# Frequency Modulated Halftoning and Dot Gain

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Abstract: A printed image often appears darker than the original due to physical and optical dot gain. Normally, the original is compensated for dot gain prior to halftoning using either an experimental dot gain curve or a dot gain model. Because of the nonlinearity of dot-gain with respect to the nominal dot percentage, the compensation may cause loss of details in the halftoned image, affecting the print quality negatively. In this presentation, we make comparisons between three dot-gain models, the Yule-Nielsen, dot overlap, and the unified dot-gain model proposed very recently. The impact of dot-gain compensation on halftoning is also studied by comparing two types of halftoning methods, the ordinary Error Diffusion and the dot-location-optimization method with consideration of edge enhancement. Applications show that the latter provides us with a fairly good tone reproduction while preserving the details.

## Introduction

Halftoning is without any doubt one of the most important parts of printing. The characteristics of the halftoning method have a great impact on the quality of the final image. There are two main types of halftoning methods, namely AM (Amplitude Modulated) and FM (Frequency Modulated). In the AM techniques the size of the dots is variable while their spacing is constant. The single dot within the halftone cell grows larger as the tone value becomes darker and smaller as the tone value becomes lighter. In the FM technique, contrary to the AM, the size of the dots is constant while their spacing varies. Both methods have their advantages and drawbacks. For example, the FM methods are generally better in reproducing the details while they suffer more from dot gain.

There are two types of dot gain, physical and optical. Physical dot gain is resulted from the physical dot extension, causing the printed dots physically bigger than their correspondences in the original digital image. The optical dot gain, on the other hand, is caused by the diffusion of light in substrate, which makes the dots appear bigger than their physical size. Consequently, the printed image appears

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darker. The effect of dot gain, both physical and optical, should therefore be taken into account one way or another. One of the most straight forward ways of doing that is to compensate the original image for dot gain prior to halftoning. That is, instead of halftoning the original image, the compensated image, which is brighter than the original, is halftoned. In order to do the compensation either an experimental dot gain curve or a reliable dot gain model is needed. In this paper we are going to test three dot gain models and investigate their accuracy representing the measured data. We will also show how the dot gain compensation of the original image can affect the final result by comparing two different FM halftoning methods, namely ordinary Error-Diffusion method and the dot-location-optimization method. Finally, we show how a simple modification of the latter FM method can increase the print quality.

## Measurements

To obtain the experimental dot gain curve, we measured 15 halftone patches of nominal (commanded) dot coverages, $\sigma_0$ = 0, 2, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 98, and 100%. These patches were halftoned using the FM method presented later in this paper and printed out at 300 *dpi* by a laser printer (*HP LaserJet 4050N*). The spectra of the patches were then measured, using a spectrophotometer (*Gretag Macbeth$^{TM}$ Spectrolino*) covering a spectral range of *380* through *730 nm* at a step of *10 nm*. The *CIEXYZ* tristimulus values were computed from the spectra. Figure 2 depicts the variation of the *Y*-values with respect to the nominal dot-percentages (dashed-dotted line).

From the experimental spectra, *R*, one can estimate the effective dot-percentage, $\sigma_{eff}$, using the Murray-Davies Equation (Murray, 1936), i.e.,

$$\sigma_{eff} = \frac{R_p - R}{R_p - R_i} \qquad (1)$$

where $R_p$, $R_i$ are the spectral reflectance values of the bare paper and the print-solid, respectively, while *R* is the spectral reflectance values of the halftone patches. The effective dot-percentage can also be determined from the experimental tristimulus values, for instance the *Y*-stimulus, according to the Neugebauer Equations (Neugebauer, 1931). In this case, Equation (1) is replaced by

$$\sigma_{eff} = \frac{Y_p - Y}{Y_p - Y_i} \qquad (2)$$

where $Y_i$, $Y_p$ and *Y* denote the *Y*-values of the full tone print, the bare paper, and the halftone patches, respectively.

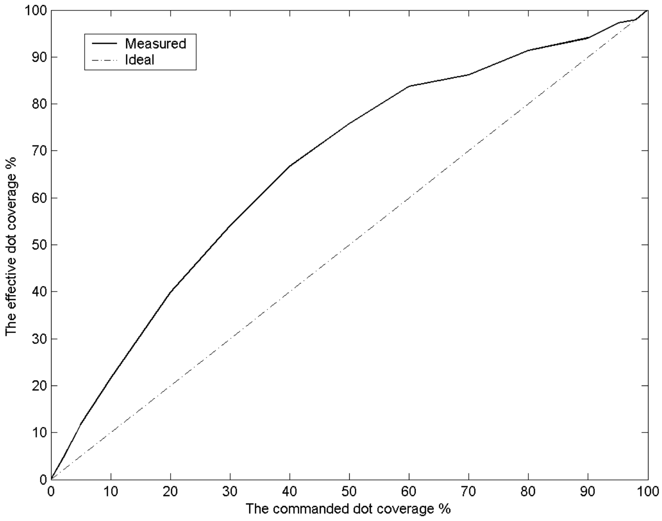


*Figure 1: The effective dot coverage vs. the commanded dot coverage. The ideal case is when there is no dot gain (the straight line).*

Since the $Y$-values are computed from the spectra, the estimations according to Eqs. (1) and (2) are identical when there exists no optical dot gain or the optical dot gain is spectral independent (Yang 2004), for example, black-white. Otherwise, applications of these two equations may result in somewhat different estimations of $\sigma_{eff}$. In the present study (gray), we compared the results computed from Eqs. (1) and (2) and noticed no considerable differences between them. Therefore, Equation (2) was used throughout the study. The correlation between the effective dot-percentage, $\sigma_{eff}$, and the nominal dot-percentage, $\sigma_0$, is shown in Figure 1.

## Dot Gain Models

**Yule-Nielsen Model**

The simple model of Murray-Davies is actually useful when optical dot gain is negligible and the fractional area is the real physical dot size in print. When we use the measured data in the Murray-Davies equation, as we did in Equation 2, the effective dot coverage does not actually correspond to the real physical dot size because in our measurement optical dot gain was also included. However, if in Equation (2) $\sigma_{eff}$ is replaced by the commanded dot coverage $\sigma_0$ then the result will be a line connecting the $Y$-values of the paper and the full tone ink, see Figure 2. Yule and Nielsen presented a modified version of Murray-Davies model to

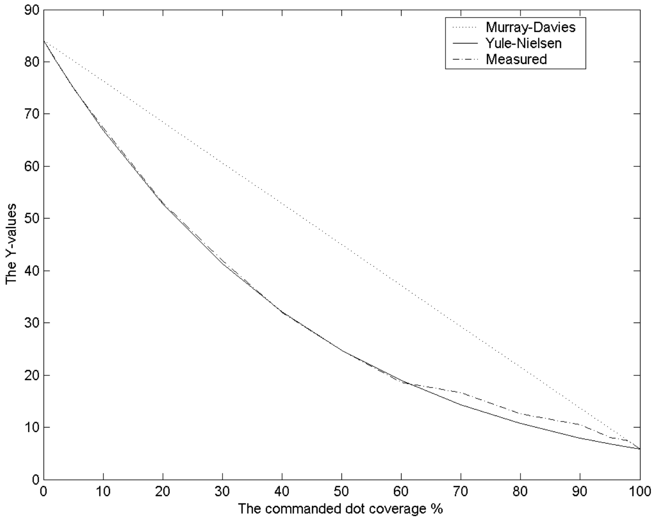


*Figure 2: The Y-values. The measured Y-values, Murray-Davies model and Yule-Nielsen Model with n=7.91.*

approximate optical dot gain. The Yule-Nielsen model is defined as (Yule, 1951),

$$R^{\frac{1}{n}} = \sigma R_i^{\frac{1}{n}} + (1-\sigma) R_p^{\frac{1}{n}} \qquad (3)$$

As mentioned, this equation is actually supposed to model the optical dot gain and therefore $\sigma$ should be the real physical dot coverage after print. The factor $n$ is a fitting factor and its real physical meaning is not so clear. Although Equation (3) is actually supposed to only model the optical dot gain, it is sometimes used to model both physical and optical dot gain. In this case $\sigma$ is the commanded dot coverage $\sigma_0$. By using the least squares error method and our measured spectra for $R_i$ and $R_p$ we found the factor $n$ that makes the best match between $R$ in Equation (3) and the measured spectra as $n$ = 7.91. The $Y$-values using the Yule-Nielsen model with $n$ equal to 7.91 is plotted in Figure 2. The $Y$-values for the measured data and the $Y$ values according to Murray-Davies model are also plotted in this figure. As can be seen the Yule-Nielsen model represents the measurement data very well for smaller commanded dot coverages but not that well for coverages bigger than 60%. Something that has to be noticed is that because of the non-linearity of Equation (3) you cannot simply replace the $R$'s in this equation by their corresponding $Y$'s, which was possible in Murray-Davies model. However, this is sometimes done and the resulting equation is called the modified

Neugebauer's equation which is shown in Equation (4).

$$Y^{\frac{1}{n}} = \sigma Y_i^{\frac{1}{n}} + (1-\sigma) Y_p^{\frac{1}{n}} \tag{4}$$

We used the least squares error method again for Equation 4 and found the best $n$ to fit the measured $Y$-values as $n = 7.52$. However, if $n = 7.91$ (the best $n$ for Equation (3)) is used in Equation (4) the relative error is less than 1.5%, which shows that although Equation (4) is not correctly derived from Equation (3) it can still be used with good acceptance, at least in this case.

**Dot Overlap Model**

The first two assumptions in this model are that printers produce ragged round rather than square black dots and printed dots are larger than the minimal covering size. The dot overlap model was proposed, discussed and used in a number of papers (Roetling, 1979), (Pappas, 1991), (Lin, 1994), (Stevenson, 1985). This model is specially useful when the effect of dot gain is taken into account within the halftoning process. It has been shown that when this model is used within the halftoning process the final image is of higher quality than the case where the effect of dot gain is taken into account prior to the halftoning process (Baqai, 1999). Here we give a brief description of Dot Overlap Model and study how this model can be used to represent our measurements.

The model is as follows,

$$p(i,j) = P(W(i,j)) = \begin{cases} 1 & \text{if } b(i,j) = 1 \\ f_1\alpha + f_2\beta - f_3\gamma & \text{if } b(i,j) = 0 \end{cases} \tag{5}$$

where $b$ denotes the halftoned image and $p$ denotes the image after applying the dot-overlap model to $b$. $W(i, j)$ consists of $b(i, j)$ and its eight neighbours, $f_1$ is the number of horizontally and vertically neighbouring dots that are black, $f_2$ is the number of diagonally neighbouring dots that are black and not adjacent to any horizontally or vertically neighbouring black dot, and $f_3$ is the number of pairs of neighbouring black dots in which one is a horizontal and the other is a vertical neighbour. $\alpha$, $\beta$ and $\gamma$ are the ratios of the areas of the shaded regions shown in Figure 3 to $T^2$, the area of each square. The parameters $\alpha$, $\beta$ and $\gamma$ can be expressed as follows:

$$\alpha = \frac{1}{4}\sqrt{2\rho^2 - 1} + \frac{\rho^2}{2}\operatorname{asin}\left(\frac{1}{\rho\sqrt{2}}\right) - \frac{1}{2} \tag{6}$$

$$\beta = \frac{\pi\rho^2}{8} - \frac{\rho^2}{2}\operatorname{asin}\left(\frac{1}{\rho\sqrt{2}}\right) - \frac{1}{4}\sqrt{2\rho^2 - 1} + \frac{1}{4} \tag{7}$$

$$\gamma = \frac{\rho^2}{2}\operatorname{asin}\left(\sqrt{\frac{\rho^2 - 1}{\rho^2}}\right) - \frac{1}{2}\sqrt{\rho^2 - 1} - \beta \tag{8}$$

where $\rho$ is the ratio of the actual dot radius to the ideal dot radius $\frac{T}{\sqrt{2}}$.

This model is actually supposed to represent the physical dot gain but still can be used to model both physical and optical dot gain. We found that $\rho = 1.1$ gives the

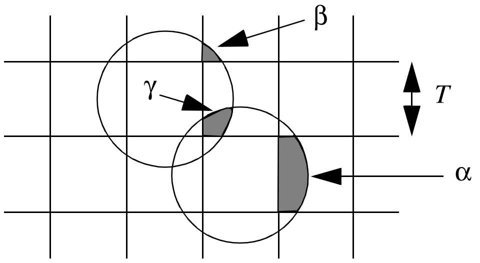


*Figure 3: Definition of* $\alpha$, $\beta$ *and* $\gamma$.

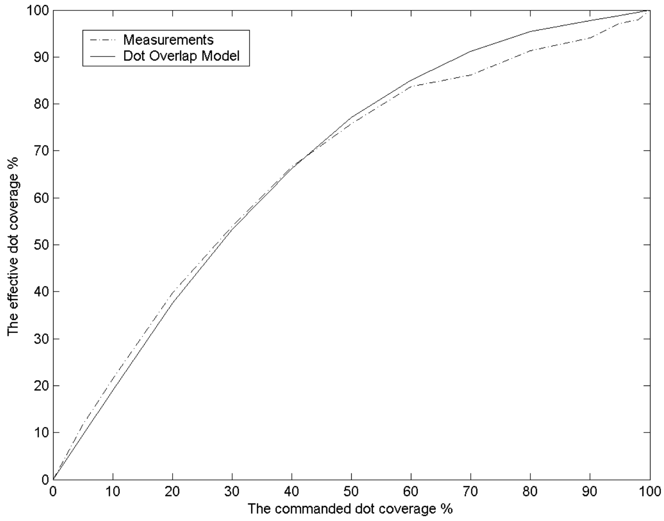


*Figure 4: The effective dot coverages vs. the commanded dot coverages using the dot overlap model for* $\rho$*=1.1. The measured effective dot coverages are also shown.*

best match to the measured effective dot coverages (shown in Figure 1) using the least squares error method. In Figure 4 the effective dot coverages approximated by the dot overlap model for $\rho = 1.1$ is shown together with the measured effective dot coverages, already shown in Figure 1.

**The unified dot gain model**

In (Yang, 2004) the author presents a model for physical dot gain. He proposes that the physical dot gain can be approximated by the following equation,

$$\Delta\sigma = \sigma - \sigma_0 = (a-1)\sigma_0(1-\sigma_0) \tag{9}$$

where $\Delta\sigma$ denotes the physical dot gain, $\sigma$ denotes the real physical dot coverage and $\sigma_0$ the commanded dot coverage. As can be seen this model is parameterized by a single parameter $a$, which depends on the printing technology, printing materials (ink and substrate) and etc. The constraint $\Delta\sigma = 0$ for $\sigma_0$=0 and 1 are automatically fulfilled in Equation (9). As will be described later in this section the parameter $a$ can be determined by using the measured data. The spectral reflectance of the print can be approximated by the following equation (Yang, 2001),

$$\begin{aligned} R(\sigma) &= R_{MD}(\sigma) - \Delta R_{opt}(\sigma) \\ &= R_{MD}(\sigma_0) - \Delta R_{phy}(\sigma_0) - \Delta R_{opt}(\sigma) \end{aligned} \tag{10}$$

where,

$$R_{MD}(\sigma_0) = R_p(1-\sigma_0) + R_p T^2 \sigma_0 \tag{11}$$

is the term computed using the Murray-Davies model. $R_p$ and $T$ in Equation 11 refer to the spectral reflectance and transmittance of the bare paper and the full tone ink, respectively. The contributions from physical and optical dot gain, $\Delta R_{phy}$ and $\Delta R_{opt}$ in Equation 10 are expressed as (Yang, 2004),

$$\Delta R_{phy}(\sigma_0) = R_p(1-T^2)(a-1)\sigma_0(1-\sigma_0) \tag{12}$$

$$\Delta R_{opt}(\sigma) = R_p(1-T)^2\sigma(1-\sigma) \tag{13}$$

The transmittance $T$ can be estimated by,

$$T = \left(\frac{R_i}{R_p}\right)^{1/2} \tag{14}$$

where $R_i$ and $R_p$ are the spectral reflectance values of the full tone ink and the bare paper, respectively.
By using the least squares error method we found $a$ = 1.49 as the best $a$ for the model to fit the measured data. The $Y$-values using this model with $a$ = 1.49 is plotted in Figure 5. As can be seen the $Y$-values fit very well the measured $Y$-values.

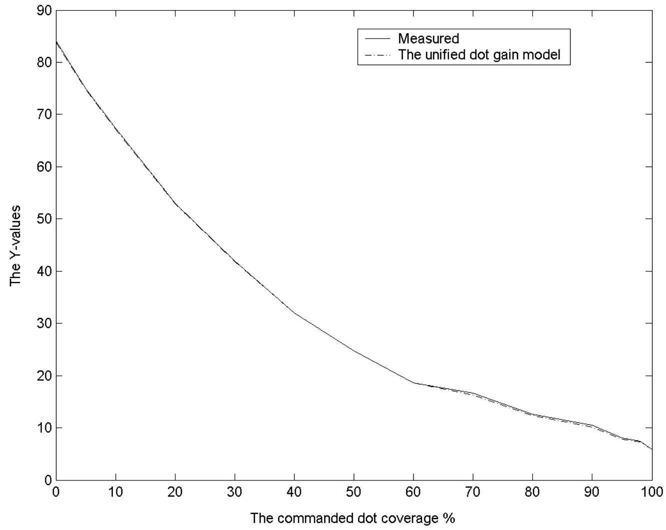


*Figure 5: The Y-values. The measured Y-values and the Y-values using the unified dot gain model with a=1.49.*

**Summary of this section**
In this section we have investigated three dot gain models. In Figure 6 the effective dot coverage for the measured data, Yule-Nielsen model, dot-overlap model and the unified dot gain model are shown. As can be seen the unified dot gain model represents the measured data more accurately than the other two models.

## Halftoning and Compensation

In this section we are going to show how the dot gain compensation impacts the final result. Before discussing the compensation and its effect on the final image we give a brief description of the FM halftoning method that is used in our experiments.

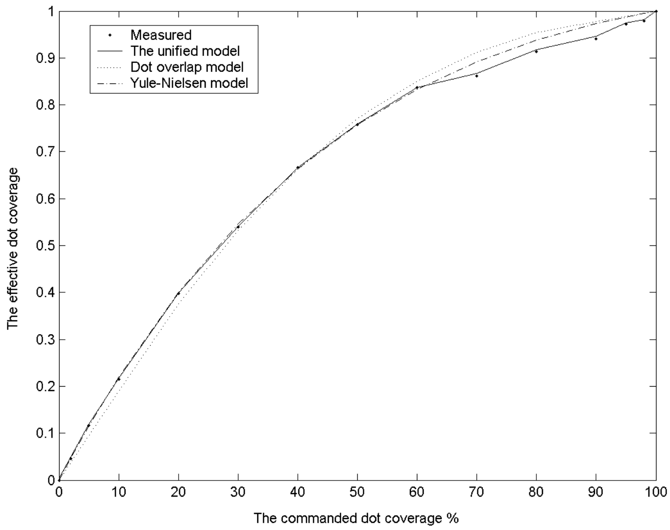


*Figure 6: The effective dot coverage vs. the commanded dot coverage. The measured data, Yule-Nielsen model, Dot overlap model and the unified dot gain model.*

**Dot-Location-Optimization FM Halftoning Method**

The FM method used in this paper has already been described in a number of publications (Gooran 2001), (Gooran, 2004). In this section it is briefly described.

The FM method is based on a successive assessment of the near optimum sequence of positions to render a halftone dot. The impact of each rendered position is then fed back to the process by a distribution function, thereby influencing subsequent evaluations. This distribution function plays a significant role in the placement of dots. We want the dots in the halftoned image to be placed as far apart as possible provided the halftoned image fulfils some conditions that connect its appearance to that of the original continuous-tone image. The problem of halftoning is the problem of placing a certain number of dots on a blank page so that the result "resembles" the original continuous-tone image. In this algorithm we begin with placing a dot at the position where the original image is darkest. Since we assume that "1" and "0" represent black and white respectively, finding the position of the darkest pixel means finding the position of the pixel that holds the largest value. This is exactly what this algorithm does at the first step, i.e. the algorithm finds the position of the largest pixel value (or the maximum) in the continuous-tone image. Then it places a dot at the same position in the binary image, which is totally white to begin with. The currently placed dot is then represented by a distribution function (filter) that affects a neighboring region of the position of the maximum in the original image. After that, the algorithm finds the position of the next maximum and place the next dot at the corresponding position in the binary image. The same feedback process is performed again for that position. The algorithm is terminated when the difference between the mean value of the original and the halftoned image is minimized. As mentioned, the filter used within the algorithm plays a significant role in the appearance of the final image. By using a filter with an appropriate size for different gray-tone regions the dots can be placed homogeneously over the entire image (Gooran, 2004).

**Dot Gain Compensation**

Since dot gain makes every print appear darker than the original its effect has to be taken into consideration one way or another. The most straight forward way of doing that is to compensate the original image for dot gain prior to the halftoning process. The compensation can therefore be done by using the dot gain curve, or similarly the curve shown in Figure 1. As described earlier the dot gain curve is obtained by measuring a number of samples. If for example we want to have 20% coverage in print the commanded dot coverage should be about 9%, see the curve in Figure 1. The coverages that are not among the measured samples can easily be found by interpolation. In this paper we use the linear interpolation. To compensate a real image the pixel-value of every pixel in the original image is

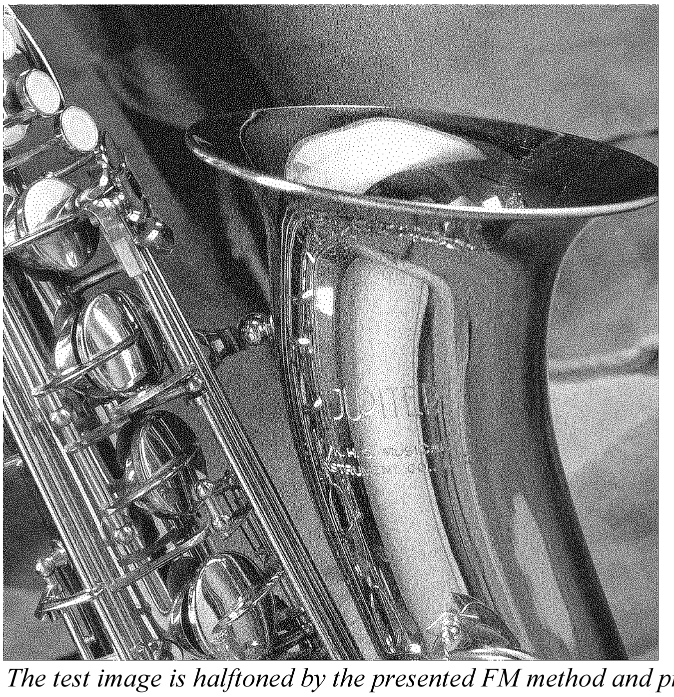

*Figure 7: The test image is halftoned by the presented FM method and printed at 300 dpi. The original image is not compensated for dot gain.*

transformed to a new value using the curve. Figure 7 shows the test image being halftoned by the presented FM method and printed at 300 *dpi*. The original image is not compensated for dot gain. As can be seen the image is slightly dark because of dot gain. Now we first compensate the original image for dot gain using the curve in Figure 1 and then halftone the compensated image using the same FM method as before. This halftoned image printed at 300 *dpi* is shown in Figure 8. The gray tones are now reproduced more correctly compared to the image shown in Figure 7. However, due to the fact that the compensation process is non-linear some details are not perceived as well as they were in Figure 7. How the details are reproduced depends very much upon the halftoning method. By studying the curve in Figure 1 we can see that a gray tone difference at 10% becomes about 20% after compensation of the darker parts of the image. The same difference of 10% becomes about 5% in the lighter parts of the image. Depending on the characteristics of the halftoning method some details can be lost when the compensated image is halftoned. In our example you can see that the small texts in the middle of the saxophone under "JUPITER" are perceived better in Figure 7. To increase the quality the halftoning method should take care of this in one way or another. In the proposed FM halftoning method we can sharpen the result by changing a filter used in the algorithm (Gooran 2004).

*Figure 8: The compensated test image is halftoned by the presented FM method and printed at 300 dpi. The original image is compensated for dot gain by using the curve in Figure 1.*

Figure 9 shows the result after halftoning the compensated test image by the presented FM halftoning method with another filter. The original image is compensated as before. As can be seen the details are perceived better in this image compared to the image shown in Figure 8, see for example the small texts under “JUPITER”.
Something worth mentioning here is that the compensation has been done for our measured data and therefore for best reproduction these images should be printed out by the same printer for which the measurements were carried out. To show more clearly how the dot gain compensation can affect the details of the image we have done another experiment. We measured the dot gain curve for the non-modified Error Diffusion method precisely as we did for the presented FM method. Figure 10 shows the original image being halftoned by Error Diffusion. In Figure 11 the image is first compensated for dot gain and then halftoned by Error Diffusion. Both images are printed at 300 *dpi*. It is very clear that some details are lost in the latter image due to the dot gain compensation, see especially the texts. This shows again the important role the halftoning methods play in the quality of the final printed image. As mentioned earlier using the dot gain model within the halftoning process, if the halftoning method allows for that, could be one solution (Baqai, 1994).

*Figure 9: The test image is halftoned by the presented FM method with another filter and printed at 300 dpi. The original image is compensated for dot gain by using the curve in Figure 1.*

## Summary and discussions

In this paper three different dot gain models have been briefly described and investigated. To see how the models work a number of measurements have been carried out. All measurements have been carried out for a laser printer (*HP LaserJet 4050N*) and the presented FM halftoning method. The patches were printed at 300 *dpi*. It has been shown that all three models work quite well but the unified method models the dot gain more accurately.

To show the dot gain compensation's impact on the final image we showed a number of examples. From the examples one could see that some details of the original might be lost when the compensated image is halftoned. We also showed how a simple modification of the presented FM halftoning method can increase the print quality. To show the impact of the dot gain compensation more clearly we also measured the dot gain curve for non-modified Error Diffusion. The test image being halftoned by the non-modified Error Diffusion before and after dot gain compensation have also been shown. From this example it is more evident that some details are lost due to the dot gain compensation.


*Figure 10: The test image is halftoned by the non-modified Error Diffusion method and printed at 300 dpi. The original image is not compensated for dot gain.*

*Figure 11: The compensated test image is halftoned by the non-modified Error Diffusion method and printed at 300 dpi. The original image is compensated for dot gain.*

The importance of the FM halftoning method is evident. A good FM halftoning technique should be able to take care of the problems caused by the dot gain compensation. One of the possible ways of doing that is to take into account the effect of dot gain within the halftoning process by using a proper model for dot gain. The presented halftoning method actually allows for such a modification. This possibility will be investigated and tested in a near future.

## References

F. A. Baqai, C. C. Taylor and J. P. Allebach
1994 "Halftoning via Direct Binary Search using a Hard Circular Dot Overlap Model," IS&T Recent Progress in Digital Halftoning II, 1999, pp. 199-207.

S. Gooran
2001 "High Quality Frequency Modulated Halftoning," Ph.D. Thesis No. 668, Linköping Studies in Science and Technology, Jan. 2001, ISBN 91-7219-913-X.

2004 “Dependent Color Halftoning, Better Quality with Less Amount of Ink,” JIST: Journal of Imaging Science and Technology, 2004, accepted to be published.

Q. Lin
1994 “Improving halftone uniformity and tonal response,” IS&T Tenth Congress on Advances in Non-Impact Printing Technologies, Louisiana, Oct. 1994, pp. 377-380.

A. Murray
1936 “Monochrome reproduction in photoengraving,” J. Franklin Institute (Philadelphia), 221, 1936, p. 721.

H. Neugebauer
1931 “Die theoretischen Grundlagen des Mehrfarbenuchdrucks, Z. tech. Phys. 36, 1931, pp. 75-89

T. N. Pappas and D. L. Neuhoff
1991 “Model-based halftoning,” IS&T / SPIE Symposium on Electronic Imaging: Science and Technology, San Jose, Feb. 1991.

P. G. Roetling and T. M. Holladay
1979 “Tone reproduction and screen design for pictorial electrographic printing,” Journal of Applied Photographic Engineering, 5(4), 1979, pp. 179-182.

R. L. Stevenson and G. R. Arce
1985 “Binary Display of hexagonally sampled continuous-tone images,” Journal of the optical Society of America A, 2(7), 1985, pp. 1009-1013.

L. Yang, S. Gooran and B. Kruse
2001 “Simulation of optical dot gain in multi chromatic tone reproduction,” JIST: Journal of Imaging Science and Technology, 45: 198, 2001.

L. Yang
2004 “A Unified Model of Optical and Physical Dot Gain in Color Reproduction of Printing,” JIST: Journal of Imaging Science and Technology, 2004, accepted to be published.

J.A.C. Yule and W.J. Nielsen
1951 “The penetration of light into paper and its effect on halftone reproduction,” Tech. Assoc. Graphic Proc. 3, 1951, pp. 65-67.