# Automatic Design of Halftone Dots

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Abstract: Designing halftone dots has been done with fine craftsmanship and details about it have never been disclosed because the method is classified knowhow of the companies or the craftspeople up to now. At this time, the mathematical method to design halftone dots without such craftsmanship is invented and it is verified that the same halftone dots like the dots offered by many prepress companies are designed only based on the parameters and equations.

The main issues are
(1) How to find out the shape of halftone dot pattern and
(2) How to number the pixels within the halftone dot.

In this paper, the algorithm about how to design the halftone dots is made clear. The relationship between characteristics of printers and the best halftone dots to suppress moiré is discussed. Finally, the properties of the best halftone dots for web offset printers and office printers are proposed.

## Introduction

Many kinds of dot patterns have been produced by many prepress companies in the graphic arts and it is well known that printed images can be upgraded by using clustered dots for halftoning, especially in case of poor adhesion of toner.

But, all clustered dots can be designed only with good craftsmanship and thus all companies cannot design the clustered dots suitable for their engines and there has been little discussion on how to eliminate moiré by designing the suitable clustered dots.

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Here, the items written below are made clear:

(1) Requirements for clustered dots
(2) Method of designing the suitable clustered dot for eliminating moiré
(3) Examples of clustered dots
(4) How to eliminate moiré by altering the specifications of halftone dots

In this paper, I will explain the algorithm to design clustered dots and to determine the parameters of clustered dots suitable for eliminating moiré.

Requirements for clustered dots

The rules to be kept when clustered dots are designed, what is called, "requirements for clustered dots" are described as follows:

(1) The 2-dimensional digital space is covered completely with no pixels to be uncovered, only by moving a dot pattern which defines a clustered dot, horizontally and vertically.
(2) Such a dot pattern can be reshaped into a rectangle.
(3) The gravity center of the clustered dot specified by the dot pattern is arranged in a line at the same intervals.
(4) Symmetry of the clustered dot is kept with respect to its gravity center as much as possible.
(5) The length of the boundary between the area adhered with the colorants and the substrate is kept as short as possible.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |
| | | | | 57 | 61 | 63 | 59 | | | | |
| | | | | 41 | 49 | 51 | 43 | | | | |
| | | 37 | 33 | 25 | 17 | 19 | 27 | 35 | 39 | | |
| | | 45 | 29 | 13 | 5 | 7 | 15 | 31 | 47 | | |
| 69 | 65 | 53 | 21 | 9 | 1 | 3 | 11 | 23 | 55 | 67 | 71 |
| 72 | 68 | 56 | 24 | 12 | 4 | 2 | 10 | 22 | 54 | 66 | 70 |
| | | 48 | 32 | 16 | 8 | 6 | 14 | 30 | 46 | | |
| | | 40 | 36 | 28 | 20 | 18 | 26 | 34 | 38 | | |
| | | | | 44 | 52 | 50 | 42 | | | | |
| | | | | 60 | 64 | 62 | 58 | | | | |
| | | | | | | | | | | | |

Figure 1. Example of a dot pattern which defines a clustered dot

Requirements (1) and (2) are necessary for installing the dot patterns in printers. Requirements from (3) to (5) are the necessities to keep uniformity of density generated by the clustered dot. For example, if the length of the boundary is not kept at a minimum, the amount of colorants adhering to the substrate is affected more owing to temperature and humidity. As the result of this, perturbation of density as streaking and banding is observed.

For all kinds of clustered dots you think are fine, these requirements are kept strictly.

Method of designing clustered dots

Designing the clustered dots is composed of the steps written below:

1. Calculation of the fundamental area

The fundamental area is the minimum area necessary to generate a clustered dot with a specified screen angle and screen ruling. The fundamental area is not always a rectangle, but the 2-dimensional digital space is covered completely with no pixels to be left, only by moving this area horizontally and vertically. An example is shown in Figure 1.

2. Numbering of each pixel in the fundamental area

After the fundamental area is determined, numbering of each pixel in it is carried out mathematically. What is the most necessary in this step is to keep the gravity center of the adhered area unmoved and to keep the adhered area symmetric with respect to its gravity center.

3. Rectangulation

As mentioned before, the fundamental area to be calculated is not a rectangle. Therefore, we reshaped the fundamental area into a rectangle. According to the method in this paper, this is achieved only by using the equation.

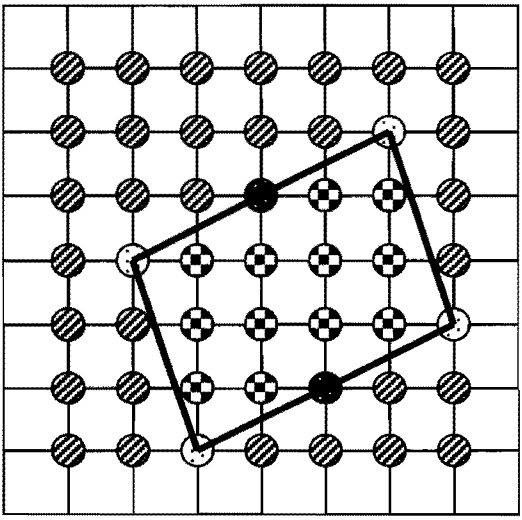
Here, I explain the details on each step as follows:

(1) Calculation of the fundamental area

The calculation is done based on "Pick's Theorem". The theorem is shown in Figure 2. This theorem shows that some fundamental areas for tiling can be calculated according to the specifications when they are determined. It is because no fundamental area can be found in case that an irrational tangent of screen angle is specified. The calculated areas are just candidates for you to select based on printed images finally. But, it is very difficult to design such fundamental area with craftsmanship. You might not get the most suitable area for your printer.

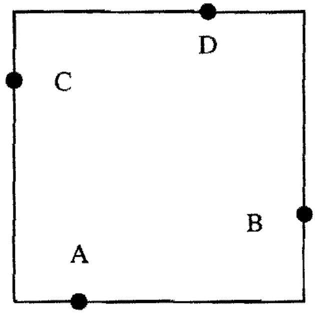
This method guarantees that the best fundamental area exists for your printer. Pick's theorem is changed into the other form to get the shapes of the area. It is shown in Figure 3.

**Pick's Theorem: For fundamental area R,**
**area(R) = #int(R) + #on(R)/2 + 1,**
**area(R): Number of pixels belonging to R**
**#int(R): Number of pixels in R**
**#on(R): Number of pixels on boundary of R**



area(R)=4*3+2*1=14
#int(R)=12,
#on(R)=2,
14=12 + 2/2 + 1

Figure 2. Explanation of Pick's Theorem




A : (c, 1)
B : (a+c, b+1)
C : (0, d+1)
D : (a, b+d+1)

AB : $ay = b(x - c) + a$ — $bx = ay + bc - a$
AC : $cy = - d(x - c) + c$ — $dx = - cy + c(d + 1)$
CD : $ay = bx + a(d + 1)$ — $bx = ay - a(d + 1)$
BD : $cy = - d(x - a) + (b + d + 1)c$ — $dx = - cy + ad + (b + d + 1)c$

screen angle=$(\tan^{-1}a./b+\tan^{-1}c/d)/2$

NOTE:
To get the fundamental area,
All pixels on lines AC and AB shall be included in the fundamental area, but the pixels on lines CD and BD shall be excluded. The pixel only on point A shall be included.

Figure 3. Explanation of parameters in Pick's Theorem

According to the notation in Figure 3, the equation of Pick's theorem is described as follows:

$$a \times b + c \times d = n \qquad \ldots \qquad (1)$$

n: number of pixels belonging to the fundamental area

This shows that the fundamental area can be calculated as follows:
1) The screen angle and the number of pixels in the fundamental area are specified.
2) All sets of (a, b, c, d) are calculated by using the equation (1) under the condition that n pixels are included in the fundamental area.
3) The calculated data (a, b, c, d) are sorted in order of the specified screen angle.

Calculation of the fundamental areas is shown as a flowchart in Figure 4. The better, which satisfies the specifications, is chosen earlier among these areas and the pixels in each fundamental area are numbered in the next step.

(2) Numbering of every pixel in the fundamental area

It is the most important to satisfy the requirements for the clustered dots written before. The flowchart to achieve the requirements is shown in Figure 5.

Rotating the fundamental area by the specified screen angle minimizes the calculated error by adopt rational values in place of irrational numbers. After this calculation, a weight value is given to each pixel in the fundamental area by using the equation as follows:

$$W=2-E\cos X-(2-E)\cos Y \quad \ldots \quad (2)$$

E: eccentricity (E=1.0: square dot, E<1.0: chain dot)
(X,Y): coordinates to identify location of each pixel in the fundamental area
W: weight value

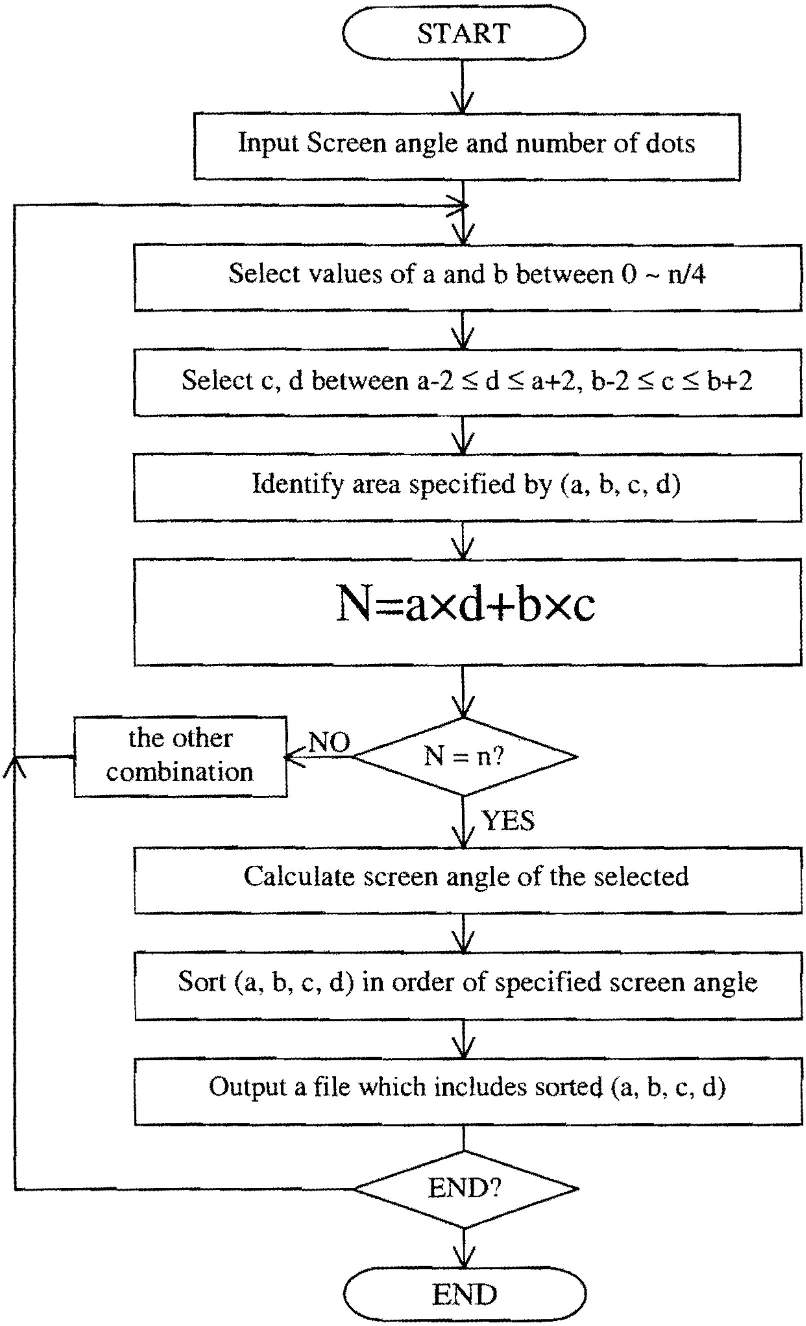


Figure 4. Algorithm on calculation of fundamental areas

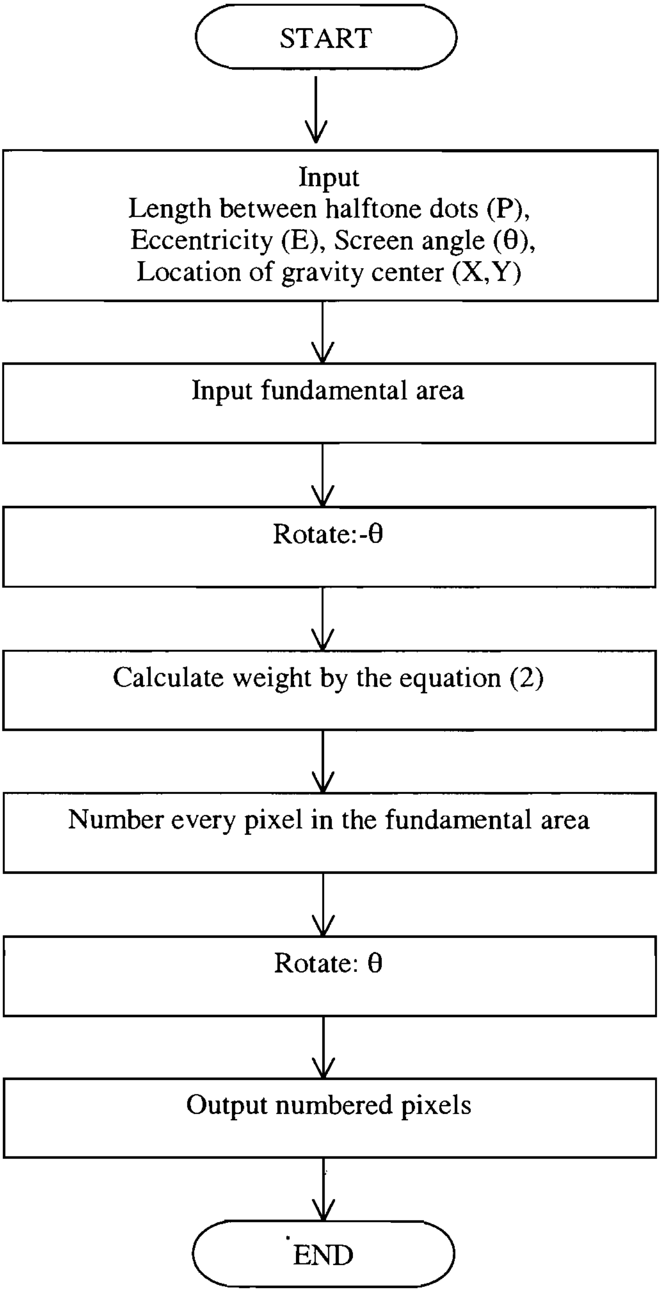



Figure 5. Algorithm on numbering every pixel in the fundamental area

After determining the area, each pixel is numbered so that a pixel with a greater weight value may have a smaller positive integer.

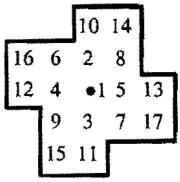



Figure 6. Example of numbering

(3) Rectangulation

The fundamental area with all numbered pixels in it is reshaped into a rectangle of the smallest size to achieve an economic circuit design.

In Figure 7-1, the location of all the reference points is described as $(ia\text{-}jc, ib+jd)$. Being possible to rectangulate the fundamental area is equal to being possible to get both the minimum value of $(ia\text{-}jc)$ under the condition $ib+jd=0$ and the minimum value of $(ib+jd)$ under the condition $ia\text{-}jc=0$.
Therefore,

$$ib+jd=0$$
$$i=-(d/b)j$$

and thus,

$$ia\text{-}jc=-(d/b)ja\text{-}jc=-j(ad+bc)/b=-j(n/b) \quad [ad+bc=n \text{ by equation (1)}]$$

In contrast,

$$j=-(b/d)i$$
$$ia\text{-}jc=ia+(b/d)ic=i(ad+bc)/d=i(n/d)$$

Here, $i$ and $j$ are integrals. Then,

$$\gcd(jd,b)=b \text{ and } \gcd(ib,d)=d$$

gcd(A,B) is equal to the greatest common divisor between A and B. For example, gcd(6,24)=6 and gcd(24,48)=24

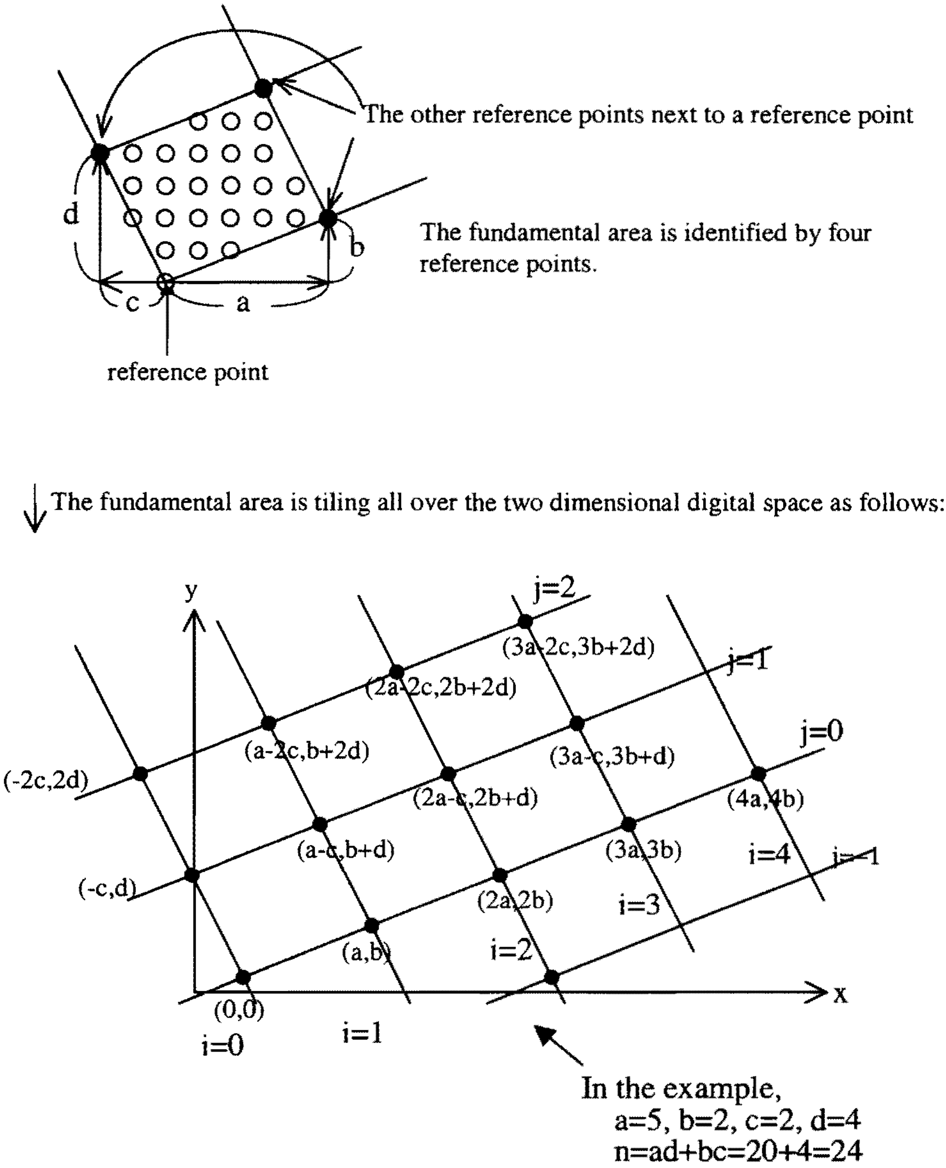



Figure 7-1. Explanation of rectangulation

Therefore, the minimum value of $-j(n/b)$ and $i(n/d)$ is searched for under the condition that both $(n/b)$ and $(n/d)$ are integers. This value is $\max(n/\gcd(b, n), n/\gcd(d, n))$. This result is obtained along the coordinate X. The same procedures are carried out along the coordinate Y and the answer as follows is obtained: This value is $\max(n/\gcd(a, n), n/\gcd(c, n))$.

Here, for the fundamental area identified by a set of parameters (a, b, c, d), finally the rectangular pattern whose size is as follows are obtained:

$$(\max(n/\gcd(b, n), n/\gcd(d, n)), \max(n/\gcd(a, n), n/\gcd(c, n)))$$

Figure 8 shows the result of rectangulating the fundamental area with all numbered pixels.

Examples of clustered dots

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 101 | 117 | 123 | 127 | | |
| | | | 89 | 67 | 63 | 75 | 93 | 109 | 111 | 119 | | |
| 129 | 121 | 105 | 79 | 55 | 43 | 51 | 59 | 85 | 95 | 103 | | |
| 125 | 113 | 97 | 71 | 39 | 29 | 27 | 35 | 47 | 69 | 81 | 87 | |
| 115 | 107 | 83 | 45 | 23 | 15 | 11 | 17 | 25 | 41 | 57 | 73 | |
| 99 | 91 | 61 | 37 | 21 | 9 | 3 | 7 | 19 | 33 | 49 | 65 | |
| | 74 | 48 | 24 | 10 | 2 | 1 | 5 | 13 | 31 | 53 | 77 | |
| | 58 | 42 | 30 | 14 | 6 | 4 | 8 | 20 | 36 | 64 | 92 | 100 |
| | 68 | 54 | 38 | 22 | 18 | 12 | 16 | 26 | 52 | 88 | 108 | 116 |
| | 86 | 80 | 70 | 46 | 34 | 28 | 32 | 40 | 72 | 96 | 114 | 124 |
| | | 102 | 94 | 84 | 62 | 50 | 44 | 56 | 78 | 104 | 120 | 128 |
| | | 118 | 112 | 106 | 90 | 76 | 60 | 66 | 82 | | | |
| | | 126 | 122 | 110 | 98 | | | | | | | |
| | | 130 | | | | | | | | | | |

Figure 7-2. Example of clustered dot designed by this method

Figure 7-2 shows an example of the clustered dots designed by this method. This dot pattern is designed for 106lpi printed images of a 1200dpi printer. The screen angle to be realized is 15.01 degrees. It takes less than a second because it is designed on a PC automatically and less than five hours to confirm its superiority based on images printed by a printer in which this pattern is installed.

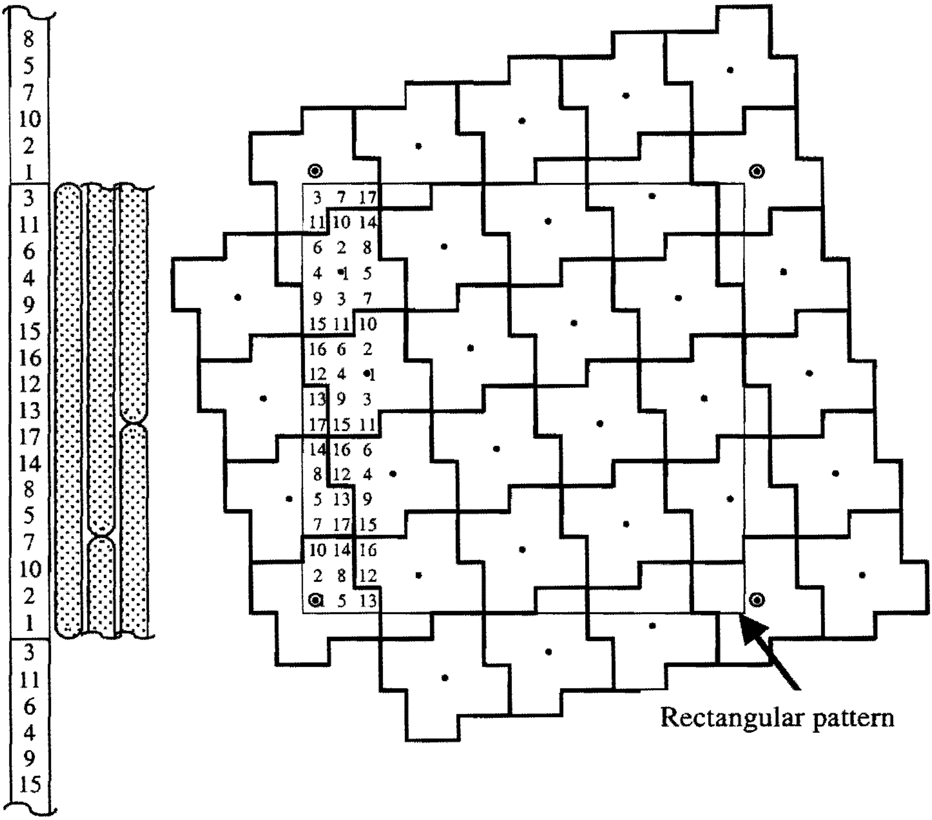



Figure 8. Example of rectangulation

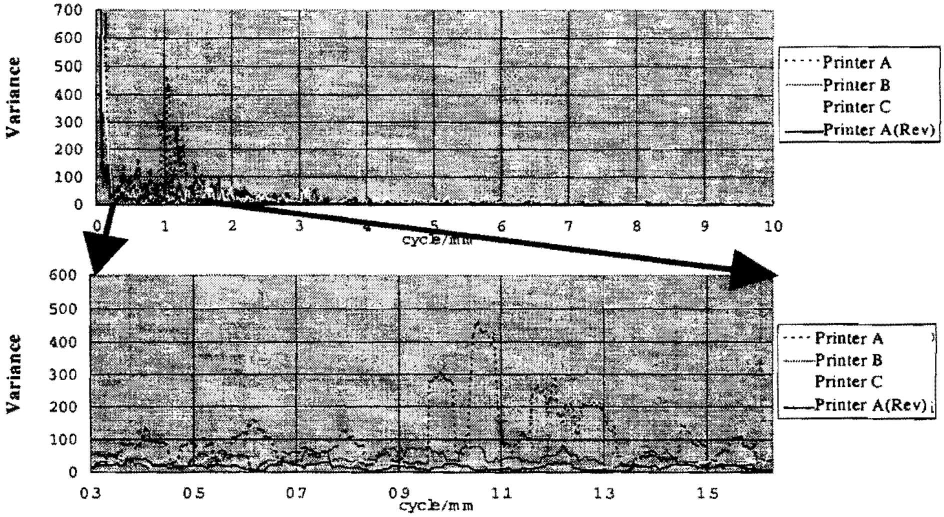



Figure 9. Analysis on perturbed density of solid color patch (Black)

How to eliminate moiré by altering the specifications of halftone dots

Moiré is a kind of resonance between the characteristics of the marking engine and the dot patterns used. Of course, moiré between these and the contents of an image occurs and sometimes moiré cannot be eliminated only by changing the dot pattern. Here, reducing moiré between the marking engine and the dot pattern used is made clear.

Figure 9 shows that a comparison of perturbed density of solid color patch (Black) printed by four printers, by using Fourier transform. As the figure shows, it is apparent that the characteristics of the marking engine are different by measuring micro-density (less than 10μm) on solid color patches. We define Figure 9 as the characteristics of marking engine.

When a dot pattern, which resonates with such characteristics, is used, moiré occurs and printed images are degraded. Especially, resonance at from 0.3 cycle/mm to 1.6 cycle/mm is very visible for human beings according to many papers on psychology and image quality. When you design the dot pattern suitable for your marking engine, the processes written below are recommended.

(1) Solid color patches are printed.

(2) Density of solid color patches is measured by micro-densitometer.

(3) Data of perturbed density are analyzed by Fourier transform.

(4) If the dot pattern specified resonates with the characteristics of marking engine, the other dot pattern with different screen ruling and angle from the specified must be designed by this method. You will know that streaking and banding are reduced based on the process. Please note that resonance at from 0.3 cycle/mm to 1.6 cycle/mm is critical for us and must be eliminated.

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