}\{1-c+cR_{sc\lambda}-\mathbf{h}_{c\lambda}\,c_l\,(1-c_l)\}\{1-m+m\,R_{sm\lambda}-\mathbf{h}_{m\lambda}\,m_l\,(1-m_l)\}$$

$$\{1-y+y\,R_{sy\lambda}-\mathbf{h}_{y\lambda}\,y_l\,(1-y_l)\}\{1-k+k\,R_{sk\lambda}\} \qquad (4)$$

(3) Modified Pollak's Equation using the spectral reflectance curves of the inks

(Nonaka's Model 2)

$$R^4_{ink} = R_{p\lambda}\{1-c+c\,R_{sc\lambda}-\mathbf{h}_{cf}\,c_l\,(1-c_l)\}\{1-m+mR_{sm\lambda}-\mathbf{h}_{mf}\,m_l\,(1-m_l)\}$$

$$\{1-y+y\,R_{sy\lambda}-\mathbf{h}_{yf}\,y_l\,(1-y_l)\}\{1-k+kR_{sk\lambda}\} \qquad (5)$$

where $c_l = 1-(1-c)(1-k)$, $m_l = 1-(1-m)(1-k)$, $y_l = 1-(1-y)(1-k)$

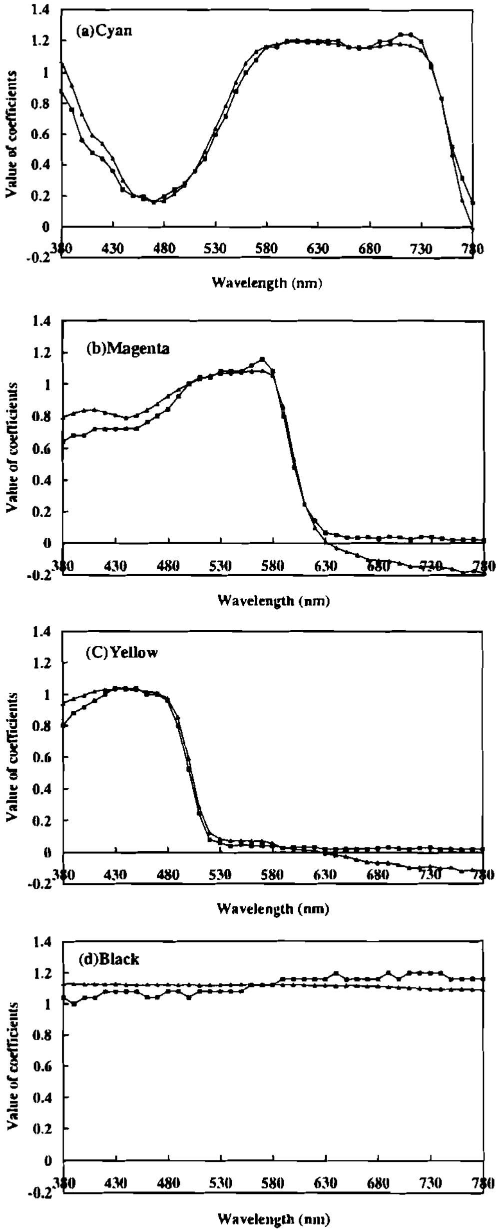


Fig.5 Comparison of $\mathbf{h}_{i\lambda}$ and $\mathbf{h}_{if}$ values of the various inks

(a) Cyan, (b) Magenta, (c) Yellow, (d) Black ■: $\mathbf{h}_{i\lambda}$ ▲: $\mathbf{h}_{if}$

Using Equations (3),(4) and (5) the tri-stimulus values of X, Y, Z can be derived, and then L* a* b* values can be calculated from the respective reflectance values of $R^4(\lambda, a)$.

$$X=K\sum_{\lambda=380}^{780} S(\lambda)\, x(\lambda)\ [\text{Spectral reflectance } (R^4_{pollak}, R^4_{spectral} \text{ or } R^4_{ink})] \qquad (9\text{-}1)$$

$$Y=K\sum_{\lambda=380}^{780} S(\lambda)\, y(\lambda)[\text{Spectral reflectance } (R^4_{pollak}, R^4_{spectral} \text{ or } R^4_{ink})] \qquad (9\text{-}2)$$

$$Z=K\sum_{\lambda=380}^{780} S(\lambda)\, z(\lambda)\ [\text{Spectral reflectance } (R^4_{pollak}, R^4_{spectral} \text{ or } R^4_{ink})] \qquad (9\text{-}3)$$

$$K= 100/\sum_{\lambda=380}^{780} S(\lambda)\, y(\lambda) \qquad (9\text{-}4)$$

$$\Delta E=[(L_i - L_{ip})^2+(a^*_i - a^*_{ip})^2+(b^*_i - b^*_{ip})^2]^{1/2} \qquad (9\text{-}5)$$

where (L_i, a^*_i, b^*_i): measured CIELAB values of halftone dot for i ink

$(L_{ip}, a^*_{ip}, b^*_{ip})$: predicted CIELAB values derived from the tri-stimulus values X, Y and Z calculated by (9-1), (9-2) and (9-3).

It is then possible to compare the calculated colorimetric values and the measured values and derived ΔE values from both values. The results of ISO 12641 patches are shown in Fig.6.

4.4 Comparison between Nonaka's Model and Rogers' Model [5]

G.L.Rogers assumed that a stream of photons travelling in the substrate was incidental on the paper at the origin. The scattering of these injected photons is the source of the dot gain phenomena.

R.G. Rogers started his analysis with the steady-state transport equation for the photon distribution and obtained the following Equation (10) and Equation (11).

$$R_{i\text{-}inked}=R_{p\lambda}T_0[1-(1-T_0)a^{1-s}] \qquad (10)$$

$$R_{i\text{-}noink}=R_{p\lambda}[1-(1-T_0)\, a\, (1-a)^{-1}\, (1-a^{1-s})] \qquad (11)$$

From Equation (10) and Equation (11) we can obtain Equation (12) as the expression of spectral reflectance averaged over a region $R_{average}(a,\lambda)$.

$$R_{average}(a, \lambda)=R_{p\lambda} [(1-a)+aT_0^2-(1-T_0)^2a (1-a^{1-s})] \qquad (12)$$

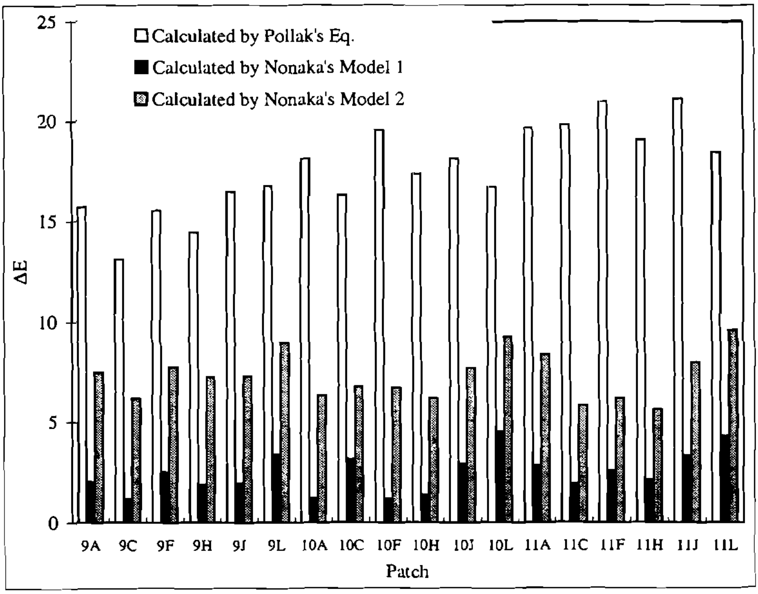


Fig.6- Color difference ΔE between the predicted values and the measured values of ISO 12641 Input Target color patches.
Left : Calculated from Pollak's Equation
Center: Calculated by using modified Pollak's Equation $R^4_{spectral}$
Right: Calculated by using modified Pollak's Equation R^4_{ink}

G.L.Rogers explains that if there is no scattering then s = 1 and for complete scattering then s = 0 and maximum optical dot gain occurs.
If the reflectance of the inked regions $R_{si\lambda}$ is equal to T_0^2 and the spectral illuminant is used, then Equation (12) can be re-written into Equation (13).
The case of s = 1 corresponds to the Murray-Davies's Equation.

$$R_{average}(a,\lambda)=R_{p\lambda}[1-a+aR_{si\lambda}-\{1-(R_{si\lambda})^{1/2}\}^2a (1-a^{1-s})] \qquad (13)$$

We can observe that if $\{1-(R_{si\lambda})^{1/2}\}^2$ is expressed by $\mathbf{h}_{i\lambda}$ and s is approximated by 0 then the term expressing the optical dot gain in Equation (13) is equal to Equation (1) or (7). Also, the term of optical dot gain can be calculated from the reflectance of the inked regions and its characteristics are common to the **Nonaka's Model 2.** The values of $\{1-(R_{si\lambda})^{1/2}\}^2$ in Equation (13) are shown in Fig.7 and can compare with the coefficients $\mathbf{h}_{i\lambda}$ and $\mathbf{h}_{if}$ shown in Fig. 5. It may be observed that the shape of these curves in Fig.5 and that of Fig.7 have close relationships.

The averaged reflectance value of four color prints for Rogers' Model R^4_{rogers} which corresponds to Equation (5) can be given by Equation (14).

$$R^4_{rogers}= R_{p\lambda}\{1-c+c\,R_{sc\lambda}-\{1-(R_{sc\lambda})^{1/2}\}^2 c_l\,(1-c_l)\}\{1-m+mR_{sm\lambda}-\{1-(R_{sm\lambda})^{1/2}\}^2 m_l\,(1-m_l)\}\{1-y+y\,R_{sy\lambda}-\{1-(R_{sy\lambda})^{1/2}\}^2 y_l\,(1-y_l)\}\,\{1-k+kR_{sk\lambda}\} \qquad (14)$$

where $c_l = 1-(1-c)\,(1-b_k)$, $m_l = 1-(1-m)\,(1-b_k)$, $y_l = 1-(1-y)\,(1-b_k)$

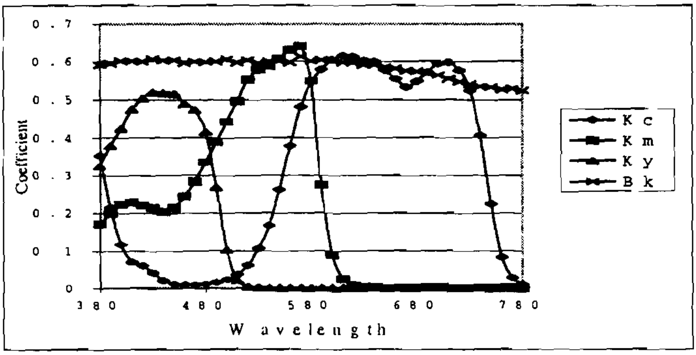


Fig.7 Rogers' coefficients $\{1-(R_{si\lambda})^{1/2}\}^2$ for the various inks

5.0 CONCLUSION

(1) In this paper, the results of the optical dot gain measurement performed by spectral analysis are presented. This shows the color aberration caused by the optical dot gain can be predicted by using the modified Pollak's Equation with the correction terms in it.

(2) The studied results show the correction terms for the optical dot gain of the monochrome halftone dots which can be expressed by the quadratic equation of dot value **a** and coefficient **h**.

$$\text{Correction term} = \mathbf{h} \times \mathbf{a}\ (1\text{-}\mathbf{a})$$

The spectral analysis for the monochrome halftones show the coefficient **h** has correlation with the measurement wavelength λ and the respective spectral reflectance characteristics of Cyan, Magenta, Yellow and Black inks.

(3) The results make it possible to expand the four color prints and to predict the tri-stimulus values of the colored halftone dots by using the modified Pollak's Equation. When **h** is expressed as the function of the ink and the measurement spectrum, **h** is expressed by $\mathbf{h}_{i\lambda}$ and is named **Nonaka's Model 1.**

(4) The **h** value is closely related to the spectral reflectance values of an ink and can be expressed as a function of the spectral reflectance characteristics of the respective inks. When the **h** value is derived from the spectral reflectance of an ink and is shown as $\mathbf{h}_{if}$ then it is **Nonaka's Model 2** .

(5) The evaluation of the effect of introducing the correction terms was achieved by the color difference ΔE between the measured values and the predicted values. The color difference ΔE derived from the Pollak's Equation and the two modified Pollak's Equations are as follows:

(1) Pollak's Equation
$33.4 > \Delta E > 6.1$ average value: 17.7
(2) Modified Pollak's Equation using spectral reflectance (Nonaka's Model 1)
$13.3 > \Delta E > 1.0$ average value: 4.9
(3) Modified Pollak's equation in which the correction term is examined by the absorption characteristics of the ink (Nonaka's Model 2)
$16.7 > \Delta E > 3.6$ average value: 8.4

In conclusion, we can show that the effect of the optical dot gain can be evaluated with Nonaka's Model. The calculated colorimetric values derived from the modified Pollak's Equation can approach the measured values adequately.

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