

A Method to Separately Model Mechanical and Optical Dot Gain Effects in Color Halftone Prints

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Keywords: Halftone, mechanical dot gain, optical dot gain,
MTF of paper

Abstract

A method is proposed to separately model the mechanical dot gain and the optical dot gain. First, an iterative algorithm is proposed to estimate the spatio-spectral transmittance of ink layer from the spatio-spectral reflectance of color halftone print measured with the reflection optical microscope attached with the liquid crystal tunable filter (LCTF). The spatio-spectral transmittance of ink layer is not affected by the optical dot gain and is only affected by the mechanical dot gain. Next, a model is proposed to estimate the effective dot coverage using the estimated spatio-spectral transmittance of ink layer. It corresponds to the analysis of mechanical dot gain. Next, a model is proposed to estimate Yule-Nielsen's n parameter using the effective dot coverage. It corresponds to the analysis of optical dot gain. Finally, the prediction accuracy of the proposed model is evaluated by the ΔE_{94} between the measured and predicted spectral reflectance of offset printing images with cyan and magenta inks. The prediction accuracy of the proposed model was significant since the

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average ΔE_{94} and the maximum ΔE_{94} of all samples between the measured spectral reflectance and the predicted spectral reflectance were 0.62 and 1.37, respectively.

Introduction

The color management system (CMS) is an important concept in order to efficiently share the color information between various digital imaging devices, such as printers, digital still cameras, scanners, and displays. In the concept of CMS, the color information is managed in the device independent color space such as the CIE XYZ value, the CIE L*a*b* value, or the spectral reflectance. Since the spectral reflectance is not influenced by the illumination environment, the CIE XYZ (or L*a*b*) value on the arbitrary illumination environment can be calculated from the spectral reflectance. To incorporate an imaging system to CMS, the color reproduction of the system needs to be comprehended. Compared to other imaging devices, it is not an easy task to predict the color reproduction of the printing system efficiently and accurately since the dot gain causes the nonlinear characteristic of the input-output relationship. Dot gain is a phenomenon in printing which makes printed paper look darker than intended. The dot gain effect can be classified to two types. One is a mechanical dot gain and the other is an optical dot gain. Due to the viscosity of ink, the shape of printed ink dot is changed compared to the intended shape. This phenomenon is called the “mechanical dot gain.” Due to the mechanical dot gain, the printed dots are generally printed larger than intended. In other words, the dot coverage actually printed is larger than that intended, where the intended dot coverage is called a “nominal dot coverage” and the actually printed dot coverage is called an “effective dot coverage.” On the other hand, the optical dot gain, also called the “Yule-Nielsen effect,” is caused by the light scattering in paper. Due to the light scattering in paper, the perceived dots are larger than actually printed dots.

Since two types of dot gain is observed simultaneously, it is difficult to separately analyze the mechanical dot gain and the optical dot gain.

In this research, a method is proposed to separately model and analyze the mechanical dot gain and the optical dot gain using the spatio-spectral reflectance data of color patches measured with a reflection optical microscope attached with a liquid crystal tunable filter (LCTF).

Spectral Neugebauer Model

The Neugebauer model [Neugebauer, 1937] predicts the CIE XYZ tristimulus values of a color halftone patch as the sum of the tristimulus values of their individual colorants weighted by their fractional dot coverages a_i . By considering instead the tristimulus values of colorants their respective reflection spectra $r_i(\lambda)$, one obtains the spectral Neugebauer model [Hersch, 2005] given by

$$r(\lambda) = \sum_i a_i r_i(\lambda), \quad (1)$$

where λ denotes wavelength, $r(\lambda)$ is the spectral reflectance of color halftone patch and i denotes the color of ink. In color prints using three primary inks, cyan, magenta, and yellow, for example, i indicates cyan c , magenta m , yellow y (primary colors), red r , green g , blue b (secondary colors), black k (tertiary color), or white p (paper, without ink). The spectra $r_i(\lambda)$ corresponds to the solid prints spectra using the ink i . The word “solid” denotes the print with 100% coverage of ink. Equation 1 is a simple linear equation with the parameters a_i . However, Equation 1 cannot precisely predict the spectra of color halftone prints

due to the dot gain effect. Even if the dot coverages a_i denote the effective dot coverage where the mechanical dot gain is considered, the prediction accuracy of Equation 1 is still poor due to the optical dot gain.

Yule-Nielsen Modified Spectral Neugebauer Model

Yule and Nielsen proposed their model to correct the prediction error caused by the optical dot gain for the black-and-white prints (Yule and Nielsen, 1951). Viggiano applied the Yule-Nielsen model to the Neugebauer model (Viggiano, 1990) and have proposed the Yule-Nielsen modified spectral Neugebauer model is given by

$$r(\lambda) = \left\{ \sum_i a_i r_i(\lambda)^{1/n} \right\}^n. \quad (2)$$

Equation 2 is a nonlinear equation which corrects the prediction error caused by the optical dot gain by a parameter n . However, the parameter n is just an empirical value and has no physical meaning. If one changes the printing conditions such as the usage of different paper, different ink, and different resolution of print, the parameter n has to be re-estimated from a lot of measurement of spectra.

Spectral Reflection Image Model (SRIM)

Ruckdeschel and Hauser (Ruckdeschel and Hauser, 1978) and Inoue et al. (Inoue et al., 1997) have proposed the same kind of prediction model having parameters which can provide the physical meaning of the dot gain effect given by

$$r(x, y) = F^{-1} \left[F \{ t(x, y) \} MTF_p(u, v) \right] r_p t(x, y) \quad (3)$$

where (x, y) denotes the spatial coordinates, (u, v) denotes the spatial frequency coordinates, $r(x, y)$ is the spatial distribution of reflectance from the halftone print, $t(x, y)$ is the spatial distribution of transmittance of ink layer, $MTF_p(u, v)$ is the modulation transfer function (MTF) of paper, r_p is the reflectance of paper, and F and F^{-1} denote the operation of Fourier transform and inverse Fourier transform, respectively. Inoue et al. named this equation as a reflection image model (RIM). Figure 1 illustrates the light transfer behavior of RIM. The RIM expresses the halftone print as a spatial distribution of reflectance where the ink dots are superimposed on paper, and it is assumed that the ink layer and paper can be optically separated. The light transfer behavior of RIM can be explained as the following steps.

1. The halftone print is illuminated by the input light.
2. The light transmits the ink layer by its transmittance $t(x, y)$.
3. The transmitted light enters into the paper.
4. The light is scattered in paper by $MTF_p(u, v)$ and reflected by the reflectance r_p .
5. The reflected light transmits the ink layer by $t(x, y)$ again before output.

In the RIM, the function $r(x,y)$ is affected by the mechanical dot gain and the optical dot gain, where the mechanical dot gain effect is expressed in the function $t(x,y)$, and the optical dot gain effect is expressed in the function $MTF_p(u,v)$.

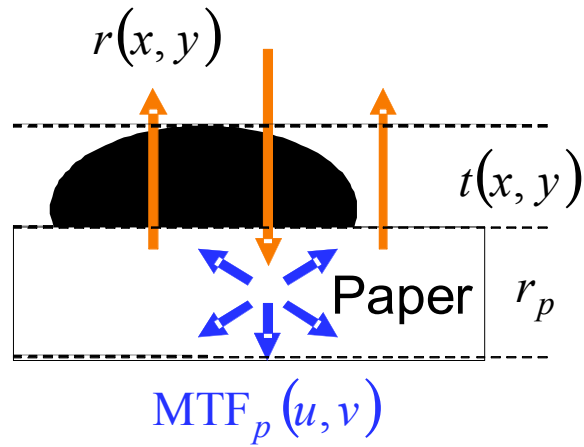


Figure 1. Light transfer behavior in the reflection image model (RIM).

The RIM can be extended to a spectral form. The spectral reflection image model (SRIM) is given by

$$r(x, y; \lambda) = F^{-1} \left[F \{ t(x, y; \lambda) \} MTF_p(u, v) \right] r_p(\lambda) t(x, y; \lambda) \quad (4)$$

where $r(x,y;\lambda)$ is the spatio-spectral reflectance distribution of the color halftone print, $t(x,y;\lambda)$ is the spatio-spectral transmittance distribution of the ink layer and $r_p(\lambda)$ is the spectral reflectance of paper. To be exact, the function $MTF_p(u,v)$ should also be a spectral form, i.e., $MTF_p(u,v;\lambda)$. We, however, assumed the

paper's MTF is independent on wavelength since the wavelength dependence of paper's MTF is not significant (Ukishima et al., 2009).

Acquisition of Components in SRIM

The SRIM shown in Equation 4 can separately analyze the optical dot gain and mechanical dot gain. However, it is difficult to obtain parameters of SRIM compared to the Yule-Nielsen spectral Neugebauer model. The SRIM consists of four components, i.e., $r(x,y;\lambda)$, $r_p(\lambda)$, $MTF_p(u,v)$, and $t(x,y;\lambda)$. The functions $r(x,y;\lambda)$ and $r_p(\lambda)$ can be easily measured with a reflection optical microscope attached with a liquid crystal tunable filter (LCTF). A problem of the SRIM is difficulty to obtain the functions $MTF_p(u,v)$ and $t(x,y;\lambda)$.

Measurement of Paper's MTF

With respect to $MTF_p(u,v)$, several researchers have proposed the methods to measure the MTF of paper. Inoue et al. have proposed a method to project sinusoidal test patterns to the paper and measure the ratio of modulation depth of these patterns, respectively (Inoue et al., 1997). Inoue et al. have also proposed another method not to project but to contact sinusoidal test target printed on film to paper to measure the ratio of modulation depth (Inoue et al., 1998). Yule et al. (Yule and Nielsen, 1951; Yule et al., 1967), Engeldrum and Pridham (Engeldrum and Pridham, 1995), and Atanassova and Jung (Atanassova and Jung, 2007) measured the line spread function (LSF) of paper from the edge spread function (ESF) obtained by the knife edge projection method. The MTF of paper was calculated from the Fourier transform of the LSF. Rogers has proposed a series-expansion bar-target technique (Rogers, 1998), where a bar-target image data is projected on paper and the response is measured. He calculated the ratio between the series-expansion coefficients of the Fourier

transform of measured data and that of ideal bar-target data in order to decide the MTF of paper.

We have also proposed a method to efficiently and accurately measure the $MTF_p(u,v)$ with the reflection optical microscope (Ukishima et al., 2009), where $MTF_p(u,v)$ is calculated by the fraction between two images of the pencil light response in Fourier domain where the two images are reflection images from the paper and the perfect specular reflector. From our measurement results, we concluded that $MTF_p(u,v)$ of various types of paper can be approximated by

$$MTF_p(u,v) \approx \frac{1}{\sqrt{1 + (2\pi d)^2 (u^2 + v^2)}}, \quad (5)$$

where d is a fitting parameter which has different value in different paper.

Difficulty of Acquisition for Spatio-Spectral Transmittance in Ink Layer

A problem in usage of SRIM is the difficulty to obtain the spatio-spectral transmittance of ink layer $t(x,y;\lambda)$ since $t(x,y;\lambda)$ cannot be directly measured with the reflection optical microscope.

As not the spatio-spectral transmittance $t(x,y;\lambda)$ but the monochrome spatial transmittance $t(x,y)$, Koopipat et al. have proposed that $t(x,y)$ can be measured with a “transparent” optical microscope (Koopipat et al., 2002). They made a microscope having two light sources where one illuminates the sample from the upper side for the reflectance measurement (Reflection mode) and the other illuminates the sample from the back side for the transmittance measurement (Transparency mode). They measured $r(x,y)$ and r_p with the reflection mode and

measured $t(x,y)$ with the transparency mode. To measure $t(x,y)$, they used the transparency image model given by

$$o(x,y) = i t_p t(x,y) \quad (6)$$

where $o(x,y)$ is the spatial intensity distribution of output light from the halftone print, i is the intensity of input light having spatial uniformity, and t_p is the transmittance of paper. One can obtain $t(x,y)$ from the measurements of $o(x,y)$, i and t_p with the transparency mode using the equation given by



Figure 2. Reflectance and transmittance images of uncoated paper:
(a) reflectance, and (b) transmittance.

$$t(x,y) = \frac{o(x,y)}{i t_p} \quad (7)$$

However, we would like to remark several problems of their method.

Problem 1 The special microscope having two light sources is needed.

Problem 2 If the paper has a high thickness, then measurements of $o(x,y)$ and t_p are difficult since little amount of light is transmitted.

Problem 3 Many types of paper have a deep fiber structure in the transmittance image especially in the uncoated paper. It means that the assumption is not valid that t_p is spatially uniform in Equation 6. Of course the reflectance of paper r_p in the RIM in Equation 3 also has the fiber structure. However, the non-uniformity level is less significant than t_p . Figure 2 shows the fiber structure of an uncoated paper in r_p and t_p .

Problem 4 If one uses $t(x,y)$ measured with the transparent microscope to predict $r(x,y)$ by Eq. (3), the prediction accuracy is significantly poor. This experimental fact is caused by a problem hiding in the RIM itself. Let a solid patch of print is considered in the RIM. It means $t(x,y) = t_{cons}$ with a constant value t_{cons} . In this case, Eq. (3) can be converted to the form given by

$$r(x, y) = r_{cons} = r_p (t_{cons})^2 \quad (8)$$

Equation 8 suggests that the reflectance r_{cons} is proportional to $(t_{cons})^2$. However, Eq. (8) is not valid in real case especially in the case that t_{cons} has a low value

corresponding that the ink has high density. It is caused by the effect of specular reflection, the light scattering effect in ink layer, and the geometrical difference of measurement between $r(x,y)$ and $t(x,y)$.

Proposed Method to Estimate Spatio-Spectral Transmittance in Ink Layer

According to the acquisition of spatio-spectral transmittance of ink layer, several problems of the conventional measurement-based method were described in the previous section. **Problem 4** is the most serious problem since it indicates that the measured $t(x,y;\lambda)$ with any measurement-based methods would not be working in the SRIM. If one uses the SRIM, one has to obtain $t(x,y;\lambda)$ as a function which is consistent with the SRIM. To solve this problem, in this section, a method is proposed to obtain $t(x,y;\lambda)$ which is consistent with the SRIM not by the measurement-based method but by an estimation-based method.

We mentioned that functions $r(x,y;\lambda)$, $r_p(\lambda)$, and $MTF_p(u,v)$ in Equation 4 can be measured with the reflection optical microscope attached with the LCTF. The spatio-spectral transmittance $t(x,y;\lambda)$ cannot only be measured with the reflection optical microscope. Therefore, if one can solve the Equation 4 with respect to $t(x,y;\lambda)$, the analytical solution $t(x,y;\lambda)$ satisfies Equation 8. However, it is difficult to mathematically solve Equation 4 with respect to $t(x,y;\lambda)$ since two transmittance functions $t(x,y;\lambda)$ are located in inside and outside of the Fourier operations, respectively. The problem is how one can obtain $t(x,y;\lambda)$. Then, we propose a iterative algorithm to estimate the approximate solution of $t(x,y;\lambda)$ using the SRIM. The proposed iterative algorithm is described in Figure 3.

Figure 4(a) shows an example of the spatio-spectral reflectance of color halftone print $r(x,y;\lambda)$ measured with a reflection optical microscope attached with the LCTF. The spectral image is displayed by converting it to the CIE RGB image. This example is a color patch of offset print with the amplitude modulation (AM) screening where the nominal dot coverages of cyan and magenta are 0.4 and 0.2, respectively. Figure 4(b) shows an example of the spatio-spectral transmittance of ink layer $t(x,y;\lambda)$ estimated by the proposed iterative algorithm from the measured $r(x,y;\lambda)$. Since the spatio-spectral transmittance $t(x,y;\lambda)$ is not affected by the optical dot gain, the ink dots look brighter and sharper. It is considered that the mechanical dot gain can directly be analyzed using $t(x,y;\lambda)$.

As the microscope system to measure the spatio-spectral reflectance shown in Figure 4(a), we used a reflection optical microscope (BX50, Olympus) attached with a LCTF (VariSpec Cis Corp., CRI) and with a monochrome CCD camera (INFINITY4-11M, Lumenera Corp., 12-bit quantization, USB 2.0). The image were captured with a resolution of 2048×2048 . An objective lens whose magnification power is $4 \times$ was used and, in this case, the vertical and horizontal pixel pitches are $1.96 \mu\text{m}$. The spectral resolution of the measurement was set to 30 nm in the interval of wavelength 430–700 nm (10 bands). To remove the specular reflection component, two polarizers were attached in front of the camera and the light source, respectively. The microscope system is shown in Figure 5. Divided by a spectral image of white reference, the measured images were converted to spatio-spectral reflectance factor $r(x,y;\lambda)$.

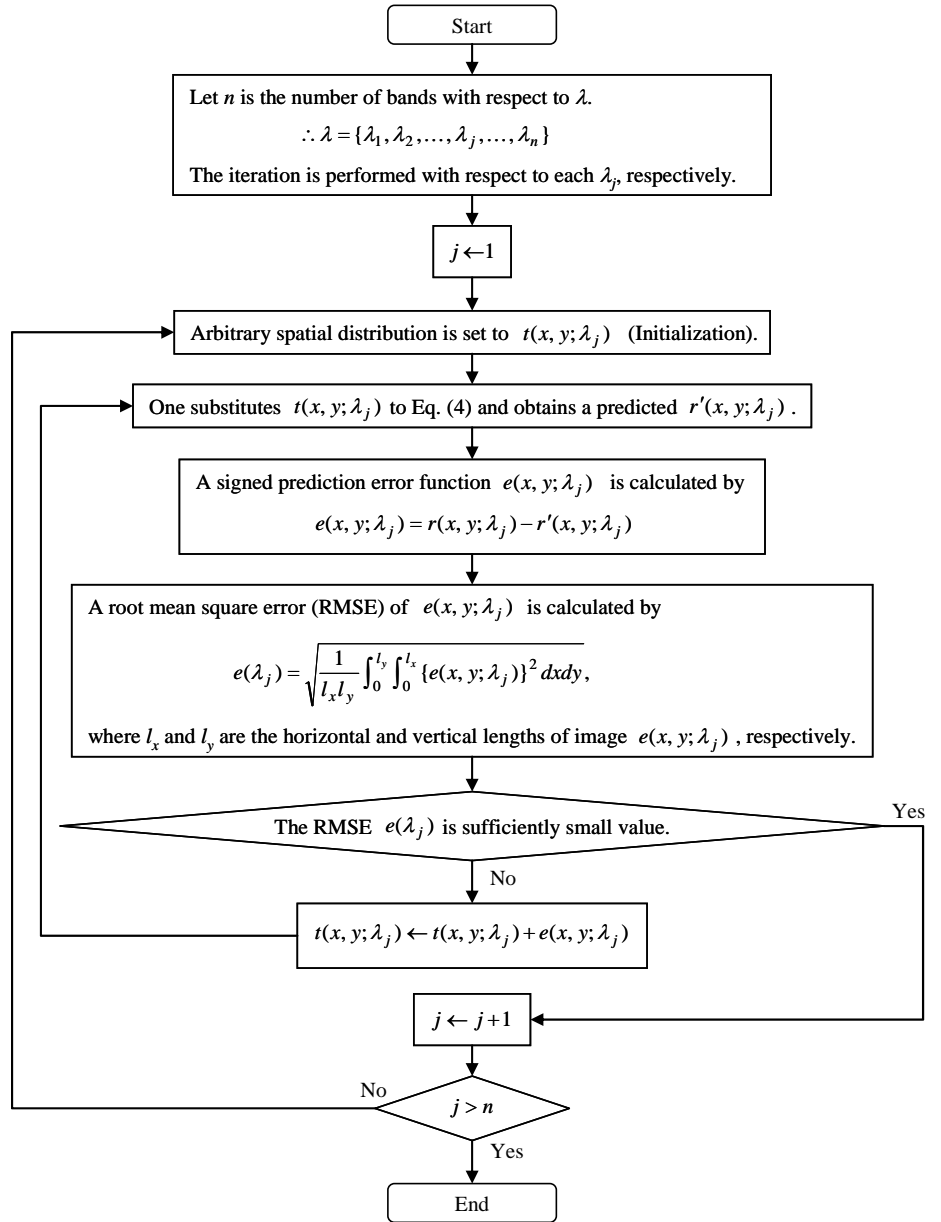
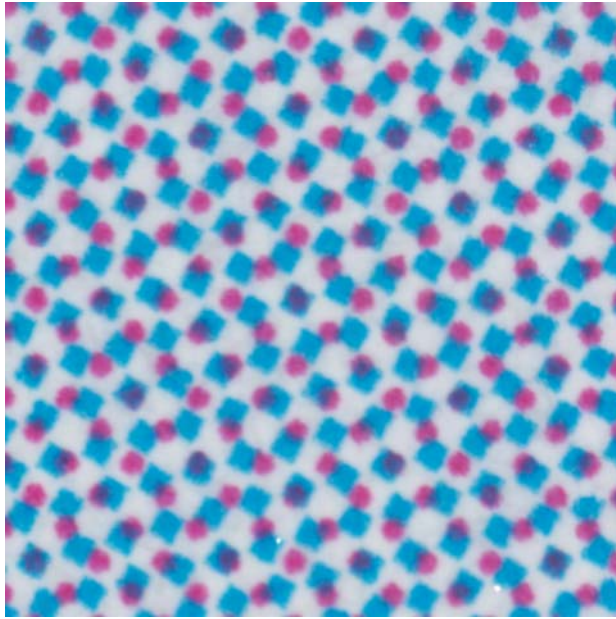
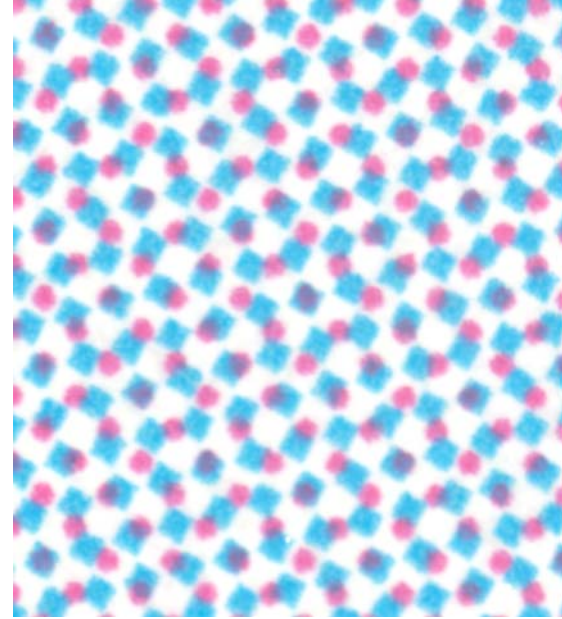


Figure 3. The proposed iterative algorithm to estimate the approximate solution of $t(x, y; \lambda)$ using the SR.



(a)



(b)

Figure 4. Measured and estimated spatio-spectral reflectance and transmittance:
(a) measured spatio-spectral reflectance (b) estimated spatio-spectral transmittance.

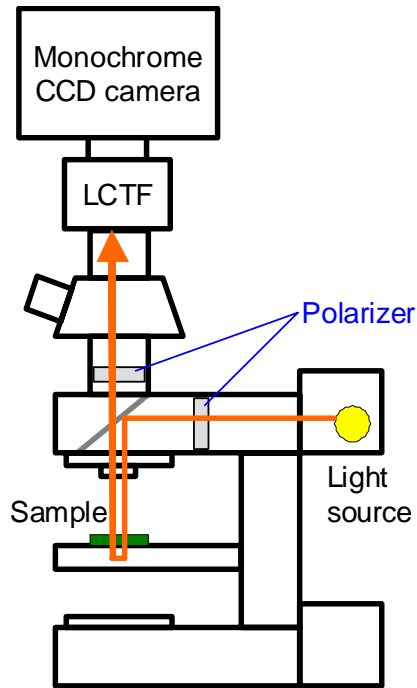


Figure 5. The reflection optical microscope attached with a liquid crystal tunable filter (LCTF).

Transmittance-Based Spectral Neugebauer Model for Analyzing Only Mechanical Dot Dain

The spectral Neugebauer model given by Equation 1 practically does not work due to the optical dot gain effect. However, the spatio-spectral transmittance of ink layer $t(x,y;\lambda)$ is not affected by the optical dot gain. It implies that the spectral Neugebauer model practically works in the domain of transmittance of ink layer. Then, we propose the transmittance-based spectral Neugebauer model given by

$$\bar{t}(\lambda) = \sum_i a_i \bar{t}_i(\lambda) \quad (9)$$

with constraints

$$0 \leq a_i \leq 1, \quad \sum_i a_i = 1 \quad (10)$$

where the suffix i contains c, m, y, r, g, b, k and p when three primary inks are used, $\bar{t}(\lambda)$ is the spatial average value of $t(x,y;\lambda)$, and $\bar{t}_i(\lambda)$ is the spatial average value of $t(x,y;\lambda)$ for the solid prints of each color i . Note that when i denotes p , $\bar{t}_i(\lambda)$ indicates the transmittance of ink layer without ink, therefore

$$\bar{t}_p(\lambda) = 1 \quad (11)$$

Validity of Model 1

In this section, the validity of the proposed transmittance-based spectral Neugebauer model is evaluated.

As the measurement sample, color patches with cyan and magenta inks printed with an offset printer on a coated paper (ISO12642, JAPAN COLOR 2007) were used. The nominal dot coverages of the patches are all cyan-magenta combinations of 0, 0.20, 0.40, 0.70, and 1.00, respectively. The total number of samples is therefore twenty-five. As the measurement system, the same microscope system shown in Figure 5 was used. The MTF of the coated paper was preliminary measured by our proposed method (Ukishima et al., 2009), and the parameter d was obtained in Equation 5 where $d = 0.030$.

The verification flow is shown in Figure 6 (Verification (1)). Using the proposed iterative algorithm, the spatio-spectral transmittance $t(x,y;\lambda)$ was estimated from the measured spatio-spectral reflectance $r(x,y;\lambda)$. Next, the average transmittance $\bar{t}(\lambda)$ was calculated from $t(x,y;\lambda)$. Next, the effective dot coverage a_i of each ink i was estimated by Equation 9 with a constrained least square method, respectively. Next, the prediction spectrum $\bar{t}'(\lambda)$ was calculated using estimated a_i and Equation 9. Finally, the predicted transmittance spectrum $\bar{t}'(\lambda)$ was compared to the correct spectrum $\bar{t}(\lambda)$. Figure 7 shows the several examples of results. The ΔE_{94} values were evaluated with respect to all sample patches between the correct and predicted spectra. The prediction accuracy was significant since the average ΔE_{94} and the maximum ΔE_{94} of all samples were 0.26 and 0.64, respectively. It can be concluded that the proposed linear equation (9) is valid in transmittance $\bar{t}(\lambda)$ space which is not be affected by optical dot gain.

Transmittance-Based Yule-Nielsen Modified Spectral Neugebauer Model for Analyzing Only Optical Dot Gain

We concluded that only the mechanical dot gain can be analyzed by Equation 9. In this section, we propose a model to analyze only the optical dot gain.

Let the optical dot gain effect is ignored in SRIM in Equation 4. It corresponds that

$$\text{MTF}_p(u, v) = 1, \tag{12}$$

Therefore, Equation 4 is converted as

$$r(x, y; \lambda) = \{t(x, y; \lambda)\}^2 r_p(\lambda) \quad (13)$$

In this case, the spatial average spectrum $\bar{r}(\lambda)$ of $r(x, y; \lambda)$ is given by

$$\bar{r}(\lambda) = \{\bar{t}(\lambda)\}^2 r_p(\lambda) \quad (14)$$

According to Equation 14, a transmittance-based Yule-Nielsen modified spectral Neugebauer model is proposed given by

$$\bar{r}(\lambda) = \left\{ \sum_i a_i \left[\{\bar{t}_i(\lambda)\}^2 r_p(\lambda) \right]^{1/n} \right\}^n \quad (15)$$

Using the parameter n , Equation 1) re-expresses the optical dot gain effect which is ignored in Equation 13. The unknown parameter is only n in Equation 15 since the effective dot coverages a_i has been already estimated using Equation 9. Therefore, it can be easily estimated by a nonlinear optimization.

Validity of Model 2

In this section, the validity of the proposed transmittance-based Yule-Nielsen modified spectral Neugebauer model is evaluated.

The verification flow is shown in Figure 6 (Verification (2)). The Yule-Nielsen's parameter n was estimated by Equation 15. As a training data for the nonlinear optimization, only one sample patch was used, where the nominal dot coverage (cyan, yellow) = (0.40, 0.40). The estimated n was equal to 1.99. Next, using the effective dot coverages a_i and the estimated Yule-Nielsen's parameter n , the prediction spectra $\bar{r}'(\lambda)$ were calculated with respect to all sample patches by Equation 15. Finally, the predicted spectral reflectance $\bar{r}'(\lambda)$ were compared to the measured spectral reflectance $\bar{r}(\lambda)$. Figure 8 shows the several examples of results. The ΔE_{94} values were also evaluated between the measured and predicted spectra. The prediction accuracy was significant since the average ΔE_{94} and the maximum ΔE_{94} of all samples were 0.62 and 1.37, respectively. It can be concluded that the proposed nonlinear equation (15) is valid.

Conclusion

In this research, a method was proposed to separately model the mechanical dot gain and the optical dot gain. The spatio-spectral transmittance of ink layer was estimated by applying the proposed iterative algorithm to the spatio-spectral reflectance of color halftone print measured with the reflection optical microscope attached the liquid crystal tunable filter (LCTF). The spatio-spectral transmittance of ink layer is not affected by the optical dot gain and is only affected by the mechanical dot gain. The effective dot coverage is estimated by applying the proposed transmittance-based spectral Neugebauer model in Equation 9 to the spatio-spectral transmittance of ink layer to analyze the mechanical dot gain. The estimated effective dot coverage is applied to the proposed transmittance-based Yule-Nielsen modified spectral Neugebauer model in Equation 15 in order to estimate the Yule-Nielsen's n parameter which quantifies the optical dot gain. The prediction accuracy of Equation 15 was significant since the average ΔE_{94} and the maximum ΔE_{94} of all samples

between the measured spectral reflectance and the predicted spectral reflectance were 0.62 and 1.37, respectively.

As future works, we would like to apply the Demichel's equation to the proposed transmittance-based Yule-Nielsen modified spectral Neugebauer model in order to predict the spectral reflectance of arbitrary input using the limited number of measurements as the training. We also would like to propose the method to predict not only the spectral reflectance but also the spatial distribution of reflectance, i.e., the spatio-spectral reflectance where the spatial distribution of reflectance of color patch is related to the granularity of halftone print.

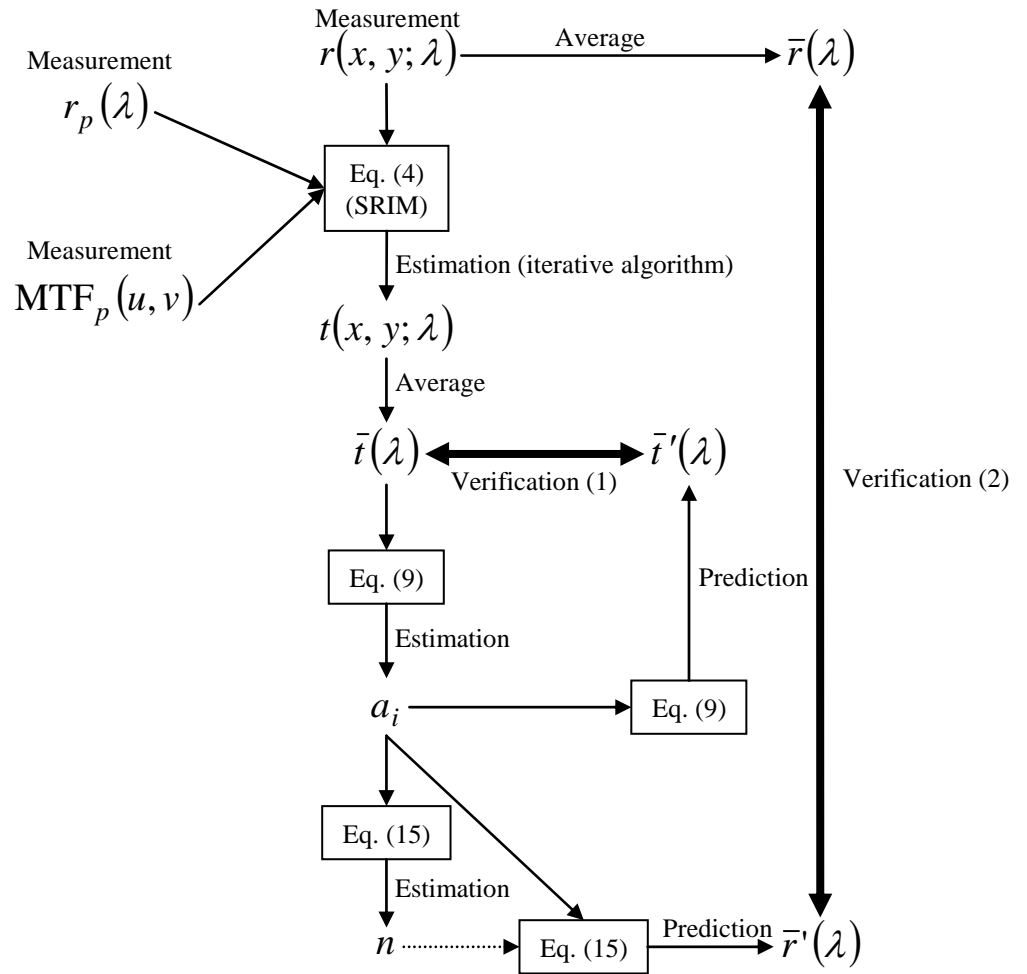


Figure 6. Flowchart of verification.

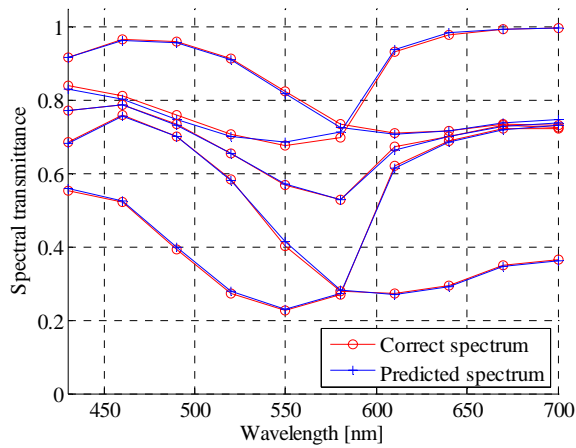


Figure 7. Prediction of average spectral transmittance.

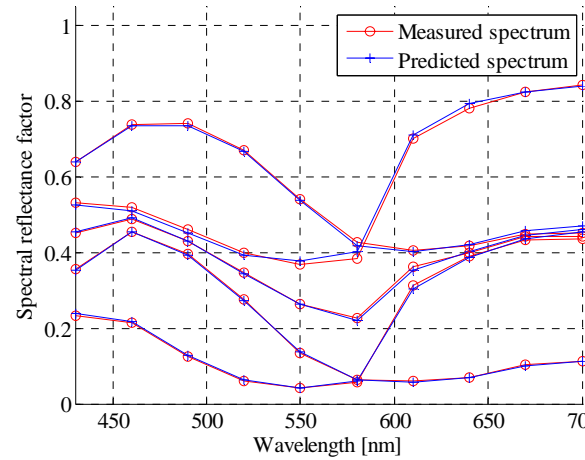


Figure 8. Prediction of average spectral reflectance.

Literature Cited

Atanassova, M. and J. Jung. 2007. "Measurement and Analysis of MTF and its Contribution to Optical Dot Gain in Diffusely Reflective Materials," *Proc. IS&T's NIP23*, pp. 428–433.

Engeldrum, P.G. and B. Pridham. 1995. "Application of turbid medium theory to paper spread function measurements," *TAGA Proc.*, 339.

Hersch, R.D. 2005. "Spectral prediction model for color prints on paper with fluorescent additives," *J. Electronic Imaging*, 14(3), pp. 33001–12.

Inoue, S., N. Tsumura, and Y. Miyake. 1997. "Measuring MTF of Paper by Sinusoidal Test Pattern Projection," *J. Imaging Sci. Technol.*, 41(1), pp. 657–661.

Inoue, S., N. Tsumura, and Y. Miyake. 1998. "Analyzing CTF of Print by MTF of Paper," *J. Imaging Sci. Technol.*, 42(6), pp. 572–576.

Koopipat, C., N. Tsumura, Y. Miyake, and M. Fujino. 2002. "Effect of Ink Spread and Optical Dot Gain on the MTF of Ink Jet Image," *J. Imaging Sci. Technol.*, 46(4), pp.321–325.

Neugebauer, H.E.J. 1937. "Die theoretischen Grundlagen des Mehrfarbendrucks," *Z. Wissen. Photog.*, 36, pp. 36–73.

Rogers, G.L. 1998. "Measurement of the modulation transfer function of paper," *Applied Optics*, vol.37, no. 31, pp. 7235–7240.

Ruckdeschel, F.R. and O.G. Hauser. 1978. "Yule-Nielsen effect in printing: a physical analysis," *Appl. Opt.*, 17(21), pp. 3376–3383.

Ukishima, M., H. Kaneko, T. Nakaguchi, N. Tsumura, M. Hauta-Kasari, J. Parkkinen, and Y. Miyake. 2009. "A Simple Method to Measure MTF of Paper and Its Application for Dot Gain Analysis," *IEICE Trans. on Fundamentals*, Vol. E92-A, No.12, pp. 3328–3335.

Viggiano, J.A.S. 1990. "Modeling the color of multi-colored halftones," *TAGA Proc.*, pp. 44–62.

Yule, J.A.C. and W.J. Nielsen. 1951. "Penetration of light into paper and its effect on halftone reproduction," *TAGA Proc.*, 65, pp. 749–758.

Yule, J.A.C., D.J. Howe, and J.H. Altman. 1967. "Effect of the spread-function of paper on halftone reproduction," *TAPPI* 50, 337.