Insight Into the Relationship Between Print Density and Ink Film Thickness

John MacPhee* and John T. Lind**

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Abstract: Based on experimental data, three new findings are presented. The first is that the Tollenaar and Ernst equation does not provide a good fit to measurements of density versus printed ink film thickness while the second is that the widely accepted theory used to explain the same relationship could not be substantiated. The third finding is that the shape of the ink mileage curve is strongly affected by the type of densitometer filter that is used. Additional experimental data is generated to show that a different equation provides a much better fit to printability data. Insight into the relationship between ink mileage and the properties of the ink and paper used to produce a print is also presented.

Introduction

The most common way of portraying the relationship between density and ink film thickness is to plot measurements of these variables of prints, made on a printability tester, as shown for example in Figure 1. Although such plots, or ink mileage curves, usually correlate very well with equation (1), known as the Tollenaar-Ernst equation (Tollenaar and Ernst, 1961), it has not been possible to relate the two parameters of this equation, D_{sr} and m, to properties of the ink and paper used. Thus, this equation provides no insight into how ink and paper interact to produce the characteristic curves shown in Figure 1 and cannot be used to predict how a given curve of density versus ink film thickness will be affected by changes in the properties of the ink and paper used. To remedy this, a study was undertaken to determine if it would be possible to derive an alternate equation embodying parameters reflecting the properties of the ink and paper used. In the course of that study, three findings were made which shed considerable new light on this relationship. The objective of this paper is to describe those findings and summarize the new insights that they provide.

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* Baldwin Technology Company, Inc.
** Graphic Arts Technical Foundation

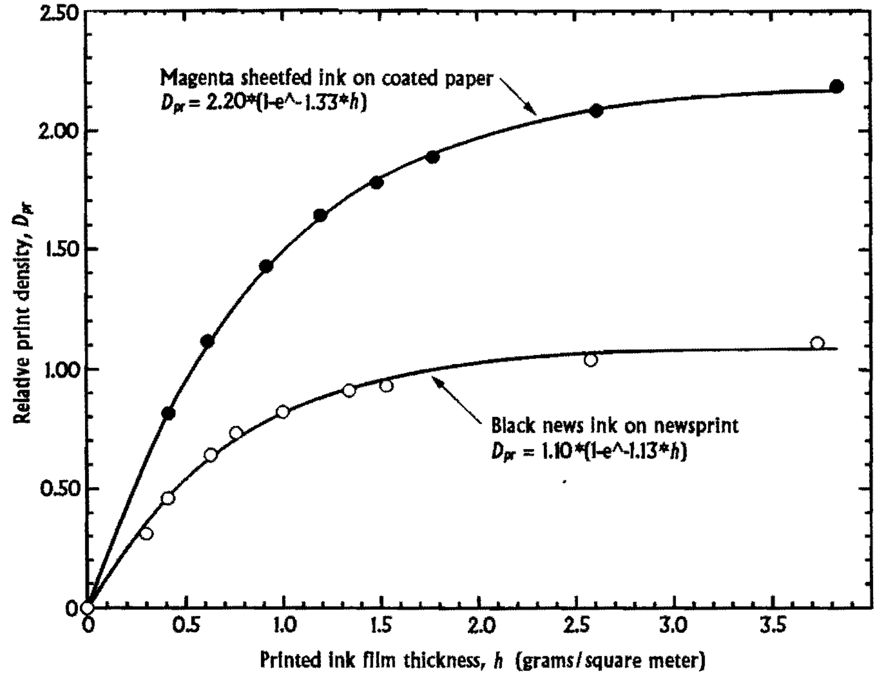

Figure 1 Typical ink mileage curves showing relationship between relative print density and printed ink film thickness on coated and uncoated papers. The most striking difference is that a given ink film thickness produces a much lower density on the uncoated paper compared to the coated paper.

$$D_{pr} = D_{sr}\left(1-\varepsilon^{-mh}\right) \tag{1}$$

where:

D_{pr} = relative print density at ink film thickness h
D_{sr} = relative saturation density, sometimes defined as D_o

R_{sr} = relative saturation reflectance = $10^{-D_{sr}}$
m = a constant, in square meters/gram
h = printed ink film thickness, in grams/meter^2

The remainder of this paper is divided into five sections. In the first, entitled *Background Information,* the widely accepted theory for explaining the shape of the subject curve is reviewed. Three sections devoted to the three findings follow this. The last section of the paper contains the conclusions reached, along with a discussion of the new insights gained and how the shape of the ink mileage curve is influenced by the properties of both the ink and paper used.

Background Information

Most of the theory needed to broadly explain the shape of the curves in Figure 1 is given in Chapters 7 and 8 of Yule's landmark text (Yule, 1967 and 2000). Although this theory is subsequently found to be unsubstantiated, it is nevertheless set forth here because its explanation entails a review of the interactions between light, a printed ink film, and paper, that affect the ink mileage curve.

This explanation will start with a review of how reflection density, of a film of ink printed on a diffusing substrate, is measured. Special emphasis will be placed on the three ways that light can reach the sensor of a reflection densitometer. Figure 2 is a diagram showing that, in its most simple form, such a densitometer consists of a source, emitting a thin beam of light, and a light detector. The densitometer is designed in such a way that the detector will, in theory, only respond to or "see" light from the source, that has been both scattered by the paper and attenuated by the ink. This is accomplished by mounting the source such that light emanating from it will form an incident angle of 45 degrees (relative to a line that is normal to the paper) and mounting the detector so as to sense only that light traveling from the paper at an angle of zero degrees, as in Figure 2. (The same effect is achieved by reversing the angles, as it is often done in practice.)

This arrangement enables some of the light that passes through the ink film, is scattered by the paper, and again passes through the ink film, to reach the detector. At the same time, assuming that the surface is optically smooth, it prevents light reflected by the paper surface from reaching the detector because the surface reflected light will be at an angle equal to the incident angle. Thus, the first way that light can reach the detector is for it to be scattered by the paper.

As shown in Figure 2, actual paper surfaces are not optically smooth, and therefore a small fraction of the surface will consist of areas that have an inclination of 22 1/2° to the horizontal. Because these areas will reflect light on a line parallel to the detector's line of sight, the detector will receive some surface reflected light and this light will not have been attenuated or filtered by the ink film. (Such light is referred to as diffuse surface reflected light.) This is the second way in which light, albeit unwanted, can reach the detector.

The third way in which light can reach the detector also involves unwanted light, i.e., light that is scattered by the ink. This is also illustrated in Figure 2. This light, incidentally, results if the ink has some opacity, or conversely, is not perfectly transparent.

Thus, the light received by the detector can be considered to consist of light coming from two different sources, one fixed and one variable, as follows:

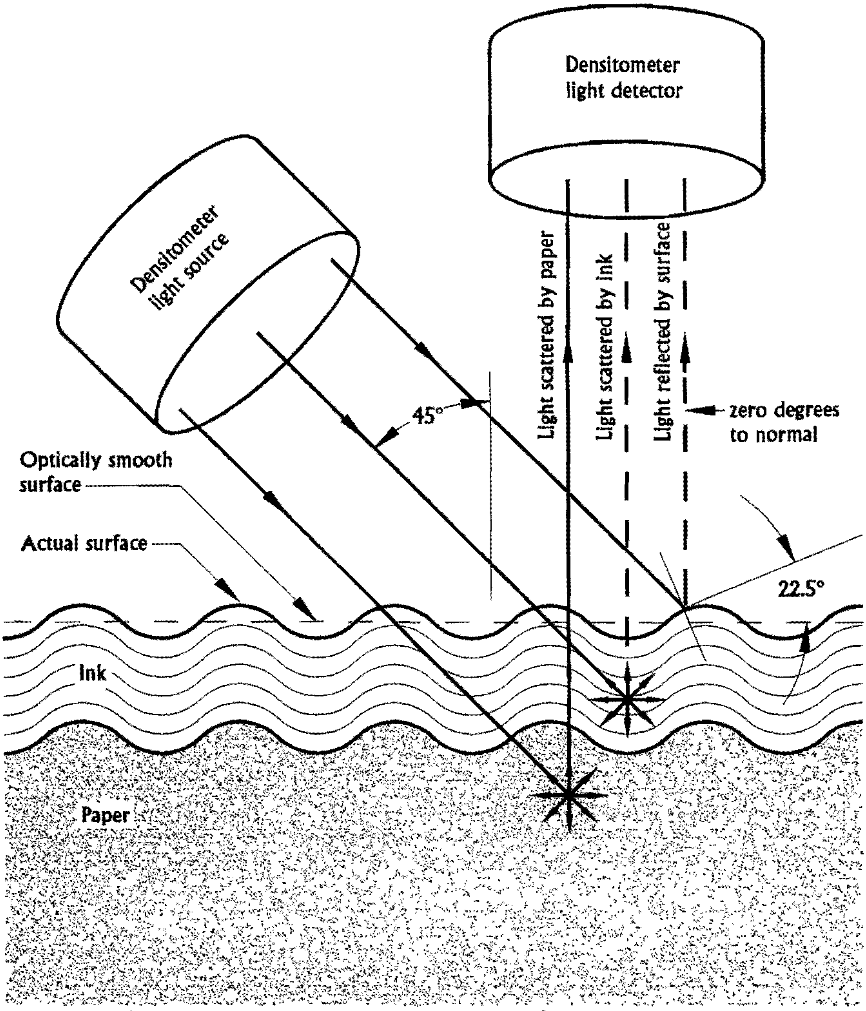

Figure 2 Diagram that portrays the three types of reflected light that contribute to the response of a densitometer.

- Variable Source: light scattered by the paper and attenuated by the ink, plus light scattered by the ink.
- Fixed Source: diffuse surface reflected light.

These two light sources can be portrayed by converting an ink mileage curve to one of reflectance versus ink film thickness, as shown in Figure 3. The respective influences of the two sources on the shape of the curves of both reflectance and density versus ink film thickness then become readily apparent. That is, the reflectance produced by the fixed source in Figure 3, designated the

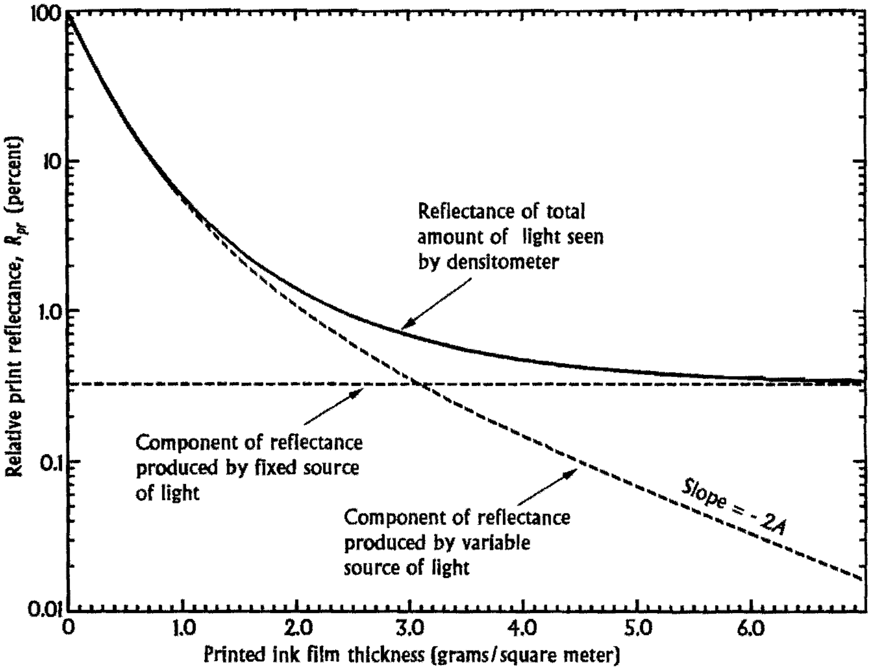

Figure 3 Ink mileage curve plotted in terms of relative reflectance. Dotted lines are plots of reflectances produced by two different sources of light. The variable source consists of light that is scattered by paper and attenuated by ink plus light that is scattered by ink. Fixed source consists of diffuse surface reflected light. Reflectance of light seen by densitometer is equal to sum of reflectances produced by the two sources. Magnitude of fixed reflectance is assumed equal to relative saturation reflectance, R_{sr}, and is equivalent to a relative saturation density, D_{sr}, of 2.49.

relative saturation reflectance, R_{sr}, constitutes a limit, below which total relative reflectance cannot fall, regardless of how thick an ink film is printed. In terms of density, this component establishes the relative saturation density, D_{sr}, above which density cannot rise, regardless of the film thickness.

Given the assumption that the fixed component is equal to the diffuse surface reflected light, R_{sr}, then its magnitude can be expressed by equation (2).

$$R_{sr} = \left| \begin{array}{l} \text{Surface reflectance for the} \\ \text{case where the print in} \\ \text{question has an optically} \\ \text{smooth surface.} \end{array} \right| * \left| \begin{array}{l} \text{Fraction of actual surface} \\ \text{area that is at a nominal} \\ \text{angle of 22.5 degrees to} \\ \text{the horizontal.} \end{array} \right| \qquad (2)$$

Assuming that the index of refraction for a typical ink film is 1.5 (based on Table VII in Lavelle, 1982), Fresnel's Law predicts that the value of the first term in equation (2) will be 4.0 percent, relative to the incident light intensity. What is needed, however, is to express this in terms of relative reflectance, that is, relative to the paper. To do this, the figure of 4.0 percent must first be corrected for the difference in densitometer geometry when viewing a smooth perfectly reflecting and perfectly diffusing material (NPES, 1993), assumed to be 0/45; versus viewing the surface elements of a rough surface that are inclined at an angle of 22.5 degrees to the horizontal, i.e., a 22.5/22.5 geometry. The corresponding correction factor will be 1.41 for these geometries. Therefore, the corresponding value of absolute reflectance will be 5.7 percent, i.e., 4 percent multiplied by 1.41. To obtain the corresponding value of relative reflectance, it is necessary to divide the figure of 5.7 percent by the absolute reflectance of the paper in question, which will generally lie somewhere between 60 percent (newsprint) and 87 percent (#1 glossy coated paper) as measured with a densitometer. (For typical values of the relative reflectance of papers, expressed in terms of density, see Table I in MacPhee and Lind, 1994.)

Thus, it can be concluded that the first term in equation (2) will be between 6.6 and 9.5 percent, depending on the paper used, with coated papers ranging toward the lower number and uncoated papers toward the higher. These values represent the upper limits on saturation reflectance, because the second term in equation (2) will always be significantly less than one; zero for an optically smooth surface and increasing in magnitude (but still quite small) for rougher surfaces. Therefore one would expect the actual values of relative saturation reflectance, R_{rs}, as given by equation (2), to be much lower than 6.6 percent for prints on coated papers and 9.5 percent for prints on uncoated papers.

The corresponding lower limits on relative saturation density, D_{sr}, are thus 1.02 (uncoated paper) and 1.18 (coated paper). Similarly, one would expect the actual values of relative saturation density to be much higher than 1.02 and 1.18.

The variable source in Figure 3 determines how rapidly the intensity of the total amount of received light decreases toward the lower limit established by the fixed source. Because this variability results from attenuation by the ink, the primary property affecting this rate of fall is the absorption coefficient of the ink, defined as A. If the light only passed through the ink film once, as happens when transmission density is measured, the curve of the variable source would obey the Beer-Lambert Law, given by equation (3).

$$R = 10^{-Ah} \quad \text{or} \quad \text{Log}_{10} R = -Ah \tag{3}$$

However, because the light must pass through the ink film twice, first to reach the paper, and then in exiting the paper to reach the densitomer, the absorption coefficient is effectively doubled. However, as shown in Figure 3, Log R

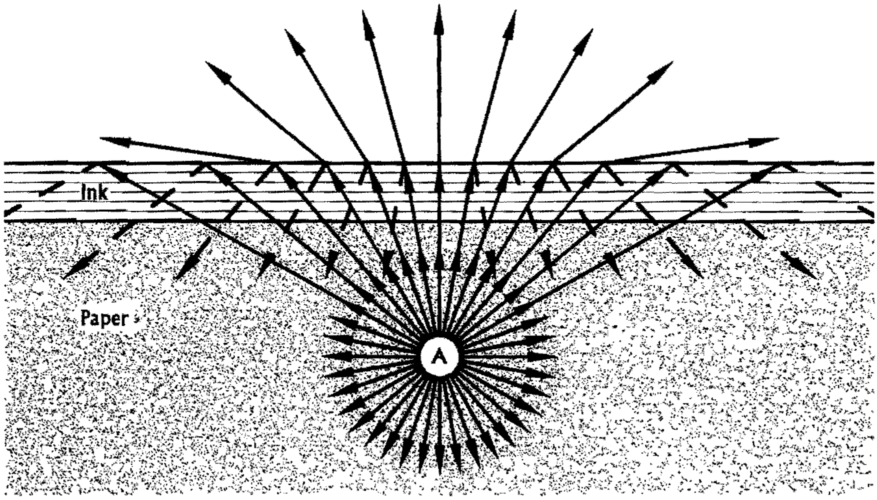

Figure 4 Diagram illustrating internal reflection of light, scattered from Point A in the paper. At the interface of ink and air, bending occurs due to the higher index of refraction of the ink. Note that as the incident angle increases, a value is reached (the critical angle) beyond which all of the light is reflected back into the ink. After Figure 3.9 in Judd and Wyszecki.

initially decreases at an even greater rate than $2A$ and then bends upward to asymptotically approach a straight line having a negative slope of slightly more than $2A$. The plot of density, corresponding to the reflectance produced by the variable source in Figure 3, behaves in a similar but inverted manner, i.e., the density curve has an initial positive slope of much greater than $2A$, which decreases to an asymptotic slope of slightly more than $2A$.

The explanation for the initial bending of the variable component of both the relative reflectance and density curves was set forth years ago as being due to the refraction of some of the scattered light back into the ink (Williams and Clapper, 1953). This additional scattering of light is referred to as internal surface reflection and is illustrated in Figure 4. According to Williams and Clapper, it has the result that at very thin ink film thicknesses some light transits the ink film more than two times before reaching the light detector of the densitometer and therefore its probability of being absorbed by the ink at those thicknesses is increased. Williams and Clapper defined this behavior with an equation wherein the reflection density is 4.0 or more times the transmission density at thin film thicknesses and asymptotically approaches a ratio of about 2.3 as film thickness increases.

To summarize, this widely accepted explanation for the shape of the curve of density versus ink film thickness could be described as follows:

1 As ink film thickness is increased, density increases at an ever decreasing rate until it reaches a plateau designated as D_{sr}, the relative saturation density. If the ink film thickness is then further increased, no corresponding increase in density will occur.

2. The curve of density versus ink film thickness can be divided into two components, one constant and one variable, as illustrated by the corresponding reflectance curves in Figure 3.

3. The magnitude of the constant component of density, which is equal to the saturation density, D_{sr} is determined by the amount of externally surface reflected light that reaches the densitometer light sensor. Principles of optics dictate that this component should be of a density much greater than about 1.18 for typical coated papers and 1.02 for typical uncoated papers. Correspondingly, the relative saturation reflectance, R_{sr}, should be much less than about 6.6 percent for coated and 9.5 percent for uncoated papers.

4. The model of Williams and Clapper is integral to the widely accepted theory for explaining the shape of the ink mileage curve, even though it does not account for scattering by ink. According to this model, the initial slope of the variable component, and hence of the overall density curve, should be four times A or more. where A is the absorption coefficient in the Beer-Lambert Law.

Test of Widely Accepted Theory

To test if the widely accepted theory could be substantiated, equation (1) was fitted to measured values of density versus ink film thickness of prints made on a printability tester in connection with previous printing experiments (MacPhee and Lind, 1994). The prints were made with the same magenta sheetfed ink and the fifteen different papers used in the printing experiments. In addition to verifying the theory, it was hoped that this exercise would provide insight into how the ink mileage curve is affected by paper properties.

Analysis of this data disclosed a significant inconsistency. Specifically, the values of saturation reflectance, R_{sr} , for the uncoated papers approached 9.5 percent, the value of the first term in equation (2). For this to be true, it would mean that the value of the second term in equation (2) for the uncoated papers approached unity, which is not realistic. Put another way, the values of the relative saturation reflectance of the uncoated papers, obtained from the best fit of equation (1), were much too high for the widely accepted theory to continue to be accepted.

This finding prompted a search for data sets covering a more extended range of ink film thickness. Three such data sets were provided by different outside laboratories and are plotted in Figure 5. (See acknowledgements of these

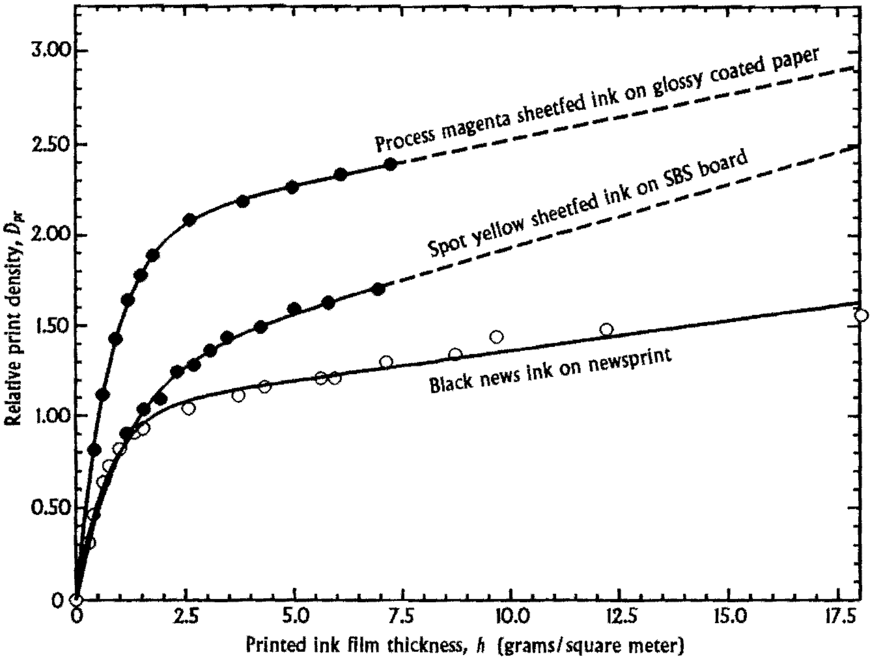

Figure 5 Typical ink mileage curves plotted over a much greater ink film thickness than normal.

contributions at end of paper.) Much to the authors' surprise, all three curves did not conform to equation (1). Rather, as can be seen, all three curves reach an inflection point at an ink film thickness in the range of 1 to 3, and then rise in a linear fashion thereafter. Based on these two inconsistencies, it was concluded that the generally accepted theory was suspect, and that additional experimental data was needed to arrive at a more acceptable explanation.

Effect of Densitomer Filter Bandwidth

The next stage of the investigation began with the preparation of prints on three different substrates: coated paper, uncoated paper, and transparent Mylar, using a traditional (low mineral oil content) magenta sheetfed ink, and an IGT printability tester. The printed ink film thickness ranged up to as high as 12 grams/square meter.

The initial analysis of these prints was directed at those on coated paper and was prompted by a prior report by another pair of investigators (Schlapfer and Keretho, 1977) that equation (1) provided a poor fit to their printability data for coated paper. The poor fit resulted because their data exhibited an initial rise that

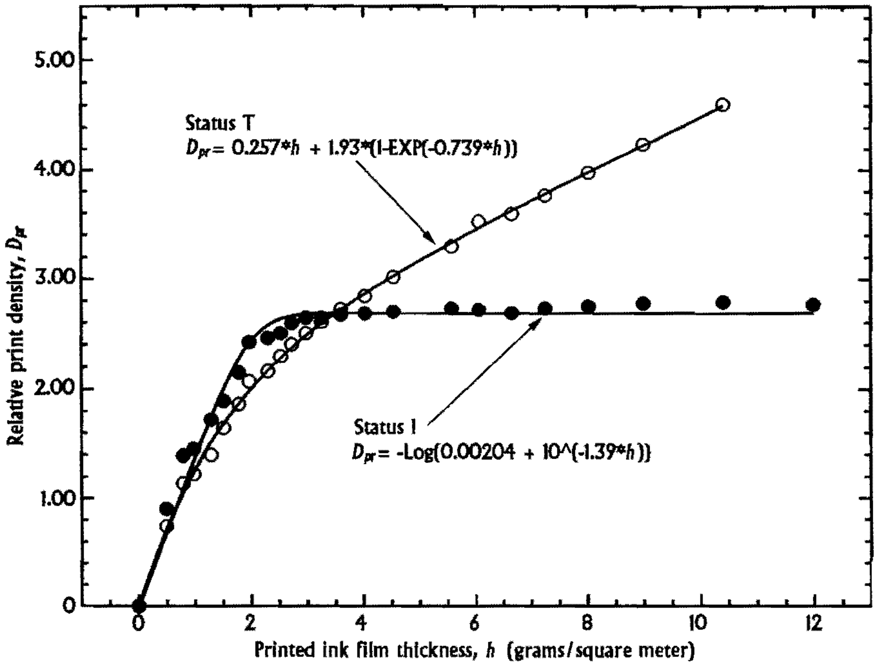

Figure 6 Two different plots of the densities of prints on coated paper showing the effect of the type of densitometer filter, Status T versus Status I, on the shape of the ink mileage curve.

was linear, whereas equation (1) predicts a rise with a constantly decreasing slope. Because this prior work was carried out in Europe, the current authors speculated that the difference in the rising part of the curves might be due to the European use of densitometers equipped with narrow bandwidth filters. To check on this, Status T and Status I density readings were taken of the new prints on coated paper. The resultant plots, versus ink film thickness, shown in Figure 6, provide proof that the shape of the curve is indeed a strong function of the bandwidth of the densitometer filter used. That is, the narrow band Status I readings produced a curve much like Schlapfer's, whereas the Status T is like those in Figure 5, which were also measured using Status T filters.

Interestingly, the prints on uncoated paper did not show similar behavior in that the Status I curve differed little from the Status T, as shown in Figure 7. This difference between the prints on coated and uncoated paper is addressed further in the section that follows.

Correlation With Kubelka Equation

A general expression for the reflection of light by a colorant layer of known absorption and scattering properties, applied to a substrate having a known

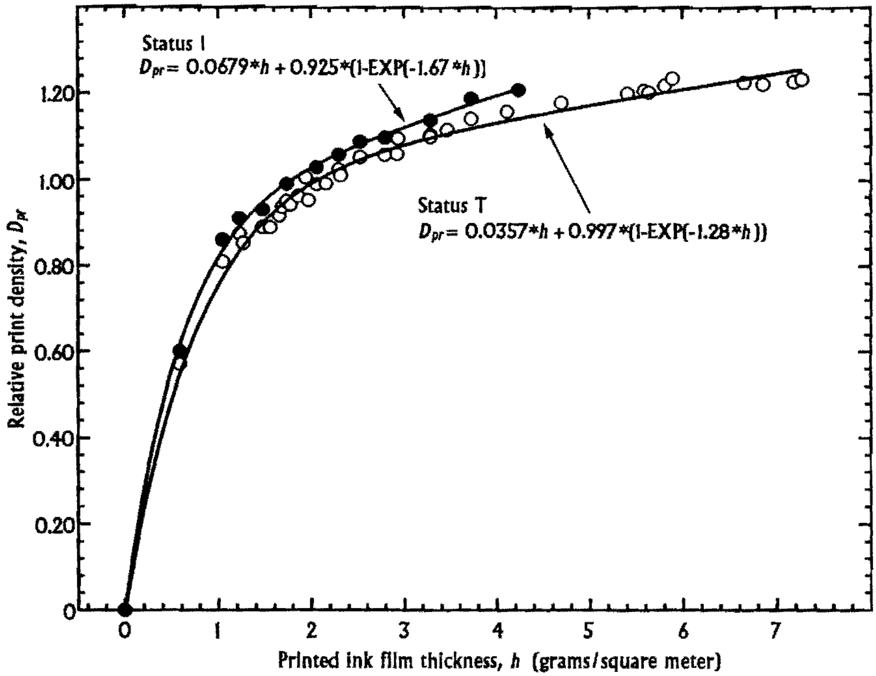

Figure 7 Two different plots of the densities of prints on uncoated paper showing the effect of the type of densitometer filter, Status T versus Status I, on the shape of the ink mileage curve.

reflectance was derived over thirty years ago (Kubelka and Monk, 1931). This model was subsequently reduced to a set of relatively straight forward equations (Kubelka, 1948). A series of calculations of Kubelka's formulae for the reflectance of a printed film, equation (4) below, and for the transmittance, equation (5) below, were carried out and the results converted to densities. The shape of typical reflection density curves thus calculated are shown in Figure 8, while Figure 9 shows a corresponding curve of transmission density versus ink film thickness.

These calculations revealed that the Status I measurements, shown in Figure 6, might correlate very well with the Kubelka-Monk model. The calculations also disclosed that for films with zero scattering, the Kulka-Monk model predicts that the initial slope of reflection density versus ink film thickness curve is twice that of transmission density versus ink film thickness. If there is scattering present, an error will be introduced, but this error will be small (less than 2.0 percent) so long as the ratio of *K/S* is greater than 50 in the case of the transmittance curve, and 2.5 in the case of the reflection curve. It was also found that the initial slopes of these curves is related to *K* in accordance with equations (6) and (7)

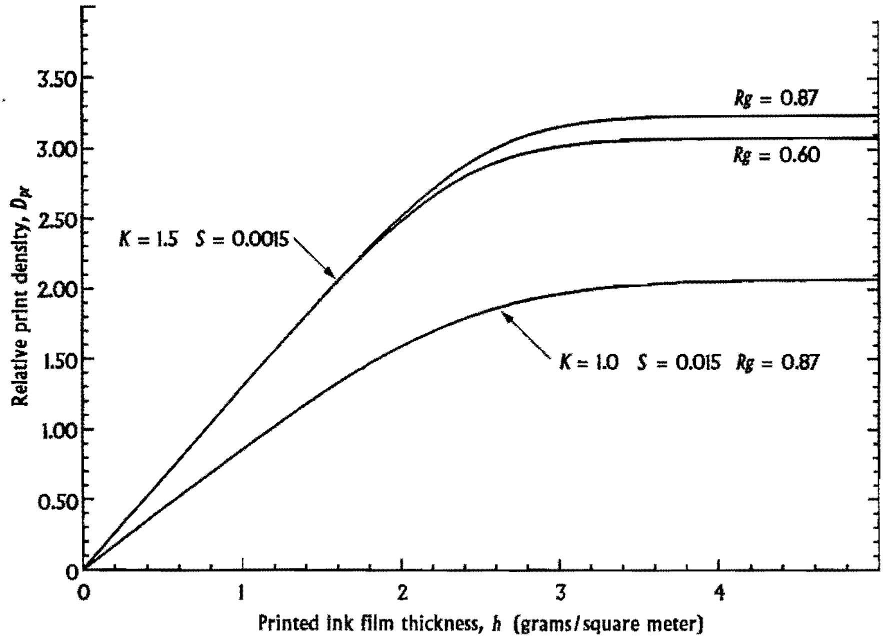

Figure 8 Plots of Kubelka's equation (4), converted to reflection density, showing shape of curve and effects of variables. Effect of variable R_g, reflectance of substrate, on saturation density is relatively small.

and that the level of the plateau of the reflection density curve (saturation density or D_{sr}) is related to *K/S* as in equation (8).

$$R = \frac{1 - Rg*[a - b*\text{ctgh}(b*S*h)]}{a - Rg + b*\text{ctgh}(b*S*h)} \quad (4)$$

$$T = b/[(a*\sinh(b*S*h) + b*\cosh(b*S*h)] \quad (5)$$

Initial slope of transmission curve = $K*0.4343$ (6)

Initial slope of reflectance curve = $K*0.8686$ (7)

$K/S = 10\text{^}(D_{sr} - 0.248)$ (8)

where

R = reflectance of colorant layer with background of reflectance R_g
R_g = reflectance of background to which layer is applied
T = transmittance of colorant layer

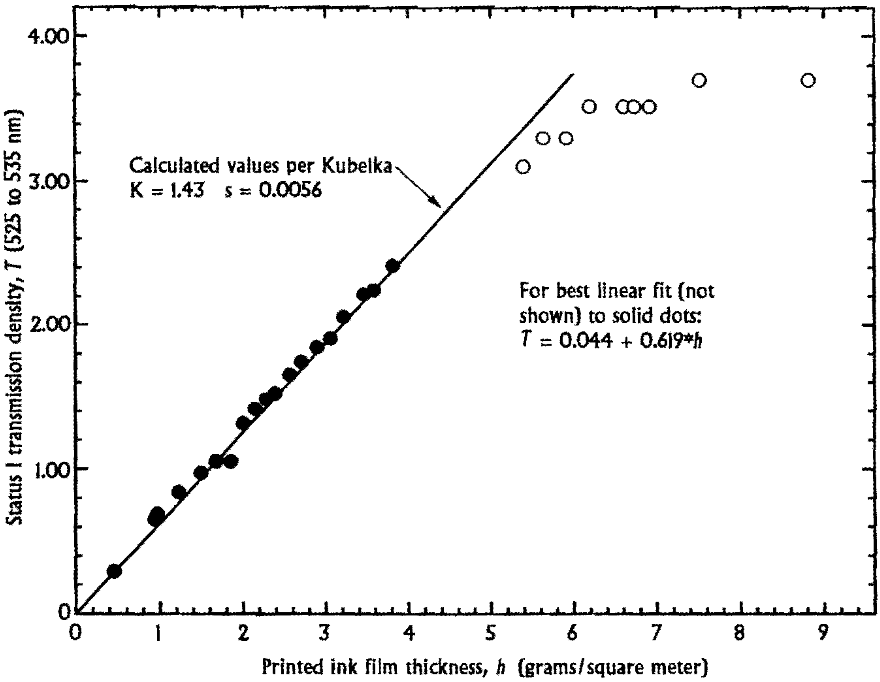

Figure 9 Plot of measured values (solid and open dots) of transmission densities of prints on Mylar made with a magenta sheetfed ink. Also shown (line) is plot of values calculated using Kubelka's equation (5).

$a = (S + K)/S$
$b = (a^2 - 1)^{1/2}$
sinh = hyperbolic sine function
cosh = hyperbolic cosine function
ctgh = hyperbolic cotangent function
h = layer (ink film) thickness
K = absorption coefficient of colorant layer, square meters/gram
S = scattering coefficient of colorant layer, square meters/gram

The calculations also brought out the fact that K in the Kubelka-Monk model differs from A in the Beer-Lambert Law due to the different bases used, i.e., the base e versus the base 10.

The fact that the ratio of the initial slopes of the reflectance and transmission density curves predicted by the model of Kubelka-Monk is a factor of two is important because it affords a way of checking if the experimental data correlates better with that model or that of Williams and Clapper, which predicts a ratio of 4.0 or more. Therefore, the initial slopes of all three plots of the

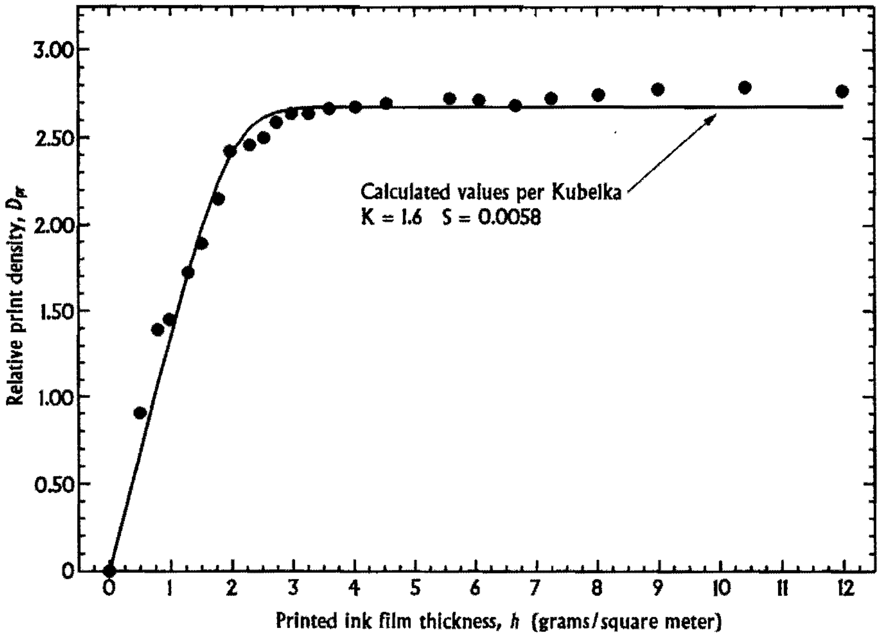

Figure 10 Fit of Kubelka's equation (4) to measured densities of prints, on coated paper, which were prepared using the same sheetfed magenta ink as in Figure 9.

measured Status I densities given in Figures 6, 7, and 9, were determined and are listed in Table I. In the case of the coated paper and Mylar prints, the ratio of the slopes is 2.25, while for the uncoated paper and Mylar the ratio is 2.6. These values were close enough to 2.0 to warrant proceeding to assume that the Kubelka-Monk model was appropriate.

Table I Initial slopes of Status I density curves and values of K extrapolated from them using equations (6) and (7). Slopes are of the curves fitted to the measured data, as shown in Figures 6, 7, and 9.

Type of curve	Substrate	Initial slope (meters^2/gram)	Calculated Value of K (meters^2/gram)
Reflection	Coated paper	1.39	1.60
Reflection	Uncoated paper	1.61	1.86
Transmission	Mylar	0.619	1.43

Accordingly, the next step was to fit Kubelka's equation (4) to the measured data for coated paper. This was done by using equations (7) and (8) to obtain estimates of *K* and *S* from the Status I data plotted in Figure 6. These values, 1.6

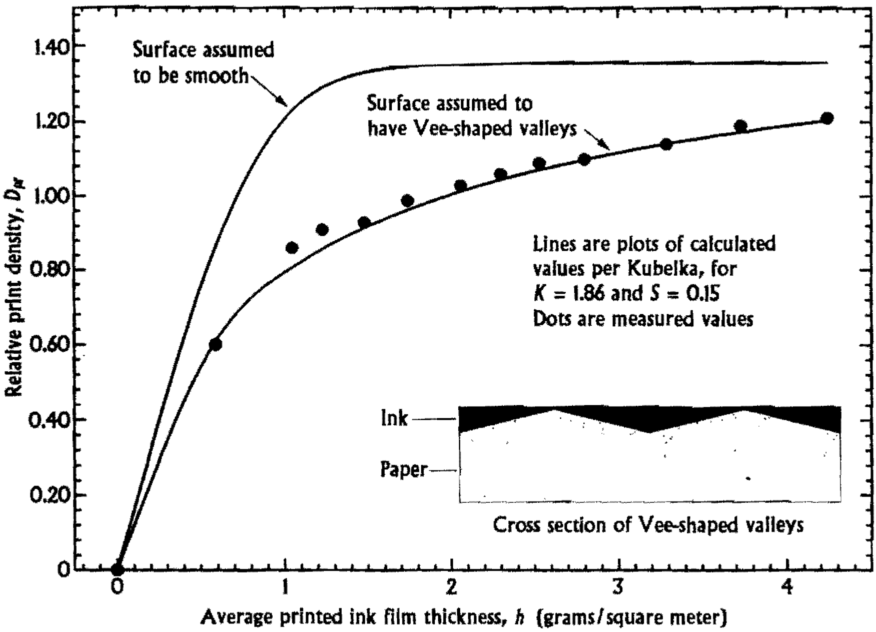

Figure 11 Fit of Kubelka's equation (4) to measured densities of prints on uncoated paper that were prepared using the same sheetfed magenta ink as in Figures 9 and 10. To achieve a good fit, it was necessary to assume that the paper surface had vee-shaped valleys in it and that, as a consequence, the lay of the ink film was uneven, as shown in the inset cross section.

and 0.0058 were then applied to equation (4) to calculate the curve shown in Figure 10. The fit to the measured data was considered good enough that further iterations were not carried out.

The Status I measured data for the uncoated paper, shown in Figure 7, presented a more difficult fitting problem for two reasons: the data does not reach a plateau, and the initial rise is not linear. One of the major differences between uncoated and coated papers is the much rougher surface of the former. With that in mind, it was theorized that the different shape of the uncoated curve might be due to uneven ink lay, due to the rougher surface. To test this theory, calculations of equation (4) were made assuming that the paper surface was in the form of parallel vee-shaped valleys. The value of K was determined from the slope of the curve in Figure 7, and the value of S was guessed. Only two trials were needed to produce the relatively good fit shown in Figure 11. To obtain this fit, however, a much larger value of S, the scattering coefficient, was necessary; 0.15 compared to 0.0058 for the coated paper. Thus, the uncoated paper interacts with the ink film in some way to significantly increase the effective value of *S*, the scattering coefficient that appears in Kubelka'a equation.

Summary and Conclusions

The findings of this study, along with the conclusions drawn from them by the authors, can be summarized as follows:

1. The shape of the curve of density versus ink film thickness obtained with current inks does not conform to the Tollenaar and Ernst equation, which dates from 1961 and which has been widely used by the industry. Furthermore, the character of the curve has two different forms, depending on whether a Status T (wide band) or a Status I (narrow band) densitometer is used to measure density. Thus, using Status T filters to read density, the curve rises with a decreasing slope until an inflection point is reached at an ink film thickness in the range of 1 – 3 grams/square meter. Beyond the inflection point, the curve then rises at a constant slope. In contrast, when using Status I filters, the curve rises linearly until the inflection point is reached. Beyond the inflection point, density reaches a plateau where no further rise occurs. In either case, the Tollenaar-Ernst equation provides a poor fit to the measured data. Nevertheless, the Tollenaar-Ernst equation is still useful for characterizing Status T data taken over the ranges of ink film thickness normally used in practice. This is illustrated by the good fits of it in Figure 1, to the same but truncated data for coated paper and newsprint that is plotted in Figure 5.

2. The widely accepted theory, as set forth in Yule's book for explaining the shape of the ink mileage curve, could not be substantiated. The primary reason for this is that this theory, which relies on the model of Williams and Clapper, predicts that the initial slope of the reflection density curve of a given ink will be much larger than twice the slope of the transmission density curve of the same ink, whereas in actuality the ratio of slopes is 2.0. It is thought that the failure of the Williams and Clapper model is due to the fact that it does not take into account the scattering of light by ink.

3. In contrast, the model of Kubelka-Monk predicts that the ratio of initial slopes will be 2.0. Furthermore, this model, in the form of Kubelka's equations, provides good correlation with Status I curves of density versus ink film thickness, as shown in Figures 9, 10, and 11. This is in spite of the fact that this model ignores both internal and external surface reflections. It is thought that the reason for this good agreement is that surface reflection can be looked upon as a type of scattering process, and therefore can be lumped together with scattering by ink into the term *S* that appears in the Kubelka-Monk equations.

4. If the model of Kubelka-Monk is accepted as being valid, then both the shape of the (Status I) reflection density curve and its dependence on ink and paper properties can be readily explained. For substrates with relatively smooth surfaces such as coated papers, i.e., R_a less than 2 microns (MacPhee and Lind, 1994), the curve consists of an initial linearly rising section that eventually

reaches a plateau, as shown in Figure 8. The slope of the rising part is equal to (K/0.4343), while the level of the plateau is dependent on the ratio K/S as in equation (8), where K and S are the absorption and scattering coefficients in Kubelka'a equation. The curve reaches a plateau at a level where the magnitude of the light scattered by the paper and seen by the densitometer is very small compared to the light scattered in other ways, e.g., by the ink, that is also seen by the densitometer. Thus, increasing the strength of the ink, and hence K, will increase the initial slope of the curve. If this change is made by increasing pigment concentration, the change in S will most likely be proportionate. Therefore, no change would be expected in the level of the plateau. In the case of uncoated paper, the same ink produces a much lower plateau and a more rounded curve. To obtain a good correlation with Kubelka's equation, it is necessary to assume uneven ink lay due to the rougher surface and to greatly increase the scattering coefficient, from 0.0056 to 0.15, a factor of 27. With regard to the latter, it is not known to what relative extent the rougher surface and the more open structure of the uncoated paper have on the need for this increase.

5. In their summary of the Kubelka-Monk model, Judd and Wyszecki point out that its most important restriction is that it applies to only one wavelength at a time. It is believed that this is the reason why this model correlated so well with the Status I data and not with the Status T, i.e., because the Status I readings more closely approach monochromatic conditions. (The poor correlation with Status T also held for the transmission densities, although this data was not included in the paper.)

Instruments Used

The density data plotted in Figures 1 and 5 were measured with an XRite densitometer equipped with Status T filters. The data plotted in Figures 6, 7, 10, and 11 were measured with an XRite spectrodensitometer. The data in Figure 9 were measured as reflectances using a Model 600 Plus Data Color Spectrophotometer. The reflectances at 530nm were then converted to density.

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