Principle of Color Reproduction in Printing by the Method of Proportional Tone Compression: The Process System by Numerical Calculation

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ABSTRACT: We already mentioned that the principle of color reproduction by superimposing halftone dots in the printing process was essentially a subtractive mixture, and that the correction of optical dot gain was needed. In this paper, we present the method of outputting the C, M, Y and Bk dot areas converted from the image signal harvested from a continuous tone color original by a scanner using proportional tone compression (Numakura-Yamatoya equation).

The Numakura-Yamatoya equation is based on the Yule-Nielsen equation in exponential form $\left(a = \frac{1-10^{-D_a/n}}{1-10^{-D_s/n}}\right)$, but the term D_a in the equation is replaced to the term K· D_s in the Numakura-Yamatoya equation. The coefficient K works as a tone compressive function in the case of using ideal printing inks that do not have extra light absorbance.

We fixed on the dot area of black by gray component replacement and we determined other C, M, Y dot areas so as to equalize the quantity of light through a color filter (R, G, B). These equations are constructed with the Pollak equation containing our corrective terms of optical dot gain. The method of determining the dot areas is to solve simultaneous quadratic equations with three unknowns by using successive approximation. This conversion contains color balance, dot gain (optical and mechanical), GCR and masking (color correction).

So far these processes have been dealt with empirically. While LUT is one of the empirical methods, the proposed treatment is to construct a numerical model of color images made up of superimposed dot areas, and fractional dot areas required are calculated numerically.

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1. Introduction

The color image reproduction method by superimposing Cyan, Magenta, Yellow, and Black halftone dots on white paper has been made up for over one hundred years. A lot of the knowledge has been stored for satisfying the severe demand on print image quality, though most of the knowledge has been made up empirically. The color image reproduction has been done by LUT which presently contains input-output data of a special case since the advent of the computer.

Basically, we considered color management which uses a color system or a color solid system as the accumulation of empirical knowledge. Accordingly, the principle of color image reproduction by superimposing dot areas on paper has not been explained clearly.

So, we found the principle by using Pollak's equation[1] that contained Demichel's equation[2]. Demichel's equation expresses the fractional dot areas which are constructed by superimposing C, M, Y, Bk halftone dots, and the fraction is derived from the multiplication law of probability.

Pollak's equation is constructed by four terms of C, M, Y, Bk and by this structure, it is expressed that the principle of color image reproduction with superimposing halftone dots is a subtractive mixture. These terms of the equation are the same forms as in Murray-Davies's equation[3] on each color but the equation does not contain the correction of optical dot gain.

The principle of color image reproduction constructed by dot areas in printing is a subtractive mixture similar to that of color photography, but the difference is that the correction of optical dot gain is needed in printing[4, 5]. Accordingly we can express color image reproduction by endowing the correction term of optical dot gain to Pollak's equation.

With this expression, in this paper, we present the method of outputting the C, M, Y and Bk dot areas converted from the image signals harvested from a continuous tone color original by the scanner using proportional tone compression (Numakura-Yamatoya equation). This conversion contains color balance, dot gain (optical and mechanical), GCR, and masking (color correction). We present the effect of simulating this process system with Excel table computing software using the measured parameters from the preproof (AGFA Pressmatch Dry).

2. Theory

Murray-Davies's equation and Yule-Nielsen's equation[6] used in this theory have been derived under the assumption that the optical reflectance of the paper as the substrate is 1. If these equations containing the optical reflectance of paper had been derived, they would be complex forms. So the optical density used in this theory is 0 or the optical reflectance of the substrate used in this theory is 1.

For comparing the calculated value with the measured value, the optical reflectance of the paper through R, G, B filters is produced to the final calculated value.

2.1 N-Y Equation

We converted R, G, B image signals harvested from a scanner to C, M, Y dot areas ($c1$, $m1$, $y1$) by applying the N-Y equation.

N-Y equation is

$$A_n = A_H + \frac{1 - 10^{-Y \cdot X_n}}{1 - 10^{-Y}}(A_S - A_H) \qquad (1)$$

A_n: fraction of dot area on any point (N) in original image

A_H: fraction of dot area on the brightest point in original image

A_S: fraction of dot area on the darkest point in original image

Y: coefficient of adjusting contrast on the area of halftone in original image

X_n: normal optical density on any point (N) in original image

Normal optical density X_n is the same value as the ratio of the optical density of the brightest point subtracted from that of any point in the original image to the optical density of the brightest point subtracted from that of the darkest point in the original image. The ratio is fixed as K.

$$K = \frac{theopticaldensityonanypointinoriginalimage - theopticaldensityonthemostbrightpointinoriginalimage}{theopticaldensityonthemostdarkpointinoriginalimage - theopticaldensityonthemostbrightpointinoriginalim} \qquad (2)$$

Because the value of the denominator in the above equation is constant to every original image, the above equation works to convert the tone of each point in the original image to relative optical density.

The value Y has been considered as D_S/n (D_S: the solid optical density on print n: corrective coefficient of optical dot gain in Yule-Nielsen's equation),

$$\frac{1 - 10^{-Y \cdot X_n}}{1 - 10^{-Y}} = \frac{1 - 10^{-(D_S \cdot K)/n}}{1 - 10^{-D_S/n}} \qquad (3)$$

We fixed to call the K as coefficient of proportional tone compression.

2.2 Proportional tone compression in original image to printing

Exponential form of Yule-Nielsen's equation is

$$a = \frac{1 - 10^{-D_a/n}}{1 - 10^{-D_S/n}} \tag{4}$$

a: fraction of dot area

D_a: the optical density of fraction of dot area a on print

So, in equation (3), D_a in equation (3) is expressed as $D_S \cdot K$. In equation (1), if it was regarded as $A_H \approx 0$, $A_S - A_H \approx 1$, equation (1) would be same as equation (4). And by the term of $D_S \cdot K$ the tone in the original image is compressed to the tone range in printing. According to the above reason, equation (1) becomes the following equation.

$$A_n = \frac{1 - 10^{-(D_S \cdot K)/n}}{1 - 10^{-D_S/n}} \tag{5}$$

By converting the R, G, B image signal on each image element in the color image original to $c1$, $m1$, $y1$ as C, M, Y dot areas with the above the equation, R, G, B is converted to C, M, Y by considering optical dot gain and proportional tone compression at the same time.

These relations are showed in Fig.1. But at the present stage, we assume use of ideal inks that do not have extra optical absorption.

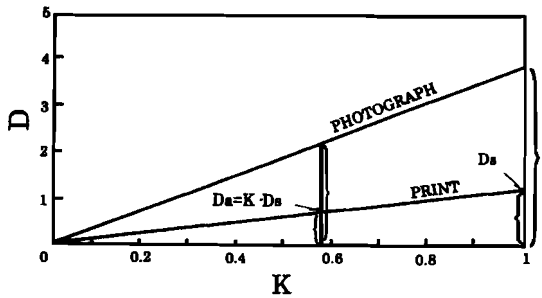

Figure 1. Schematic description of the principle of proportional tone compression.

2.3 Management of gray balance

Before using equations (7), (8), and (9) shown subsequently, it is needed that a set of fractional dot areas $(c1, m1, y1)$ are adjusted to have gray balance.

Gray balance means that the color original image had gray balance, and if K calculated from the optical density of the image signal harvested through R, G, B filters from an achromatic color point on a highlight area and shadow area and the optical density of any point in the original image were the same values, it would be needed that the values of $D_S \cdot K$ that were products of K of each color (C, M, Y) by the solid optical density of each color were the same value in print.

For adjusting the values as same, in the case of standardizing C, K calculated from image signals harvested are fixed as K_{C1}, K_{M1}, K_{Y1}. These are

$$\begin{aligned} K_C &= K_{C1} \\ K_M &= (D_{SC}/D_{SM}) \cdot K_{M1} \\ K_Y &= (D_{SC}/D_{SY}) \cdot K_{Y1} \end{aligned} \qquad (6)$$

D_{SC}, D_{SM}, D_{SY}: solid optical density of each C, M, Y print

These relations are showed in Fig.2.

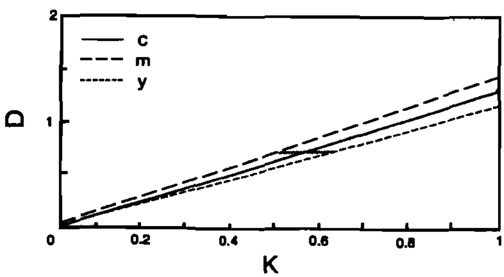

Figure 2. Schematic description of the principle of gray balance treatment.

As it can be seen from Fig.2, in the case of a lower optical density of a color than the optical density of a standardized color, it happens that K of the color having the lower value is higher than 1 for establishing gray balance. This shows that in this area of the original, gray balance cannot be established in print.

Therefore in the case that the value of K is larger than 1, we must give up on establishing gray balance.

2.4 Deriving method of each fraction of dot area (C, M, Y) in the case of using ideal inks not having extra optical absorption

We express $c1$, $m1$, $y1$ by using equation (5), these are:

$$c1 = \frac{1 - 10^{-D_{SC} \cdot K_C / n_C}}{1 - 10^{-D_{SC} / n_C}} \quad (7)$$

$$m1 = \frac{1 - 10^{-D_{SM} \cdot K_M / n_M}}{1 - 10^{-D_{SM} / n_M}} \quad (8)$$

$$y1 = \frac{1 - 10^{-D_{SY} \cdot K_Y / n_Y}}{1 - 10^{-D_{SY} / n_Y}} \quad (9)$$

n_C, n_M, n_Y: each coefficient of optical dot gain in Yule-Nielsen's equation on C, M, Y print

K_C, K_M, K_Y: each coefficient of proportional tone compression on each image element of color original image to C, M, Y print

Parameters except proportional tone compression are entered as the values taken from print with fixed printing paper, printing ink, and printing conditions. These conditions make it possible for connecting the process to press. The values of the parameters vary according to the paper, ink and press machine used in printing.

2.5 Determination of fractional black dot area with GCR (Gray Component Replacement)

The next step is determination of the black curve. We have two methods of determining the black curve. One is called full black that replaces C, M, Y to Bk as much as possible, and the another method is called skeleton black that replaces C, M, Y to Bk from the middle tone part to the shadow part of the original image. The determining method of skeleton black has optionality.

Determination of the quantity of black should be done by replacing an equal density harvestable from $D_{SC} \cdot K_C$, $D_{SM} \cdot K_M$ and $D_{SY} \cdot K_Y$. If the quantity of optical density is D_{bk1}, the fractional dot area calculated by Yule-Nielsen's equation should be

$$bk1 = \frac{1 - 10^{-D_{bk1} / n_{Bk}}}{1 - 10^{-D_{SBk} / n_{Bk}}} \quad (10)$$

Generally, the optical density replaced is D_{bk1}, D_{bk1} should be

$$D_{bk2} = min(D_{SC} \cdot K_C, D_{SM} \cdot K_M, D_{SY} \cdot K_Y)$$
$$D_{bk1} = \alpha \cdot (D_{bk2} - \beta) \qquad (11)$$

Therefore, if you determine α and β suitably, you should get the dot area of full black or skeleton. α works to decide the ratio of replacing optical density, and β works to decide the point of entering black dot area firstly in tone of the original image.

The following limited conditions are needed to determining those values:

$$D_{bk1} \geq 0$$
$$D_{bk1} \leq D_{bk2}$$
$$\alpha \leq \frac{D_{SC}}{D_{SC} - \beta}$$
$$0 \leq \beta \leq 1$$

We show the black curve of the optical density of the above mentioned in Fig.3.

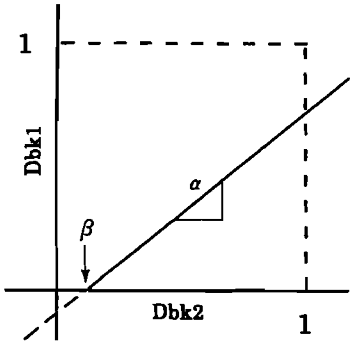

Figure 3. Schematic description of black curve. (α)is a coefficient of black curve. (β) is a tone position where black halftone dot is begun to enter.

2.6 Determination of black curve in the case of ideal inks not having extra optical absorption

In the case of ideal inks not having extra optical absorption, we must convert a set of dot areas ($c1$, $m1$, $y1$) to a set of dot area ($c2$, $m2$, $y2$, $bk1$) by using $bk1$ already fixed, and with considering optical dot gain in superimposed dot areas to paper, and with keeping to the same color. In this case, the correction of optical dot gain that happens by superimposing dot areas is needed.

Pollak's equation used here is as follows for red filter optical reflectance.

$$R = (1 - c + c \cdot R_{SC}) \cdot (1 - m + m \cdot R_{SM}) \cdot (1 - y + y \cdot R_{SY}) \cdot (1 - bk + bk \cdot R_{SBk}) \qquad (12)$$

For green filter optical reflectance or for blue filter optical reflectance, R must be converted to G or B in the above equation.

We define the value of the subtracted measured reflectance from calculated reflectance by equation (12) to each R, G, B color as quantity of optical dot gain.

In the case of C print, Yule-Nielsen's equation expressed by optical reflectance is $(1-c+c \cdot 10^{-D_{SC}/n_C})^{n_C}$. And in the case of the print superimposed with c and bk, reflectance of that print is $\{1 - c + c \cdot R_{SC} - k_C \cdot (c + bk - c \cdot bk) \cdot (1 - c - bk + c \cdot bk)\} (1 - bk + bk \cdot R_{Bk})$ by using Pollak's equation (12) fixed $m = 0$, $y = 0$ in it and introduced our correction term to it[5].

Therefore you must search the correction coefficients n_C, n_M, n_Y, n_{Bk} of Yule-Nielsen's equation and corresponding to our corrected coefficients k_C, k_M, k_Y, k_{Bk} in our quadratic equation before calculating by equation (12). The reason for introducing our correction term to the term of Pollak's equation is as follows: We regard the terms of Pollak's equation as continuous ink films having correspond optical density of C, M, Y, Bk print, and the progressive print has not been superimposed with another color.

Therefore it is thought that if we use the corrected values determined from progressive prints as it is, because of not superimposing another color on it, the quantity of correction should become excessive to that of placing another color on it.

Therefore producing the values of upper colors to the correction value determined from progressive print is needed. If you observe the print superimposed by C, M, Y and Bk dot areas through R, G, B filters, because each C, M, Y dot area is superimposed by Bk dot area, it should be observed that each size of C, M, Y dot area becomes larger.

Therefore we must use the correction value at that size of dot area becoming larger by black halftone dot. In the case of reflectance through a red filter, the correction term $k_C \cdot c \cdot (1 - c)$ that is not superimposed by Bk halftone dot yet must be marked as $k_C \cdot (c + bk - c \cdot bk) \cdot (1 - c - bk + c \cdot bk)$ after superimposing Bk halftone dot. We already mentioned these in the previous paper[5]. We show the scheme of this conversion in Fig.4.

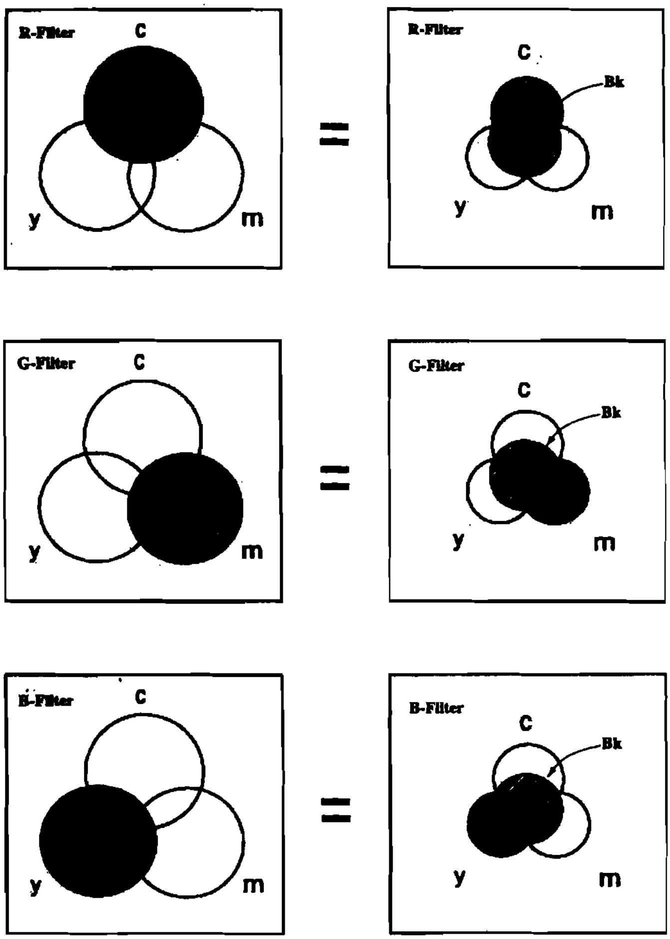

Figure 4. Schematic description of equivalent reflectance between C, M, Y dot and combined C, M, Y dot with fixed black dot in the case of ideal inks.

If we show this matter with red filter reflectance, it is as follows:

$$\begin{aligned} R &= \{1 - c + c \cdot R_{SC} - k_C \cdot (c + bk - c \cdot bk) \cdot (1 - c - bk + c \cdot bk)\} \cdot (1 - m + m \cdot R_{SM}) \\ &\quad \cdot (1 - y + y \cdot R_{SY}) \cdot (1 - bk + bk \cdot R_{SBk}) \\ &= (1 - c + c \cdot R_{SC}) \cdot (1 - m + m \cdot R_{SM}) \cdot (1 - y + y \cdot R_{SY}) \cdot (1 - bk + bk \cdot R_{SBk}) \\ &\quad - k_C \cdot (c + bk - c \cdot bk) \cdot (1 - c - bk + c \cdot bk) \cdot (1 - m + m \cdot R_{SM}) \cdot (1 - y + y \cdot R_{SY}) \\ &\quad \cdot (1 - bk + bk \cdot R_{SBk}) \end{aligned} \tag{13}$$

The addition of a black halftone dot in the case of ideal inks not having extra optical absorption is expressed as the following equations:

$$
\begin{aligned}
(1-c1+c1\cdot 10^{-D_{SC}/n_C})^{n_C} &= \{1-c2+c2\cdot R_{SC}-k_C\cdot(c2+bk1-c2\cdot bk1) \\
&\quad \cdot(1-c2-bk1+c2\cdot bk1)\}\cdot(1-bk1+bk1\cdot R_{SBk}) \qquad (14)\\
(1-m1+m1\cdot 10^{-D_{SM}/n_M})^{n_M} &= \{1-m2+m2\cdot G_{SM}-k_M\cdot(m2+bk1-m2\cdot bk1) \\
&\quad \cdot(1-m2-bk1+m2\cdot bk1)\}\cdot(1-bk1+bk1\cdot G_{SBk}) \qquad (15)\\
(1-y1+y1\cdot 10^{-D_{YC}/n_Y})^{n_Y} &= \{1-y2+y\cdot 2B_{SY}-k_Y\cdot(y2+bk1-y2\cdot bk1) \\
&\quad \cdot(1-y2-bk1+y2\cdot bk1)\}\cdot(1-bk1+bk1\cdot B_{SBk}) \qquad (16)
\end{aligned}
$$

R_{SC}, R_{SB_K} : red filter optical reflectance of C, Bk solid print

G_{SM}, G_{SB_K} : green filter optical reflectance of M, Bk solid print

B_{SY}, B_{SB_K} : blue filter optical reflectance of Y, Bk solid print

Because these equations are quadratic equations on *c*2, *m*2, *y*2, we can solve them by the formula for the relation between root and coefficient. But you must calculate with adopting a minus sign on double sign in the formula.

Each equation (14), (15), (16) becomes a quadratic equation on each *c*2, *m*2, *y*2 by ordering. They are as follows:

$$
\begin{aligned}
&k_C\cdot(1-bk1+bk1\cdot R_{SBk})\cdot(1-bk1)^2\cdot c2^2+(1-bk1+bk1\cdot R_{SBk})\cdot\{k_C\cdot(1-bk1)\\
&\cdot(2bk1-1)-(1-R_{SC})\}\cdot c2+(1-bk1+bk1\cdot R_{SBk})\cdot\{1-k_C\cdot(1-bk1)\cdot bk1\}\\
&-(1-c1+c1\cdot 10^{-D_{SC}/n_C})^{n_C}=0 \qquad (17)\\
&k_M\cdot(1-bk1+bk1\cdot G_{SBk})\cdot(1-bk1)^2\cdot m2^2+(1-bk1+bk1\cdot G_{SBk})\cdot\{k_M\cdot(1-bk1)\\
&\cdot(2bk1-1)-(1-G_{SM})\}\cdot m2+(1-bk1+bk1\cdot G_{SBk})\cdot\{1-k_M\cdot(1-bk1)\cdot bk1\}\\
&-(1-m1+m1\cdot 10^{-D_{SM}/n_M})^{n_M}=0 \qquad (18)\\
&k_Y\cdot(1-bk1+bk1\cdot B_{SBk})\cdot(1-bk1)^2\cdot y2^2+(1-bk1+bk1\cdot B_{SBk})\cdot\{k_Y\cdot(1-bk1)\\
&\cdot(2bk1-1)-(1-B_{SY})\}\cdot y2+(1-bk1+bk1\cdot B_{SBk})\cdot\{1-k_Y\cdot(1-bk1)\cdot bk1\}\\
&-(1-y1+y1\cdot 10^{-D_{SY}/n_Y})^{n_Y}=0 \qquad (19)
\end{aligned}
$$

If we consider quadratic equation (17) as $\epsilon\cdot c2^2+\zeta\cdot c2+\eta=0$, $c2$ should be as follow.

$$
c2=\frac{-\zeta-\sqrt{\zeta^2-4\epsilon\cdot\eta}}{2\epsilon} \qquad (20)
$$

With the same way, we can get *m*2, *y*2.

2.7 Management of masking (color correction)-method of successive approximation

Fractional dot areas $c2$, $m2$, $y2$ derived from equation (17), (18), and (19) are the values in the case of ideal inks not having extra optical absorption.

Therefore you need to apply the management of masking to actual inks. The management of masking becomes possible by using Pollak's equation. We use our correction term by introducing it to the term of Pollak's equation for correcting optical dot gain. We already showed that effect of our correction on optical dot gain was almost the same to that of Yule-Nielsen in the previous paper.

Here, we convert a set of fractional dot areas $c1$, $m1$, $y1$ in the case of ideal inks not having extra optical absorption to a set of fractional dot areas ($c3$, $m3$, $y3$, $bk1$) in the case of actual inks having extra optical absorption on keeping the same color. This conversion becomes possible by using Pollak's equation having the correction term on optical dot gain to halftone print constructed with each halftone dot of C, M, Y, Bk.

We showed the scheme of these relations in Fig.5.

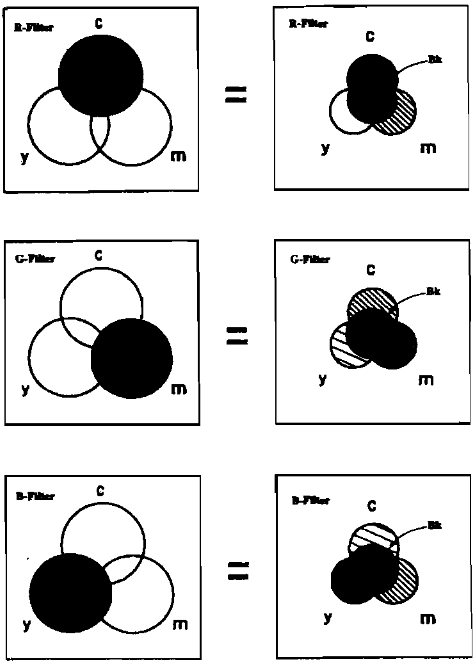

Figure 5. Schematic description of equivalent reflectance between C, M, Y dot and combined C, M, Y dot with fixed black dot in the case of actual inks.

We express our correction terms as k_C, k_M, k_Y corresponding to Yule-Nielsen's n_C, n_M, n_Y, and the fractional dot area managed on masking with consideration of converting to three fractional dot areas of actual inks having extra optical absorption are expressed as $c3$, $m3$, $y3$.

These are expressed as follows:

$$(1 - c1 + c1 \cdot 10^{-D_{SC}/n_C})^{n_C} = \{1 - c3 + c3 \cdot R_{SC} - k_C \cdot (c3 + bk1 - c3 \cdot bk1) \cdot (1 - c3 - bk1 + c3 \cdot bk1)\}$$
$$\cdot (1 - m3 + m3 \cdot R_{SM}) \cdot (1 - y3 + y3 \cdot R_{SY}) \cdot (1 - bk1 + bk1 \cdot R_{SBk}) \quad (21)$$

$$(1 - m1 + m1 \cdot 10^{-D_{SM}/n_M})^{n_M} = (1 - c3 + c3 \cdot G_{SC})$$
$$\cdot \{1 - m3 + m3 \cdot G_{SM} - k_M \cdot (m3 + bk1 - m3 \cdot bk1) \cdot (1 - m3 - bk1 + m3 \cdot bk1)\} \cdot (1 - y3 + y3 \cdot G_{SY})$$
$$\cdot (1 - bk1 + bk1 \cdot G_{SBk}) \quad (22)$$

$$(1 - y1 + y1 \cdot 10^{-D_{SY}/n_Y})^{n_Y} = (1 - c3 + c3 \cdot B_{SC}) \cdot (1 - m3 + m3 \cdot B_{SM})$$
$$\cdot \{1 - y3 + y3 \cdot B_{SY} - k_Y \cdot (y3 + bk1 - y3 \cdot bk1) \cdot (1 - y3 - bk1 + y3 \cdot bk1)\} \cdot (1 - bk1 + bk1 \cdot B_{SBk}) \quad (23)$$

$R_{SC}, R_{SM}, R_{SY}, R_{SB_K}$: each red filter optical reflectance of C, M, Y, Bk solid print
$G_{SC}, G_{SM}, G_{SY}, G_{SB_K}$: each green filter optical reflectance of C, M, Y, Bk solid print
$B_{SC}, B_{SM}, B_{SY}, B_{SB_K}$: each blue filter optical reflectance of C, M, Y, Bk solid print

The equations above are simultaneous quadratic equations with three unknowns, so commonly we cannot solve them.

But generally as shown in Table 1, the optical reflectance of a solid print on each color (C, M, Y) has the same pattern of extra optical absorptions. So we can get good approximated answers ($c3$, $m3$, $y3$) by searching $c3$, $m3$, $y3$ in order.

For getting $c3$ from equation (21), we use $m2$, $y2$ as an alternative to $m3$, $y3$ as approximated values.

The value of (1 – m3 + m3 R_{SM}) in the equation (21) does not vary much because the value of R_{SM} is 0.85 and 0.85 is close to 1. Though $m3$ would vary, the value (1 – m3 + m3 R_{SM} does not vary much. Therefore, if $m3$ is replaced by $m2$, the value of (1 – m2 + m2 R_{SM} is not changed much.

As the same, the value of (1 – y3 +y3 RSY is always 1, because the value R_{SY} is 1. If you enter any number to $y3$, the value of the term is always 1. Therefore, if $y3$ is replaced by $y2$, the value of that term does not changed.

As mentioned above, the value of $c3$ is fixed at a high approximation by replacing $m3$, $y3$ with $m2$, $y2$ in equation (21). Replacement is done on equation (21) and organization is done about $c3$. The result is as follows:

$$P_C = (1 - m2 + m2 \cdot R_{SM}) \cdot (1 - y2 + y2 \cdot R_{SY}) \cdot (1 - bk1 + bk1 \cdot R_{SBk})$$
$$Q_C = (1 - c1 + c1 \cdot 10^{-D_{SC}/n_C})^{n_C}$$
$$k_C \cdot P_C \cdot (1 - bk1)^2 \cdot c3^2 + P_C \cdot \{k_C \cdot (1 - bk1) \cdot (2bk1 - 1) - (1 - R_{SC})\} \cdot c3$$
$$+P_C \cdot \{1 - k_C \cdot (1 - bk1) \cdot bk1\} - Q_C = 0 \quad (24)$$

This equation can be regarded as $\epsilon \cdot c3^2 + \zeta \cdot c3 + \eta = 0$

$$\epsilon = k_C \cdot P_C \cdot (1 - bk1)^2$$
$$\zeta = P_C \cdot \{k_C \cdot (1 - bk1) \cdot (2bk1 - 1) - (1 - R_{SC})\}$$
$$\eta = P_C \cdot \{1 - k_C \cdot (1 - bk1) \cdot bk1\} - Q_C$$
$$c3 = \frac{-\zeta - \sqrt{\zeta^2 - 4\epsilon \cdot \eta}}{2\epsilon} \quad (25)$$

From equation (25), the value of $c3$ is fixed.
For getting $m3$ from equation (22), we use $c3$ fixed already and use $y2$ as an alternative to $y3$ as approximated values.

The value of $(1 - y3 + y3\ G_{SY})$ in equation (22) does not vary much because the value of G_{SY} is 0.95 and 0.95 is close to 1. Though $y3$ would vary, the value $(1 - y3 + y3 \cdot G_{SY}$ does not vary much. Therefore, if $y3$ is replaced by $y2$, the value of $(1 - y2 + y2 \cdot G_{SY})$ is not changed much.

As mentioned above, the value of $m3$ is fixed at a high approximation by replacing $y3$ with $y2$ in equation (22).

Replacement is done on equation (22) and organization is done about $m3$. The result is as follows:

$$P_M = (1 - c3 + c3 \cdot G_{SC}) \cdot (1 - y2 + y2 \cdot G_{SY}) \cdot (1 - bk1 + bk1 \cdot G_{SBk})$$
$$Q_M = (1 - m1 + m1 \cdot 10^{-D_{SM}/n_M})^{n_M}$$

とすると式(22)は,

$$k_M \cdot P_M \cdot (1 - bk1)^2 \cdot m3^2 + P_M \cdot \{k_M \cdot (1 - bk1) \cdot (2bk1 - 1) - (1 - G_{SM})\} \cdot m3$$
$$+P_M \cdot \{1 - k_M \cdot (1 - bk1) \cdot bk1\} - Q_M = 0 \quad (26)$$

With the same method as fixing $c3$, $m3$ can be fixed by using equation (25).

For getting $y3$ from equation (23), we use $c3$, $m3$ fixed already with a high approximation. We use $y2$ as an alternative to $y3$ as approximate values.

We organizes equation (23) about $y3$, The result is as follows:

$$P_Y = (1 - c3 + c3 \cdot B_{SC}) \cdot (1 - m3 + m3 \cdot B_{SM}) \cdot (1 - bk1 + bk1 \cdot B_{SBk})$$
$$Q_Y = (1 - y1 + y1 \cdot 10^{-D_{sY}/n_Y})^{n_Y}$$
$$k_Y \cdot P_Y \cdot (1 - bk1)^2 \cdot y3^2 + P_Y \cdot \{k_Y \cdot (1 - bk1) \cdot (2bk1 - 1) - (1 - B_{SY})\} \cdot y3$$
$$+P_Y \cdot \{1 - k_Y \cdot (1 - bk1) \cdot bk1\} - Q_Y = 0 \quad (27)$$

By the above methods, we can fix a set of fractional dot areas ($c3$, $m3$, $y3$, $bk1$) for reproducing the color of any element in the original color image by the method of proportional tone compression.

2.8 Correction of mechanical dot gain

We can approximate mechanical dot gain on press by quadratic equations having l_C, l_M, l_Y, l_{Bk} as their coefficients. The fractional dot areas outputted under consideration of mechanical dot gain are as follows.

$$\begin{aligned} c &= c3 - l_C \cdot c3(1 - c3) \\ m &= m3 - l_M \cdot m3(1 - m3) \\ y &= y3 - l_Y \cdot y3(1 - y3) \\ bk &= bk1 - l_{Bk} \cdot bk1(1 - bk1) \end{aligned} \quad (28)$$

The correction term showed above express rate of increase on halftone dot shape, and the correction term of equation (21), (22), (23) express optical reflectance.

2.9 Calculation for comparing with measured value

With above equations, we can determine a set of fractional dot areas (c, m, y, bk). Till now we have regarded the optical reflectance of paper as 1, but here we introduce the reflectance of paper.

Reflectance R, G, B through R, G, B filters containing the value of paper and their optical densities D_C, D_M, D_Y are as follows:

$$R = R_P \cdot [\{1 - c + c \cdot R_{SC} - k_C \cdot (c + bk - c \cdot bk) \cdot (1 - c - bk + c \cdot bk)\} \cdot (1 - m + m \cdot R_{SM}) \cdot (1 - y + y \cdot R_{SY}) \cdot (1 - bk + bk \cdot R_{SBk})] \tag{29}$$

$$G = G_P \cdot [(1 - c + c \cdot G_{SC})\{1 - m + m \cdot G_{SM} - k_M \cdot (m + bk - m \cdot bk) \cdot (1 - m - bk + m \cdot bk)\} \cdot (1 - y + y \cdot G_{SY}) \cdot (1 - bk + bk \cdot G_{SBk})] \tag{30}$$

$$B = B_P \cdot [(1 - c + c \cdot B_{SC}) \cdot (1 - m + m \cdot B_{SM}) \cdot (1 - y + y \cdot B_{SY} - k_Y \cdot (y + bk - y \cdot bk)(1 - y - bk + y \cdot bk)) \cdot (1 - bk + bk \cdot B_{SBk})] \tag{31}$$

$$D_C = \log_{10} \frac{1}{R} \tag{32}$$

$$D_M = \log_{10} \frac{1}{G} \tag{33}$$

$$D_Y = \log_{10} \frac{1}{B} \tag{34}$$

R_P, G_P, B_P: optical reflectance of paper through R, G, B filters

With these processes, the color original image is outputted by being compressed on tone, managed on gray balance and masking, corrected on optical dot gain of superimposed halftone dots and considered on mechanical dot gain as fractional dot area.

2.10 Simulation

By using the value of parameter derived from measuring preproof (AGFA:Pressmaich Dry), and with Excel, we simulate on the part of achromatic color in the original. We showed the parameters used in Table 1 and showed the process of simulating by successive approximation in Table 2.

Optical Density of Solid Print				Reflectance of Solid Print				Coefficients of Optical and Macanical Dot Gain			
	R	G	B		R	G	B		n	k	l
SC	1.31	0.42	0.20	SC	0.05	0.38	0.63	C	5	1	0
SM	0.07	1.35	0.54	SM	0.85	0.04	0.29	M	4.5	1	0
SY	0.00	0.02	1.25	SY	1.00	0.96	0.06	Y	4	1	0
SBk	1.46	1.47	1.51	SBk	0.03	0.03	0.03	Bk	6	1	0
Optical Density of Paper				Reflectance of Paper				Parameters of Bk Dot Area Inserted			
Dp	0	0	0	Rp	1	1	1	α	0.4	β	0

Table 1. Used parameters which were measured on and derived from the preproof (AGFA Pressmatch Dry).

Kc1	Km1	Ky1	Dcc	Dcm	Dcy	Kc	Km	Ky	nc	nm	ny	c1	m1
0	0	0	1.31	1.35	1.25	0.000	0.000	0.000	5	4.5	4	0.000	0.000
0.1	0.1	0.1	1.31	1.35	1.25	0.100	0.087	0.105	5	4.5	4	0.129	0.130
0.2	0.2	0.2	1.31	1.35	1.25	0.200	0.184	0.210	5	4.5	4	0.251	0.252
0.3	0.3	0.3	1.31	1.35	1.25	0.300	0.291	0.314	5	4.5	4	0.365	0.365
0.4	0.4	0.4	1.31	1.35	1.25	0.400	0.388	0.419	5	4.5	4	0.473	0.471
0.5	0.5	0.5	1.31	1.35	1.25	0.500	0.486	0.524	5	4.5	4	0.575	0.571
0.6	0.6	0.6	1.31	1.35	1.25	0.600	0.582	0.629	5	4.5	4	0.670	0.664
0.7	0.7	0.7	1.31	1.35	1.25	0.700	0.679	0.734	5	4.5	4	0.760	0.751
0.8	0.8	0.8	1.31	1.35	1.25	0.800	0.776	0.838	5	4.5	4	0.845	0.832
0.9	0.9	0.9	1.31	1.35	1.25	0.900	0.873	0.943	5	4.5	4	0.925	0.908
1	1	1	1.31	1.35	1.25	1.000	0.970	1.000	5	4.5	4	1.000	0.979

y1	Kc*Dcc	Km*Dcm	Ky*Dcy	nbk	α	β	Dbk1	Dsbk	bk1	Rcc	Gcm	Bcy	Rcbk
0.000	0.000	0.000	0.000	6	0.4	0	0.000	1.44	0.000	0.05	0.04	0.06	0.03
0.142	0.131	0.131	0.131	6	0.4	0	0.052	1.44	0.046	0.05	0.04	0.06	0.03
0.273	0.262	0.262	0.262	6	0.4	0	0.105	1.44	0.092	0.05	0.04	0.06	0.03
0.395	0.393	0.393	0.393	6	0.4	0	0.157	1.44	0.134	0.05	0.04	0.06	0.03
0.506	0.524	0.524	0.524	6	0.4	0	0.210	1.48	0.180	0.05	0.04	0.06	0.03
0.612	0.655	0.655	0.655	6	0.4	0	0.262	1.48	0.223	0.05	0.04	0.06	0.03
0.708	0.786	0.786	0.786	6	0.4	0	0.314	1.48	0.265	0.06	0.04	0.06	0.03
0.798	0.917	0.917	0.917	6	0.4	0	0.367	1.48	0.308	0.06	0.04	0.06	0.03
0.883	1.000	1.000	1.000	6	0.4	0	0.400	1.48	0.332	0.05	0.04	0.06	0.03
0.960	1.000	1.000	1.000	6	0.4	0	0.400	1.48	0.332	0.05	0.04	0.06	0.03
1.000	1.000	1.000	1.000	6	0.4	0	0.400	1.48	0.332	0.05	0.04	0.06	0.03

kc	km	ky	εc	ζc	ηc	εm	ζm	ηm	εy	ζy	ηy	c2	m2
1	1	1	1.00	-1.95	0.00	1.00	-1.96	0.00	1.00	-1.84	0.00	0.00	0.00
1	1	1	0.88	-1.76	0.19	0.88	-1.76	0.19	0.87	-1.73	0.17	0.11	0.11
1	1	1	0.76	-1.58	0.31	0.77	-1.58	0.31	0.75	-1.53	0.29	0.22	0.22
1	1	1	0.66	-1.44	0.40	0.66	-1.44	0.40	0.65	-1.38	0.36	0.33	0.33
1	1	1	0.58	-1.30	0.45	0.58	-1.30	0.45	0.55	-1.21	0.40	0.43	0.43
1	1	1	0.52	-1.18	0.48	0.51	-1.18	0.48	0.47	-1.08	0.43	0.54	0.53
1	1	1	0.45	-1.07	0.50	0.45	-1.07	0.50	0.40	-0.96	0.43	0.64	0.63
1	1	1	0.38	-0.98	0.51	0.38	-0.98	0.51	0.34	-0.85	0.43	0.73	0.73
1	1	1	0.35	-0.92	0.52	0.35	-0.92	0.52	0.30	-0.78	0.44	0.82	0.81
1	1	1	0.35	-0.92	0.54	0.35	-0.92	0.54	0.30	-0.78	0.46	0.88	0.88
1	1	1	0.35	-0.92	0.54	0.35	-0.92	0.58	0.30	-0.78	0.47	0.96	0.94

y2	bk1	Rcc	Rgm	Rcy	Rcbk	Gcc	Gcm	Gcy	Gcbk	Bcc	Bcm	Bcy	Bcbk
0.00	0.00	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.11	0.05	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.21	0.09	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.31	0.14	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.41	0.18	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.51	0.22	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.61	0.26	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.70	0.31	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.79	0.33	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.88	0.33	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03
0.91	0.33	0.05	0.85	1.00	0.03	0.38	0.04	0.96	0.03	0.83	0.29	0.06	0.03

Pc	Qc	εc	ζc	ηc	c3	Pm	Qm	εm	ζm	ηm	m3	Py	Qy
1.00	1.00	1.00	-1.95	0.00	0.00	1.00	1.00	1.00	-1.90	0.00	0.00	1.00	1.00
0.94	0.74	0.85	-1.71	0.16	0.10	0.89	0.74	0.81	-1.63	0.11	0.07	0.87	0.74
0.88	0.55	0.73	-1.48	0.26	0.18	0.79	0.55	0.65	-1.35	0.18	0.14	0.76	0.55
0.83	0.40	0.62	-1.31	0.33	0.29	0.70	0.40	0.52	-1.11	0.22	0.22	0.66	0.40
0.78	0.30	0.52	-1.14	0.36	0.38	0.62	0.30	0.42	-0.91	0.23	0.29	0.58	0.30
0.72	0.22	0.44	-1.00	0.38	0.48	0.54	0.22	0.33	-0.75	0.22	0.38	0.48	0.22
0.68	0.16	0.37	-0.88	0.38	0.57	0.47	0.16	0.25	-0.61	0.21	0.42	0.41	0.16
0.63	0.12	0.30	-0.77	0.38	0.66	0.40	0.12	0.18	-0.48	0.20	0.49	0.35	0.12
0.60	0.09	0.27	-0.70	0.38	0.75	0.35	0.08	0.16	-0.41	0.18	0.58	0.29	0.08
0.58	0.07	0.26	-0.70	0.39	0.83	0.32	0.07	0.14	-0.38	0.18	0.64	0.26	0.07
0.58	0.06	0.28	-0.69	0.41	0.89	0.29	0.06	0.13	-0.34	0.18	0.70	0.23	0.06

								Successive Approximation				Newton Raspheon			
εy	ζy	ηy	y3	lc	lm	ly	lbk	c	m	y	bk	c	m	y	bk
1.00	-1.94	0.00	0.00	0	0	0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.78	-1.58	0.08	0.06	0	0	0	0	0.10	0.07	0.06	0.05	0.10	0.07	0.06	0.05
0.63	-1.28	0.15	0.12	0	0	0	0	0.19	0.14	0.12	0.08	0.20	0.15	0.12	0.09
0.48	-1.03	0.17	0.19	0	0	0	0	0.29	0.22	0.19	0.14	0.29	0.22	0.19	0.14
0.38	-0.83	0.18	0.25	0	0	0	0	0.38	0.29	0.25	0.18	0.39	0.29	0.25	0.18
0.29	-0.68	0.18	0.31	0	0	0	0	0.48	0.36	0.31	0.22	0.49	0.36	0.31	0.22
0.22	-0.53	0.17	0.37	0	0	0	0	0.57	0.42	0.37	0.26	0.58	0.42	0.37	0.26
0.17	-0.42	0.15	0.44	0	0	0	0	0.66	0.49	0.44	0.31	0.67	0.49	0.44	0.31
0.13	-0.34	0.14	0.50	0	0	0	0	0.75	0.50	0.50	0.33	0.76	0.56	0.50	0.33
0.12	-0.30	0.13	0.57	0	0	0	0	0.83	0.64	0.57	0.33	0.84	0.63	0.57	0.33
0.10	-0.27	0.12	0.58	0	0	0	0	0.89	0.70	0.58	0.33	0.91	0.70	0.59	0.33

Table 2. Sample of simulation with spreadsheet software (Excel: Mac) by using our successive approximation method in the case of the preproof (AGFA Pressmatch Dry).

We showed fractional dot areas (c, m, y, bk) to K, and showed optical density derived by equations (31) sim (33) on comparing to measured value in Fig. 6. The values are matched fairly with optical density of pre replacement of black.

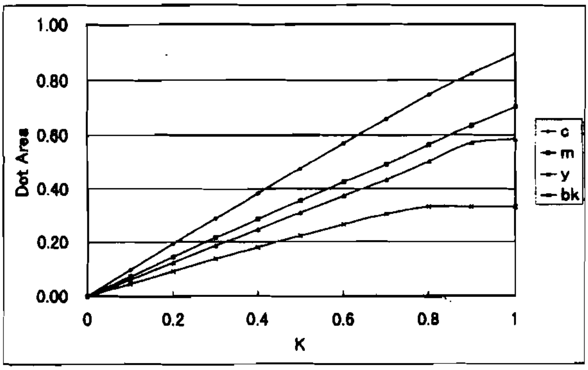

Figure 6. Combination of halftone dots that is determined by simulating with our system in the case of successive approximation method.

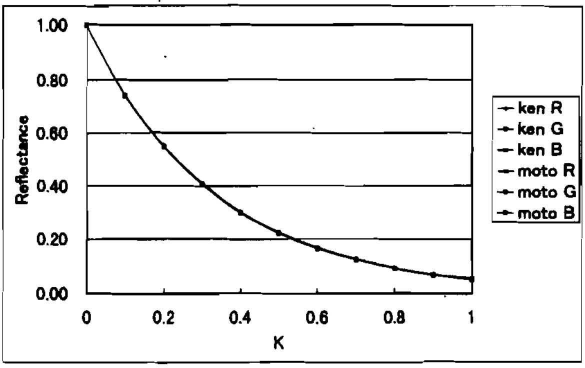

Figure 7. Optical density curves for comparing fixed reflectance first by using N-Y equation with resulted reflectance from our approximation method. (moto Dr, moto Dg, moto Db) are fixed first by N-Y equation. (ken Dr, ken Dg, ken Db) are results of our method.

3. Conclusion

With the above method, we were able to convert the color image signal inputted by a scanner to halftone dots of C, M, Y, and Bk by mathematically modelling all image processing process in printing.

Because this theory uses equation, it contains some assumptions. We used Pollak's equation. This equation contains the assumption that each fractional dot areas constructed by superimposing (17 parts) are expressed by the theorem of product in probability (Demichel's equation) and optical density of print superimposed by each color solid is expressed sum density of each solid print. And our method of correcting optical dot gain on print superimposed with halftone dots contains the assumption that Bk and C, M, Y contain almost the same optical dot gain.

In printing optical density is varyied by unstable ink transfer, you would have anxiety if applying our theory to actual printing. But in this theory, the principle of color reproduction by halftone dots is mathematically modeled exactly, so to variation effected this theory would carry out as mainframe.

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