A Regression-Based Model of Colorimetric Tone Reproduction for Use in Print Standards

William B. Birkett* and Charles Spontelli**

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Abstract: In previous work on improving print standards, we recommended that a mathematical model of "ideal" tone reproduction (TR) be incorporated into future print standards. (Birkett and Spontelli, 2004) This paper describes our efforts to create such a model.

Our work builds on that of Murray-Davies (Murray, 1936) and Yule-Neilsen (Yule and Neilsen, 1951). They modeled densitometric measurements of black and white halftone ink ramps. Our work models colorimetric measurements of color multi-ink ramps. The model has many practical applications, including process characterization, pressroom calibration, and color matching.

We explain the development of the model, beginning with multi-ink ramps. Colorimetric xyz measurements are a non-linear function of the ink ramp parameter. A simple transform borrowed from the xyz to Lab calculation makes the functions nearly linear for many data sets. The transformed data is then modeled using a Hermite spline. The Hermite spline has four parameters, which correspond directly to essential print characteristics, and are easy to visualize.

To validate our work, we developed a computer algorithm for fitting the model to real data. We've measured the quality of fit for an assortment of publicly available characterization data. We recommend this model for use in future industry print standards.

*Doppelganger, LLC, www.doplganger.com
**Bowling Green State University, Visual Communication Technology

Introduction

Numerous models have been proposed to describe the halftone printing process. An excellent review of popular "models for halftone color reproduction" from 1936 to 1999 can be found in a paper on the subject. (Wyble and Berns, 2000) The various models fall into two primary categories, first-principal models or regression based models. First-principal models "attempt to simulate the physical process." (Wyble and Berns, 2000) These interpretations of halftone printing are attempting to answer fundamental questions about the physical behavior of the process. They are used to test theories about the physical processes involved.

Ours is a regression-based model. We are not overly concerned with describing these physical processes. For the purpose of providing enhanced print standards, it is only necessary that our model accurately represent the color halftone printing process. We are simply modeling the colorimetric output of the color halftone printing process as a function of percent dot input.

Our research builds on the work of Murray-Davies (Murray, 1936) and Yule-Neilsen (Yule and Neilsen, 1951). They studied the tone reproduction of black-and-white halftone printing, using visual reflectance measurements. Beginning in 1936, the Murray-Davies first-principals model (Murray, 1936) described the relationships between input (percent dot) and printed output (density). The Murray-Davies model is based on the assumption that the reflectance of a tone scale decreases linearly from the white point to the black point. This model is accurate for very coarse screen rulings, but not for the finer screen rulings currently used in offset printing. Measured tone values are darker (lower reflectance) than predicted by Murray-Davies. The extent to which they are darker is referred to as dot-gain or "tone value increase" (TVI). A later model proposed by Yule-Neilsen (Yule and Neilsen, 1951) has an additional parameter called the n-value, which better fits the non-linear behavior of real printing.

We extend the work of Murray-Davies and Yule-Neilsen in two important ways. First, these earlier studies only modeled output as a single reflectance value. We model the colorimetric x, y, z, reflectance output of the color halftone printing process. Second, the previously mentioned work measures the percent-dot output of a single ink ramp. We measure the output of individual C, M, Y, K ramps and a multi-ink "isometric" color ramp. The "isometric" color ramp is composed of overprinting equal values of cyan (C), magenta (M), and yellow (Y). This quantifies important C, M, and Y overprint characteristics, critical to modeling the gray balance of halftone color reproduction.

Objectives

The major objective of this study is to create a mathematical model of colorimetric tone reproduction (CTR) that would be useful in advancing print standards. CTR is determined by measuring the colorimetric reflectance (xyz) output of a printed color ramp as a function of CMYK input tone values (dot-percent). This model should conform closely to existing practice (real printing with currently employed materials, equipment, and methods). It should have a high correlation to physical observation of real printing (e.g. TR001). The model must be a smooth function to "iron-out" any "bumps" or inconsistencies introduced by the production process.

The model's complexity must be sufficient to capture the essential qualities of the process with minimal information and processing overhead. The model should be formula based. This will permit "vectorization" – using a formula for computing colorimetric output values for any tone input.

To enhance its understanding and increase the likelihood of its adoption by printers, the model should also be familiar. The model should generate CTR curves with recognizable attributes. This colorimetrically based model is intended to support our call for recommended improvements to print standards. (Birkett and Spontelli, 2004)

Assumptions

With the application of tone reproduction (TR) curves in the CtP process, just about any type of printing tone reproduction is possible. Trying to model the myriad of approaches currently used to calibrate printing systems would be futile. In fact, one of our major objectives – advancing print standards – is to reduce the industry's use of these "home-grown" printing references. Our model has been used to match many different device characterizations (TR001, TR004, etc.) However, we recommend it be used to fit the colorimetric tone reproduction of "natural printing" (printing with no TR curves applied). This characterization identifies the "natural" or baseline CTR of a process. Our model works very well in describing these "natural" printing processes. It may not fit as well to processes where arbitrary tone curves have been applied. The smooth simple curves modeled from "natural" printing would be best used in any print standard.

Model Development

Since our model is intended for use in print standards, we began by looking at what was needed for that purpose. Most standards specify tone reproduction of single ink ramps (cyan, magenta, yellow, black) using density measurements. Density is an appropriate measure for process control, but print standards need to be specified colorimetrically. Furthermore, to specify gray balance, we need to model the 3/C (cyan, magenta, yellow) overprint characteristics.

These requirements expand the scope of prior work in two directions. First, we are modeling ink ramps of more than one color. Second, the output of our model is colorimetric.

To illustrate, let's consider the "isometric" ramp of the familiar ANSI CGATS TR001 characterization data. (ANSI, 1995). This data was obtained by measuring an IT8.7-3 test target. There are six data points, each with equal percentages of cyan, magenta and yellow (0%, 10%, 20%, 40%, 70% and 100%). Figure 1 (below) shows xyz reflectance values plotted against percent dot.

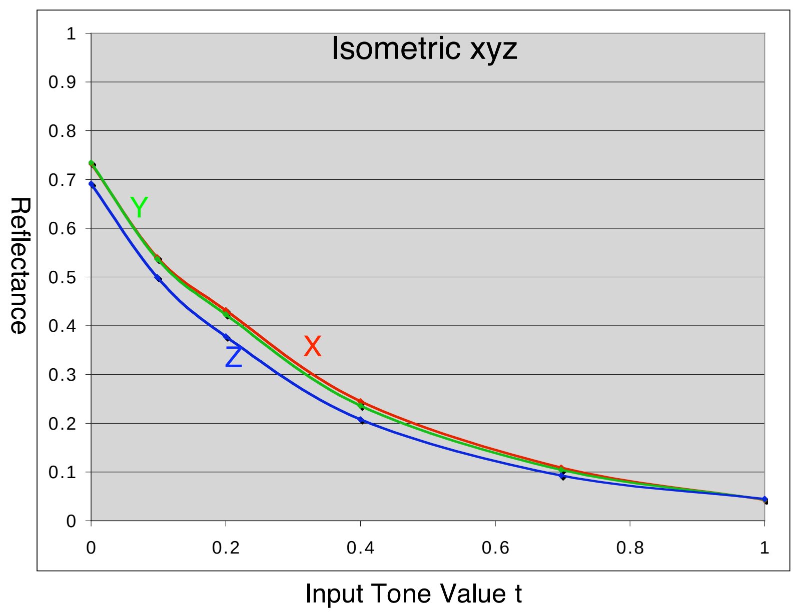


Figure 1

These curves exhibit a bowed and irregular shape. The irregularity of this data is probably due to non-linearities in the analog platemaking. We wouldn't want to build these flaws into a print standard. It would be better for our model to be a smooth curve, fit to these points as well as the smooth curvature allows.

The non-linearity of the xyz functions is very much the same problem that Yule and Neilsen confronted in 1951. Their solution was to transform the measurements in a way that produced a linear function. They discovered a formula that accomplished this for a variety of printing conditions, known today as the Yule-Neilsen equation.

The Yule-Neilsen equation works equally well with colorimetric data, but we chose, instead, to borrow a transform used in the computation of CIE Lab values. There are two reasons we took this path. First, we wanted our measurements to be visually uniform, which is a quality of the CIE Lab color space. Second, we felt the need for more control of the curve shapes than we could obtain with the Yule-Neilsen transform.

Appendix B shows the math used to compute Lab values from XYZ values. In our work, we typically use lower case xyz values, which are derived from the XYZ values and the viewing illuminant. The transform of the y-value to L is the transform we chose to linearize our data. We put the x and z values thorough the same transform. We call these transformed xyz values L_x, L_y and L_z. L_y, of course, is the same as L, but we give it a subscript for clarity. Figure 2 shows L_x, L_y and L_z graphed against tone value.

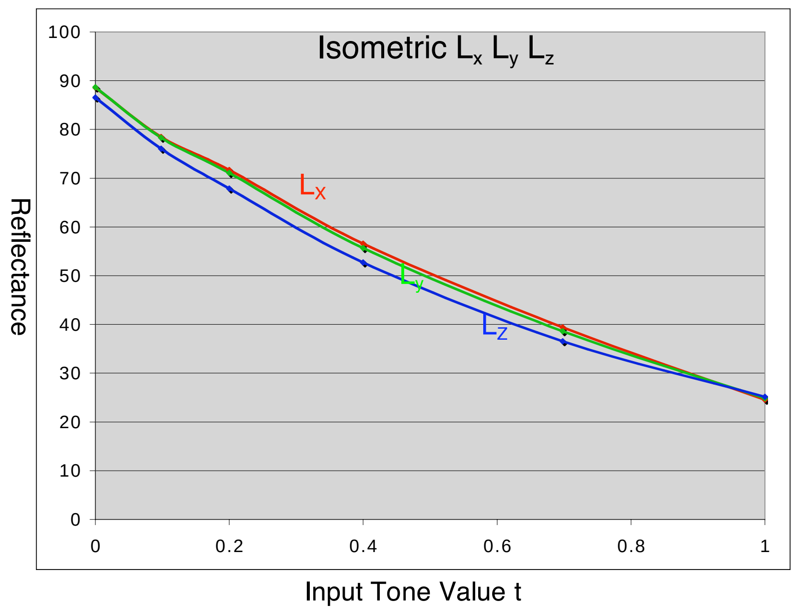


Figure 2

These curve are much more linear. In fact, we have observed that the process color ramps in "natural" printing are generally very linear when transformed to L_x L_y L_z. The isometric ramp of this data set has a bow, which is typical.

The third step in our model is to fit a Hermite spline segment to the $L_X L_Y L_Z$ data. The Hermite spline is a smooth function that joins two endpoints, with control of the slope at each endpoint. Figure 3 shows spline curves fit to the TR001 data. These curves are inherently smooth, and fit very well to a variety of measured data. Each curve requires four parameters – the values and slopes at the endpoints. Since we are modeling three curves (L_X L_Y L_Z), there are twelve parameters for each ramp.

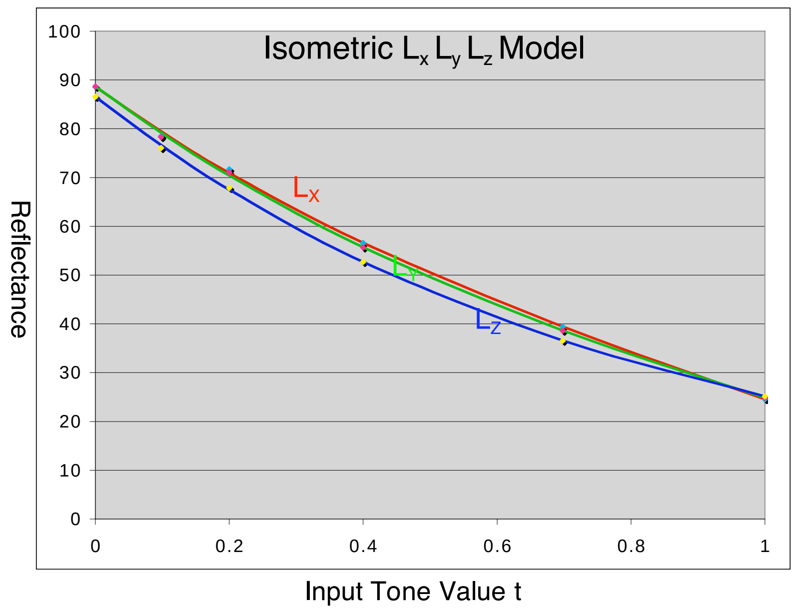


Figure 3

Methodology

The first step in creating a regression-based model for color halftone printing was experimenting with methods of transforming the printing data so it was represented by linear functions. Linear functions are more easily and accurately modeled by regression techniques.

Looking at the printing process, we see image files composed of tones as the input to the process. Tone is described in terms of percent-dot value. Values range between 0% and 100% – 0% being no ink printed – and 100%

being the maximum ink printed. The process input values are the tonal values (0 to 1) for each printing ink (typically CMYK).

The measurements needed to compute CTR curves are of pure color ramps. For instance, a cyan ramp might consist of 21 steps of pure cyan from 0% to 100% in 5% increments. We want to model the colorimetric measurements of this tone ramp with an equation that fits the measured points accurately. This is completed for each process color individually, for a total of 12 equations. Additionally, a fifth multi-ink ramp consisting of equal amounts of cyan, magenta and yellow inks (“isometric” ramp) provides very useful gray balance information. Appendix A describes the derivation of multi-ink ramps.

Process output values are color measurements of printing corresponding to the input values. These measurements are made with a spectrophotometer and are commonly specified as CIE Lab values. But, Lab is not the color space best suited for our objectives. Plotting Lab as a function of percent dot input results in some very unfamiliar looking color tone reproduction curves. (Figure 4) A print color standard described with these types of CTR curves would be foreign to the printers who would want to use them.

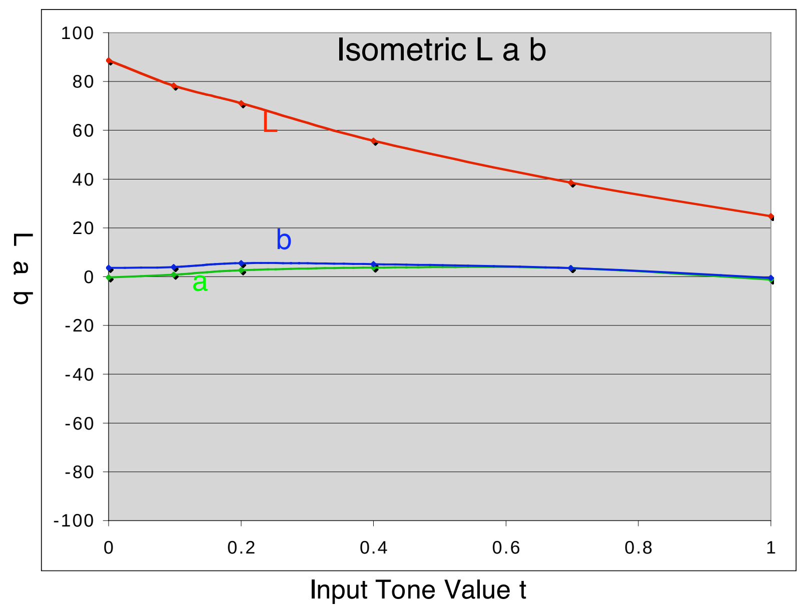


Figure 4

We find, it is better to use xyz values, which are directly related to Lab by formulas. (Appendix B) xyz values roughly correspond to red, green and blue reflectance measurements. The curves are intuitively more familiar. As the percent tone input increases, the output reflectance de-creases. We feel, they would be more easily understood in a print standard than the graphs of Lab functions. Color TR curves can be computed easily using xyz data. (Figure 1) The color outputs of this printing process are xyz values. These values also range between 0 and 1. A perfect white is denoted by a 1 and 0 denotes a perfect black. This is convenient for computation and comparison to input values which are also in the same 0 – 1 range. The mathematics for this modeling is simplified by the fact that all inputs and outputs are between 0 and 1. Figure 1 represents the tonal inputs and xyz colorimetric outputs of the "isometric" multi-ink ramp.

Although these color TR curves appear more intuitive than the Lab graph, their bowed and twisted character make them more complicated to model than a more linear function. We were looking for a function to represent color halftone printing that was both familiar and linear.

We were able to create new more linear functions in the form of L_x, L_y, and L_z. These new functions were created, by making use of the calculation of the L value in the CIE Lab model. We observed that the L value was very linear as a function of input tone value. (Figure 4) Our function L_y is simply the usual CIE calculation for L. Functions L_x and L_z are fashioned by using the colorimetric x or z values in place of y in the CIE calculation of L. (Appendix B) The resulting functions are much more linear than the xyz functions. (Figure 2)

Optimal Model

The optimal model achieves a balance between simplicity and accuracy. We want to describe the essential characteristics of the process, yet remove any measurement errors or peculiar behaviors.

Because we must take into account legacy images, tone curves cannot be specified in an arbitrary way. The current systems that create images and legacy images must both be taken into account. Printers have little control over the images they receive. To be useful, our model must be able to describe printing that uses these types of images.

The Yule-Neilsen model describes some processes quite well. (Yule and Neilsen, 1951) But in other cases, it doesn't. This isn't too surprising, when you consider that this model has just three control values, highlight, shadow and n-value. To obtain a good fit to real data, some additional controls are needed. Our regression-based model was created to satisfy this necessity. We fit a Hermite spline segment to the measured printing data using a least squares fit. (Appendix C)

Hermite Spline

The Hermite spline is a cubic polynomial arranged in an intuitive way. The spline's control parameters are the values at the end-points, and the slopes at the end-points

Suppose: L(t) = L-value (as a function of t)
t = ink ramp parameter (range from 0 to 1)

V0 = L(0) (L-value at 0% dot or highlight value)
V1 = L(1) (L-value at 100% dot or shadow value)
S0 = L′(0) (slope at 0% dot or highlight contrast)
S1 = L′(1) (slope at 100% dot or shadow contrast)

Hermite Blending Functions:

$$H0 = 2t^3 - 3t^2 + 1$$
$$H1 = -2t^3 + 3t^2$$
$$H2 = t^3 - 2t^2 + t$$
$$H3 = t^3 - t^2$$

Model: $L(t) = H0\,V0 + H1V1 + H2S0 + H3S1$

The spline parameters (V0, V1, S0, S1) are easy to visualize. They correspond to the familiar concepts of value and contrast. The slopes at t=0 and t=1 are commonly known as “highlight contrast” and “shadow contrast.”

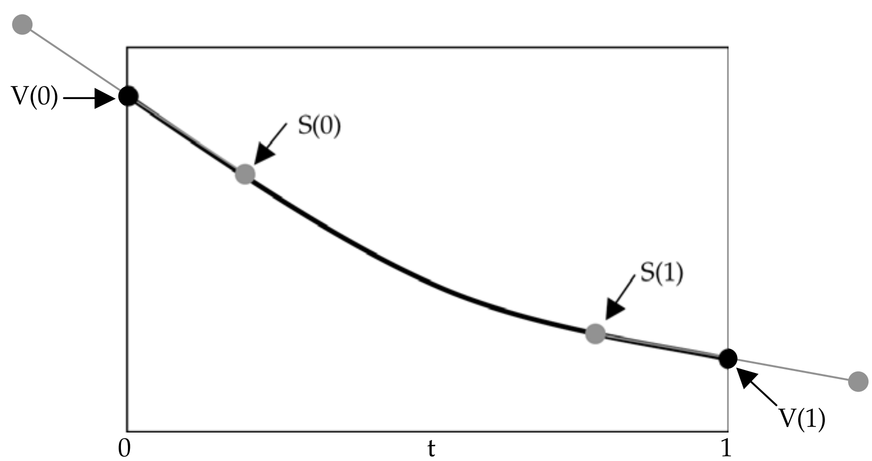


Figure 5

Figure 5 illustrates the Hermite parameters. V(0) is the value at the left endpoint (t=0), and S(0) is the slope or contrast. V(1) is the value at the right endpoint (t=1), and S(1) is the slope or contrast.

In our model, the curve endpoints are the x, y, or z reflectance of paper white V(0) and the solid ink/s V(1) of the color ramp. We compute the slope or "contrast" at each end of the curve, to best fit the measured data. We wrote software which varies the slopes of V(0) and V(1) until a minimum overall error to the measured data is achieved. (Appendix C)

To help visualize the overall behavior of a printed ramp, it is common to consider parameters at the midtone (50% tone input). We can derive two additional midtone quantities from the Hermite spline parameters. We call these additional measures "bow" and "twist". Bow is stated as percent and is similar to tone value increase (TVI). Values greater than 0 are darker and less than 0 lighter. Twist is measure of the slope or contrast of the curve pivoting at the midpoint. A "twist" measure greater than 1.0 illustrates a steeper slope or higher midtone contrast. A measure less the 1.0 indicates lower midtone contrast. A twist of 1.0 indicates a straight line. Calculations for "bow" and "twist" are presented in Appendix D.

Test Results

We tested our model using 177 data sets of various offset and gravure printing characterizations. All of the reference data sets published by ANSI-CGATS, FOGRA, IFRA and ECI were tested. We also tested data from our own sheet-fed and web-press runs, done with the help of local printers. This collection of data sets contained a wide of range press types, papers and screening.

To determine the accuracy of our model, we calculated the ΔE between the actual measured colorimetric printed values and our modeled values. Average ΔE was calculated for each of the five color ramps (C, M, Y, K, and "isometric"). The model demonstrated a high degree of accuracy across this very diverse range of printing conditions. The majority of average ΔE values are below 1.0. They range from a low 0.18 ΔE to a high of 3.4 ΔE. A sampling of the test results is shown in Table 1, below.

Average ΔE of Measured vs. Modeled Values

	Black ΔE	Cyan ΔE	Magenta ΔE	Yellow ΔE	Isometric ΔE
CGATS TR001	0.373	0.434	0.531	0.644	0.456
CGATS	0.611	0.554	1.040	0.939	0.709

TR004					
FOGRA 27	0.130	0.208	0.442	0.354	0.313
FM SCREENING	0.4525	0.8350	0.8882	1.1794	1.0336
HALFTONE DOT PROOF	0.386	0.375	0.536	0.605	0.458
GRAVURE	0.883	0.353	1.039	0.896	0.955
HEATSET WEB OFFSET	0.467	0.375	0.503	0.752	0.674
LINEAR OFFSET	0.363	0.430	0.467	0.471	0.502
IFRA 28	0.103	0.112	0.207	0.196	0.149

Table 1

Conclusions and Discussion

In our prior work, (Birkett and Spontelli 2004) we demonstrated that colorimetric tone reproduction (CTR) can be used to visually match one printing or proofing process to another. We were able to achieve a very close visual match between processes that had similar base colorants. (i.e. press-to-press or halftone dot proof-to-press) If the individual CMYK and "isometric" ramps are a close colorimetric reflectance match, the two printed samples will be in close visual agreement. We also recommended that current industry print specifications (SWOP, GRACoL, etc.) (SWOP, 2001) (GRACoL, 2002) be updated to include colorimetrically based CTR curves. This will enable printers to calibrate their platemaking and presswork to match a reference. Current practice requires only matching loosely defined density and tone measurements.

Characterizing process color printing, with these five recommended CTR curves, results in a very accurate description of the printing. This CTR characterization has more than the necessary precision needed to calibrate a printing system to match a reference standard specified with our approach. Replacing traditional density and dot-gain calculation with colorimetric measurement adds precision and modernizes the calibration process. Measuring the overprinting "isometric" ramp characterizes the overprinting and gray balance of the process.

We found that matching only the individual CMYK color ramps can result in an overall "weight" or saturation difference between the reference and the calibration. The interactions that occur between overprinting inks are significant. This overprint attribute can be notably different between a halftone dot proof and a press-sheet, for example. We find that measuring the "isometric" ramp is essential to an accurate characterization.

Attempting a press-to-proof match, as in this example, requires the additional overprint details (ink trapping and paper interactions, etcetera) assessed through the multi-color “isometric” ramp.

Our regression-based model was successful in correctly modeling individual printed ramps of CMYK and the overprinted multi-ink “isometric” ramp. With this modeling technique, we were able to achieve our objectives of creating a simple, smooth mathematical representation of “real printing” that would be familiar to printers.

To update current print standards, we recommend adding smooth modeled CTR curves based on colorimetry. This update should be based on fifteen straightforward equations. These equations would describe the CTR of the individual CMYK and “isometric” color ramps. They would calculate a colorimetric output for any tone (percent-dot) input. The colorimetric $L_xL_yL_z$ output of each of these five ramps would provide a precise “vectorized” characterization of the standard. These would serve as a specific reference for printers trying to match the standard. These would replace the outmoded dot-gain, print contrast, and solid ink density specifications that are used currently. Any reference printing modeled in this way would provide an unambiguous guide to printers seeking to match an industry standard.

Appendix A - Multi-Ink Ramps

We're all familiar with simple ink ramps. For instance, a typical black ramp is a set of patches with values of 0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, 90% and 100% dot. The patches are printed, measured with a densitometer, and the measurements paired with the %-dot values to form a discreet function.

0	0.06
10	0.13
20	0.20
30	0.27
40	0.36
50	0.45
60	0.56
70	0.71
80	0.87
90	1.10
100	1.47

A multi-ink ramp allows for more than one ink to be varied. This is done by introducing a set of parametric equations into the definition of the test patches. For instance, a ramp consisting of equal amounts of cyan, magenta and yellow would be defined by:

C = 100t
M = 100t
Y = 100t

where t is the ramp parameter, with a range of 0 to 1. A set of 11 equally spaced patches would look like this:

t	C	M	Y
0	0	0	0
0.1	10	10	10
0.2	20	20	20
0.3	30	30	30
0.4	40	40	40
0.5	50	50	50
0.6	60	60	60
0.7	70	70	70
0.8	80	80	80
0.9	90	90	90
1.0	100	100	100

The patches in this ramp would typically be brownish or purplish in color. We could measure them using a densitometer using a visual filter to get a single density value. However, our goal is a colorimetric model, so we'll take L a b measurements instead. The discreet function looks like this:

t	C	M	Y	L	a	b
0	0	0	0	94.3	0.2	-1.7
0.1	10	10	10	84.3	1.6	0.9
0.2	20	20	20	75.8	2.7	3.2
0.3	30	30	30	68.0	2.9	6.3
0.4	40	40	40	60.4	3.3	7.5
0.5	50	50	50	52.6	4.0	9.2
0.6	60	60	60	45.6	3.8	9.0
0.7	70	70	70	39.3	2.6	9.2
0.8	80	80	80	33.8	1.9	5.3
0.9	90	90	90	29.0	-0.2	2.2
1.0	100	100	100	25.4	-3.1	0.5

The colorimetric measurement of the ramp has three components, L, a and b, but the function has just one variable, t. Yes, we are varying C, M and Y, but in a very specific way, defined by the parametric equations:

C = 100t

$$M = 100t$$
$$Y = 100t$$

The parametric equations for C, M, Y, and K could have many forms, but for our purpose, a simple linear equation for each is sufficient.

$$C = C_1t + C_0$$
$$M = M_1t + M_0$$
$$Y = Y_1t + Y_0$$
$$K = K_1t + K_0$$

Appendix B – XYZ to Lab and $L_xL_yL_z$

These are equations for converting XYZ values to Lab (Lindbloom, 2003) and $L_xL_yL_z$ values.

X_r, Y_r, Z_r are for the reference illuminant

$$x_r = \frac{X}{X_r}$$

$$y_r = \frac{Y}{Y_r}$$

$$z_r = \frac{Z}{Z_r}$$

$$f_x = \begin{cases} \sqrt[3]{x_r} & x_r > \varepsilon \\ \dfrac{\kappa x_r + 16}{116} & x_r \le \varepsilon \end{cases}$$

$$f_y = \begin{cases} \sqrt[3]{y_r} & y_r > \varepsilon \\ \dfrac{\kappa y_r + 16}{116} & y_r \le \varepsilon \end{cases}$$

$$\varepsilon = 216/24389 \cong 0.008856$$
$$\kappa = 24389/27 \cong 903.3$$

$$f_z = \begin{cases} \sqrt[3]{z_r} & z_r > \varepsilon \\ \dfrac{\kappa z_r + 16}{116} & z_r \le \varepsilon \end{cases}$$

From the intermediate values above, we can compute both Lab and $L_xL_yL_z$:

$$L = 116 f_y - 16$$

$$a = 500(f_x - f_y)$$

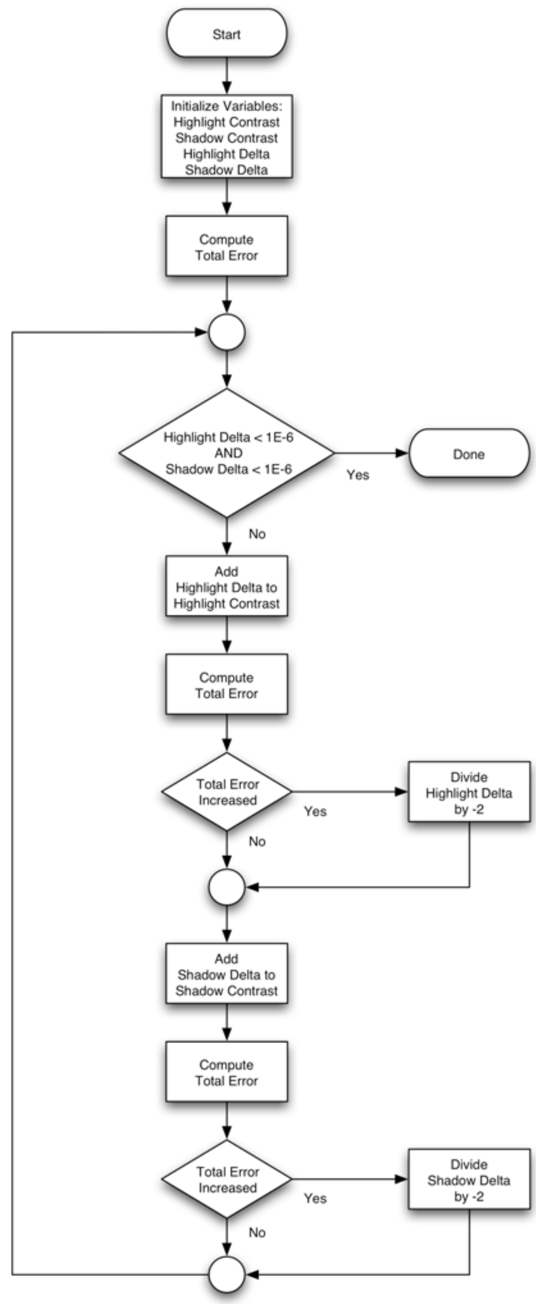
$$b = 200(f_y - f_z)$$

$$L_x = 116 f_x - 16$$

$$L_y = 116 f_y - 16$$

$$L_z = 116 f_z - 16$$

Appendix C - Regression Algorithm Flowchart



The Hermite spline model has four parameters, highlight value (V(0)), highlight contrast (S(0)), shadow value (V(1)), and shdow contrast (S(1)). From these four parameters, it is possible to derive the midtone value (V(0.5)) and midtone contrast (S(0.5)).

The Hermite basis functions are:

$$H0 = 2t^3 - 3t^2 + 1$$
$$H1 = -2t^3 + 3t^2$$
$$H2 = t^3 - 2t^2 + t$$
$$H3 = t^3 - t^2$$

To get the midtone value, evaluate these functions at t=0.5:

$$H0 = 0.25 - 0.75 + 1 \quad = 0.5$$
$$H1 = -0.25 + 0.75 \quad = 0.5$$
$$H2 = 0.125 - 0.5 + 0.5 \quad = 0.125$$
$$H3 = 0.125 - 0.25 \quad = -0.125$$

Evaluate the function:

$$V(0.5) = (V(0) + V(1))/2 + (S(0) - S(1))/8$$

To get the midtone contrast, take the derivative of the basis functions,

$$d(H0)/dt = 6t^2 - 6t$$
$$d(H1)/dt = -6t^2 + 6t$$
$$d(H2)/dt = 3t^2 - 4t + 1$$
$$d(H3)/dt = 3t^2 - 2t$$

and evaluate at t = 0.5:

$$d(H0)/dt = 1.5 - 3 \quad = -1.5$$
$$d(H1)/dt = -1.5 + 3 \quad = 1.5$$
$$d(H2)/dt = 0.75 - 2 + 1 \quad = -0.25$$
$$d(H3)/dt = 0.75 - 1 \quad = -0.25$$

Evaluate the derivative,

$$S(0.5) = 3 * (V(1) - V(0))/2 - (S(0) + S(1))/4$$

The value and slope at the midtone are derived from the Hermite parameters, and are just another way of looking at the same information.

To make these values more meaningful, they can be compared to the values of a straight line connecting the two endpoints. First, compute the value and contrast of the line:

V(0.5)(linear) = (V(0) + V(1))/2

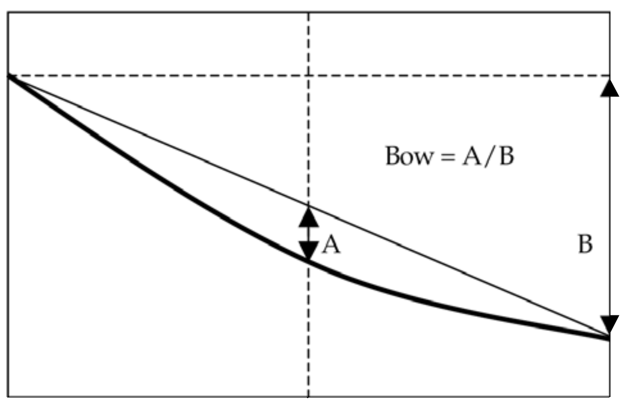
S(0.5)(linear) = (V(1) - V(0))

For the midtone value, divide the difference by the value range to get:

Bow = (V(0.5) - V(0.5)(linear))/(V(1) - V(0))

Bow = (S(0) - S(1))/((V(1) - V(0)) * 8)

Bow is similar to midtone TVI. Values greater than 0 are fuller, and values less that 0 are sharper. The figure below illustrates bow geometrically.



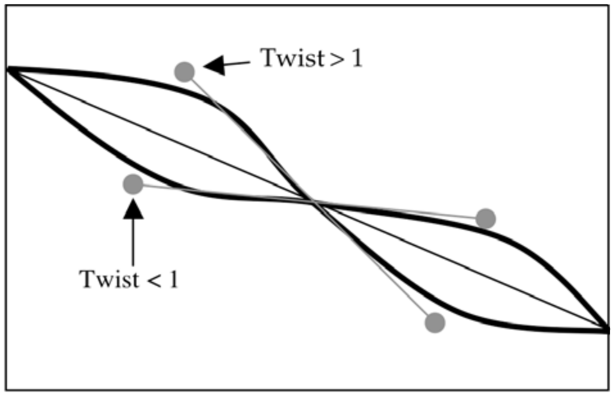
A straight line curve has a bow of 0.

For the midtone contrast, compute the ratio of the spline slope to the linear slope:

Twist = S(0.5)/S(0.5)(linear)

Twist = 3/2 - (S(0) + S(1))/((V(1) - V(0)) * 4)

This value is called twist, because curves with values significantly greater than or less than one will be S-shaped. Geometrically, it will appear that the curve was “twisted” around the midpoint. This is illustrated in the figure below.



The curve with the steeper slope has a twist greater than one, while the other curve has a twist less than one. A straight-line curve has a twist of one, by definition.

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Literature Cited

American National Standards Institute (ANSI) Committee for Graphic Arts Technologies Standards (CGATS)
1995 Graphic Technology - Color Characterization Data for Type 1 Printing ANSI CGATS TR 001.

Birkett, William B. and Spontelli, Charles
2004 "Improving Print Standards by Specifying Tone Reproduction for the Overall Process," TAGA Proceedings, pp. 453-472.

Committee for Graphic Arts Technologies Standards (CGATS)
2004 "Graphic technology — Color characterization data for printing on Grade 1 and Grade 2 gloss coated paper based on GRACoL commercial offset aims," Draft Technical Report, CGATS DTR 004-2003.

GRACOL
2002 "General Requirements for Applications in Commercial Offset Lithography," (IDEAlliance, Alexandria, VA), ver. 6.0, pp. S4.

Lindbloom, Bruce J.
2003 "Useful Color Equations," Retrieved March 14, 2005, from Bruce Lindbloom's Web Site: http://www.brucelindbloom.com/

Murray, Alexander
1936 "Monochrome Reproduction in Photoengraving," Journal of the Franklin Institute, 221: pp. 721-744.

Yule, John A. C. and Neilsen, W. J.
1951 "The Penetration of Light into Paper and its Effect on Halftone Reproductions," TAGA Proceedings, 3: pp. 65-76.

SWOP - Specifications for Web Offset Publication
2001 "SWOP for the New Millennium" Ninth edition

Wyble, David R. and Berns, Roy S.
2000 "A Critical Review of Spectral Models Applied to Binary Color Printing," Color Research and Application (John Wiley & Sons, Inc. Hoboken, NJ), Vol. 25, pp. 4-19.