

# Mathematical Models of Ink Mileage Curves of Dry Toners in Electrophotography

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## Abstract

An ink mileage curve is a plot of the reflection density of an ink film printed on a substrate as a function of the ink film thickness. It is helpful to predict how much ink is needed to achieve a target density and has been studied for many years. It was found that it was affected by both paper properties and ink properties. However, previous research was done for liquid inks used in conventional printing processes, such as offset and gravure printing. Non-impact printing (NIP) technologies have grown rapidly in the last several decades. Electrophotography (EP), as one of the major NIP technologies, mostly uses dry toners. Their ink mileage behaviors are still not well understood. In this study, four different substrates were printed on a dry-toner color production EP press, a Xerox iGen3. The print layout contains patches with different CMYK tonal values from 10% to 100%. Toner amounts on cyan patches were measured using an analytical method. Printed patches and unprinted paper samples as well as dry toners were dissolved in nitric acid and the copper concentrations in the solutions were analyzed by a Zeeman graphite furnace atomic absorption spectrometer (AAS). Analytical results were calculated to determine the toner amounts on paper for different tonal values. Their corresponding reflection densities were also measured. All the data were plotted in OriginPro<sup>®</sup> 8 software and four mathematical models were used for curve fitting. It is found that C-S model fits the experimental data of the two uncoated papers slightly better than the other three models. None of the four models fit the experimental data of the two coated papers well, while the linear model is found to fit the data well. So linear fitting is best in the practical density region for the two coated papers. Ink mileage curves obtained from curve fitting can be used to estimate how much ink is required to achieve a target density and hence the ink mileage can be calculated. The effect of paper properties on ink mileage was studied. It is found that the rougher the paper surface is, the higher ink film weight is required, and then the lower the ink mileage is.

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## **Introduction**

An ink mileage curve is a plot of the reflection density of an ink film printed on a substrate as a function of the ink film thickness. Reflection density increase with the increase of ink film thickness until it approaches a value called saturation density. In practice, ink film thickness is difficult to measure; therefore, ink film weight, as the mass of ink film per square meter, has been used to represent ink film thickness, and its unit is gram per square meter, or gsm.

An ink mileage curve is helpful to predict how much ink is needed to achieve a target density and has been studied for many years. However, previous research was done for liquid inks used in conventional printing processes, such as offset lithography (Tollenaar and Ernst, 1962, Kornerup et al., 1964, Oittinen, 1972, Calabro and Mercatucci, 1974, Calabro and Savagnone, 1983, Blom and Conner, 1990, Chou and Harbin, 1991, MacPhee and Lind, 2002) and gravure printing (Xu et al., 2005, 2006, 2007, 2008). It was found that ink mileage was affected by both paper properties and ink properties.

Non-impact printing (NIP) technologies have grown rapidly in the last several decades. Electrophotography (EP), as one of the major NIP technologies, has been used for commercial production printing. EP presses mostly use dry toners instead of liquid inks. Their ink mileage behaviors are still not well understood. The goal of this study was to print different substrates on a dry-toner EP press, measure their ink film weights and reflection densities, and use different models to fit the experimental data to find a best model.

## **Experimental**

### *Substrates*

In this study, four commercial paper substrates were selected. Two of them are uncoated, 24# Glatfelter (GL) and 28# Navigator (NV), while the other two are coated, 80# Elite Gloss (EG) and 10 pt. Supreme Gloss (SG) from Xerox. The surface roughness was measured using a Mitutoyo Surface Roughness Tester Model 211. The arithmetic mean deviation of the roughness file, Ra, was measured with evaluation length of 2.5 cm. 75° Gloss was measured using a Gardco Novo-Gloss® Glossmeter. Brightness and cast values were measured using an X-Rite SpectroDensitometer 528.

### *Printing*

The substrates were printed on a Xerox iGen3™ 110 with EFI Fiery® Color Server, located at Muncie Novelty in Muncie, Indiana. Xerox Dry Inks were used. Line screen was 175 lpi. Printing speed was 40 ipm.

The print layout contains patches with different CMYK tonal values from 10% to 100%, as shown in Figure 1.



**Figure 1.** Print layout.

### *Ink Film Measurement*

Toner amounts on cyan patches with different tonal values were measured using an analytical method. Samples of unprinted papers, printed patches, and dry toners were dissolved in nitric acid by boiling. The copper concentrations of all obtained solutions were analyzed by a Zeeman graphite furnace Atomic Absorption Spectrometer (Model 4110 ZL) with AA WinLab Analyst Version 2.5 software. Analytical results were calculated to determine the transferred toner amounts for different tonal values.

$$w = \frac{c_1 - c_2}{c_3} \quad (1)$$

where  $w$ : ink film weight (gram per square meter, or gsm)  
 $c_1$ : element concentration in printed paper (gsm)  
 $c_2$ : element concentration in unprinted paper (gsm)  
 $c_3$ : element concentration in dry toner (wt%)

The relative reflection densities at different tonal values with reference to unprinted paper were measured using an X-Rite SpectroDensitometer 528.

### *Curve Fitting*

All the data were analyzed in OriginPro<sup>®</sup> 8 software. Four mathematical models proposed by previous researchers (Tollenaar and Ernst, 1962, Oittinen, 1972, Calabro and Mercatucci, 1974, Calabro and Savagnone, 1983) were used for curve fitting. However, some modifications were made so that the regression coefficients have the same meanings.  $D_s$  is the saturation density of an ink film at infinite ink film thickness. Parameter  $m$  determines the steepness of density curve in the region of very thin film and is called “density smoothness” (Tollenaar and Ernst, 1962). Parameter  $n$  is the ink film weight exponent and was added to improve curve fitting.

$$\text{T - E} \quad D = D_s (1 - e^{-mw}) \quad (2)$$

$$\text{Oittinen} \quad D = D_s (1 - e^{-mw^n}) \quad (3)$$

$$\text{C - M} \quad \frac{1}{D} = \frac{1}{D_s} + \frac{1}{mw} \quad (4)$$

$$\text{C - S} \quad \frac{1}{D} = \frac{1}{D_s} + \frac{1}{mw^n} \quad (5)$$

where  $w$ : ink film weight  
 $D$ : relative reflection density  
 $D_s, m, n$ : regression coefficients

Sum of squares of residuals (SSR) was used to tell how good the curve fitting was, in which the residual was equal to the experimental value minus the value calculated from the fitting model. The smaller the SSR value is, the better the curve fitting is.

## **Results and Discussion**

The results of measured paper properties are listed in Table 1. Coated EG and SG papers are much smoother than uncoated GL and NV papers, therefore higher gloss values. NV paper has very high brightness which is out of the measuring range of the instrument used. GL paper has lower brightness than the other three. NV paper shows a blue white color so its cast value is very high. GL paper shows a yellow white color while EG and SG papers both have a neutral white color.

It is found that for the two uncoated papers, all four models fit the data well, which is evidenced by small values of SSR as shown in Table 2. The

comparison of their SSR values shows that the curve fitting results obtained using C-S model are slightly better than those using other models. The SSR values for GL paper are higher than those for NV paper. As shown in Table 1, GL paper is much rougher than NV paper, which introduced higher variation into ink film weight measurement for GL paper.

*Table 1. Paper Properties*

Paper	Basis Weight g/m <sup>2</sup>	Roughness Ra, $\mu\text{m}$	Gloss 75°, %	Brightness %	Cast %
Glatfelter (GL)	105	2.959	12.3	77.8	5
Navigator (NV)	90	1.976	9.4	-	15
Elite Gloss (EG)	120	0.534	70.0	94.9	4
Supreme Gloss (SG)	219	0.334	82.1	90.6	3

*Table 2. Sum of Squares of Residuals*

	GL	NV
T-E	0.03174	0.00811
Oittinen	0.02886	0.00799
C-M	0.03316	0.00734
C-S	0.02611	0.00657

The curve fitting results for the uncoated GL and NV papers using C-S model are shown in Figure 2. It shows that less amount of toners is required for NV paper than for GL paper in order to achieve the same reflection density. GL paper is rougher than NV paper, so more toners are needed to fill out the valleys on the surface before an ink film layer with a smooth surface and certain film thickness can be obtained.

None of the four models was found to fit the data of the two coated papers well. The available experimental data seem to be located within the linear region which is in the beginning of an ink mileage curve. Linear curve fitting results show the R<sup>2</sup> values are 0.990 for EG and 0.982 for SG, respectively. The ink mileage curves of the two coated papers obtained from linear curve fitting are compared in Figure 3. It shows that less amount of toners is required for EG paper than for SG paper in order to achieve the same reflection density. The reason is not clear and needs to be investigated. Both of the papers are coated

with low roughness and high gloss, but other important properties are unknown, such as porosity, due to lack of measuring instrument. Previous studies showed that porosity also plays an important role in ink mileage behaviors.

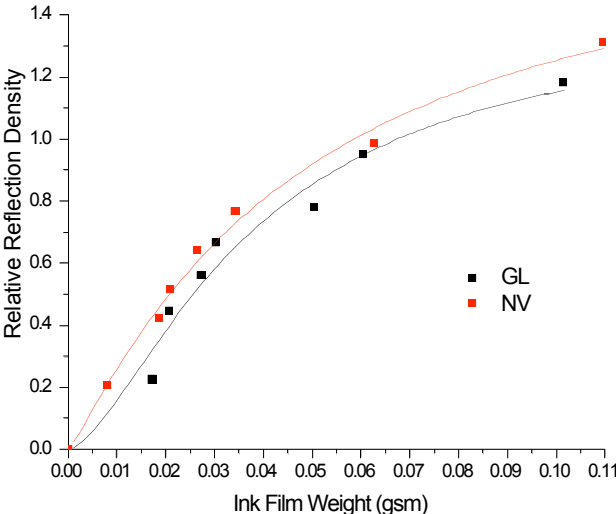


Figure 2. Ink mileage curves of uncoated GL and NV papers with C-S model.

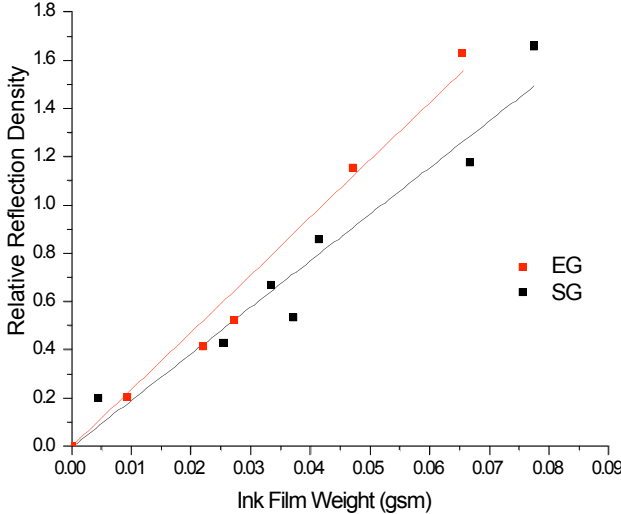
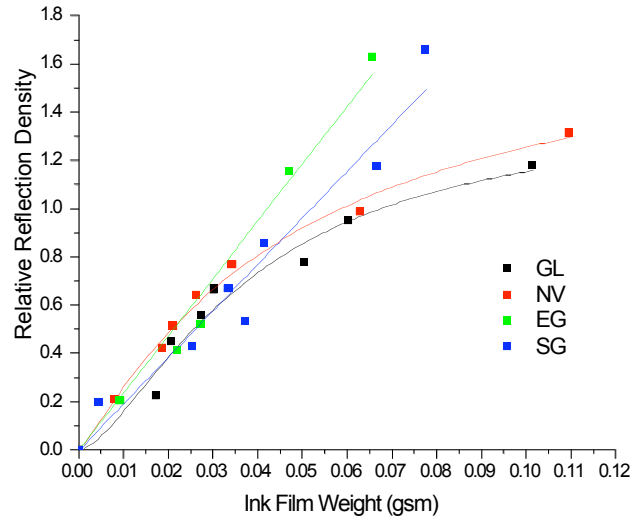


Figure 3. Ink mileage curves of coated EG and SG papers with linear fitting.

The ink mileage curves of all the four papers are compared in Figure 4. In the thin ink film region, ink mileage behaviors of the four papers are similar. However, the linear regions of the ink mileage curves of the two coated papers extend to a wide density range, while the ink mileage curves of the two uncoated paper gradually approach their saturation densities.



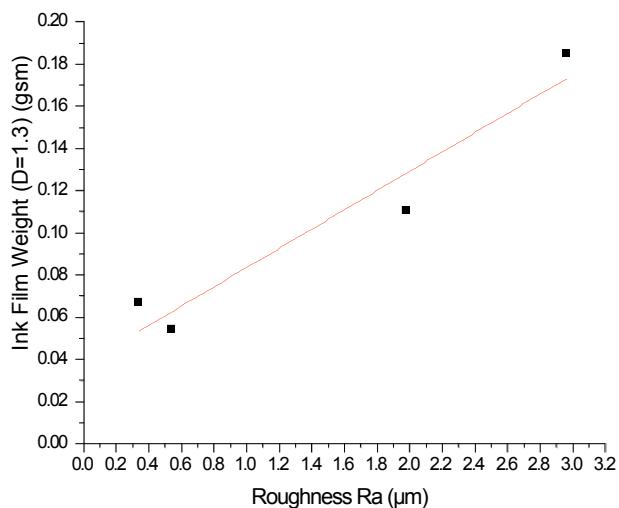
**Figure 4.** Ink mileage curves of all four papers.

Ink mileage curves can be used to predict how much ink is needed to achieve a target density. For example, if a cyan density of 1.30 is desired, the ink film weight can be obtained from the ink mileage curve and hence the ink mileage for that paper can be calculated, as shown in Table 3.

**Table 3.** Ink Mileage Prediction at a Target Cyan Density of 1.30

Substrate	Ink Film Weight gsm	Ink Mileage m <sup>2</sup> /g
GL	0.1850	5.40
NV	0.1109	9.02
EG	0.0546	18.31
SG	0.0673	14.85

The relationship between calculated ink film weight and paper surface roughness is shown in Figure 5. The rougher the paper surface is, the higher ink film weight is required, and then the lower the ink mileage is.



**Figure 5.** The relationship between roughness and ink film weight at cyan density of 1.30.

### Conclusions

C-S model fits the experimental data of the two uncoated papers slightly better than the other three models.

None of the four models fit the experimental data of the two coated papers well, while the linear model is found to fit the data well. So linear fitting is best in the practical density region for the two coated papers. With higher densities, a nonlinear model should be used to fit the data.

Ink mileage curves obtained from curve fitting are helpful to estimate the ink requirement for a target density and hence the ink mileage can be calculated. The effect of paper properties on ink mileage was studied. It is found that the rougher the paper surface is, the higher ink film weight is required, and then the lower the ink mileage is.

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## Literature Cited

- Blom, B. E. and T.J. Conner.  
1990 "Optical Density and Ink Film Thickness; A Comparison of Models," *TAGA Proceedings*, pp. 213–225.
- Calabro, G. and F. Mercatucci.  
1974 "Method for Evaluating Newsprint Printability," *Proceedings of 12th International Conference of Printing Research Institute (IARIGAI)*, pp. 155–159.
- Calabro, G. and F. Savagnone.  
1983 "Method for Evaluating Printability," *Advanced Printing Science and Technology 17*, Chap. 23, pp. 358–380.
- Chou, S. M. and N. Harbin.  
1991 "Relationship between Ink Mileage and Ink Transfer," *TAGA Proceedings*, pp. 405–432.
- Kornerup, A., P. Fink-Jensen, and C.O. Rosted.  
1969 "Tristimulus Values of Prints and Mileage of Printing Inks," *Die Farbe*, Vol. 18, pp. 29–64.
- MacPhee, J. and J.T. Lind.  
2002 "Insight into the Relationship between Print Density and Ink Film Thickness," *TAGA Proceedings*, pp. 479–496.
- Oittinen, P.  
1972 *Graphic Arts in Finland*, Vol. 1, No. 2, pp. 11–22.
- Tollenaar, D. and Ernst, P. A. H.  
1962 "Optical Density and Ink Layer Thickness," *Proceedings of International Conference of Printing Research Institute (IARIGAI)*, pp. 214–234.
- Xu, R., A. Pekarovicova, P.D. Fleming, and V. Bliznyuk.  
2005 "Physical Properties of LWC Papers and Gravure Ink Mileage," *TAGA Proceedings*, pp. 485–497.
- Xu, R., A. Pekarovicova, P.D. Fleming, and X.Q. Wang.  
2006 "Study of Ink Mileage Curve of Gravure Printing," *TAGA Proceedings*, pp. 443–452.
- Xu, R., A. Pekarovicova, P.D. Fleming, Y.J. Wu, and M.X. Wang.  
2007 "A method for evaluating ink mileage in gravure printing," *TAPPI Journal*, Vol. 6, No. 12, pp. 27–32.
- Xu, R., Y.J. Wu, A. Pekarovicova, P.D. Fleming, and M.X. Wang.  
2008 "The effects of paper coating on gravure ink mileage curve," *TAGA Journal of Graphic Technology*, Vol. 4, No. 3, pp. 117–125.