Working Toward a Color Space Built on DE2000

John Seymour

Keywords: uniform color space, CIEDE2000

Abstract

CIEDE2000 is the most recent in a series of color difference formulas. It has become the standard color difference formula, despite two of its shortcomings. The first shortcoming is that it does not have an associated color space. The second shortcoming is that it is a very complicated formula. This ongoing work addresses both of these issues. The goal is to provide a color space where color differences very similar to CIEDE2000 can be computed using the Euclidean distance formula, and where the calculations are significantly simpler.

This paper provides an alternative for the L* value, introducing a much simpler computation that is a quite serviceable approximation to ΔE_{00} color differences in the L* direction. This is called L_{00}.

Background, (BC – before CIELAB)

In 1834, Ernst Weber devised the concept of "just noticeable difference". He researched our abilities to sense the difference between two weights, and concluded that humans are able to detect a difference in weight, provided the difference was more than about 3% of the weight. He assumed this to be generally applicable to all our senses, but perhaps with a constant other than 3%.

Gustav Fechner (1860) provided a mathematically equivalent form of Weber's law, which stated that our perception of physical quantities is proportional to the logarithm of the stimulus.

The Weber-Fechner law is generally applicable. The approximate validity of this is attested to by the fact that we measure sound intensity logarithmically (in decibels), the fact that each note on the piano keyboard is roughly 5.9% higher in pitch than the next one lower, and the fact that raises in salary are generally thought of in terms of percentages, rather than absolute amounts.

QuadTech

The law is clearly an approximation, particularly at the lowest levels of perception. To take an extreme example, the perceptual difference between 100% and 3% reflectance is literally white to black, whereas that same ratio between 0.01% and 0.001% reflectance is imperceptible.

This limiting factor is at least in part due to simple physics. A silicon photo detector has what is called "dark current", where heat can spontaneously perform the same action as visible light even in the presence of no light. The eye exhibits this same phenomenon. A person does not perceive absolute black when he has adapted to an absolute dark room.

Lens flare is another issue that limits all imaging systems from seeing "pure black". A small black area may register as 2% reflectance when surrounded by black, but may register as 5% when contaminated by scattered light from a white surround. The human eye is no different from any other imaging system.

Note that most experiments in perception are carried out with the eye "adapted" to a gray surround, which is to say, when the center of the field of view has been contaminated by a fixed amount of scattered light.

If a logarithmic response makes sense, then it is reasonable to add an offset before the logarithm function to account for the human visual system equivalent of dark current and lens flare. Delbouef did just this is 1873, arriving at the following formula:

$$D = 10 - 6.1723\text{Log}_{10}(40.7h + 1) \quad (1)$$

Where h is relative lightness, on a scale from 0 to 1, and D is the perceptual lightness. The constants in this formula are determined so that $D = 0$ means perfect white and $D = 10$ means pure black, upside from how it normally quantified today.

Richter and Witt borrowed this formula for the development of the German standard Color Chart (DIN 6164) in 1953. They aligned the system with Munsell so that black became zero and white became 10.

$$V = 6.1723\text{Log}_{10}\left(40.7\frac{Y}{Y_n} + 1\right) \quad (2)$$

In this case, V is the equivalent of the Munsell value, Y is the Y tristimulus value, and Y_n is the normalizing factor.

Stanley Smith Stevens proposed an alternative to the Weber-Fechner law, also intended to approximate the relationship between stimulus and sensory perception. He first developed this to model human vision in 1953, and published the general rule in 1957. His law says that perception is proportional to the stimulus raised to some power, generally a power less than 1.0.

Numerous researchers into vision have applied this to approximate the lightness scale, including Priest et al., Moon and Spencer, Saunderson and Miner, Ladd and Pinney, Glasser, and Wyszecki.

Psycho-physical experiments with the gray scale from the Munsell Book of Color became the basis for another line of research that eventually led to CIELAB. The following table summarizes the various formulas that had been proposed prior to CIELAB (1976).

Year	Authors	Equation
1873	Delbouef	$10 - 6.1723\mathrm{Log}_{10}(40.7h + 1)$
1920	Priest, Gibson, and Nicholas	$100\sqrt{Y}$
1933	Munsell, Sloan, and Godlove	$10\sqrt{1.474Y - 0.00474Y^2}$
1940	Foss, Nickerson, and Granville	$100 + 40\log_{10}(Y)$
1943	Newhall, Nickerson, and Judd	$1.2219V - 0.2311V^2 + 0.2395V^3 - 0.021009V^4 + 0.0008404V^5$
1943	Moon and Spencer	$100Y^{0.426}$
1944	Saunderson and Miner	$2.357Y^{0.352} - 1.52$
1953	Richter and Witt	$61.723\log_{10}(40.7Y + 1)$
1955	Ladd and Pinney	$1.658Y^{1/3} - 1.636$
1958	Glasser	$25.29Y^{1/3} - 18.38$
1964	Wyszecki	$25Y^{1/3} - 17$

***Table 1** – Collection of formulas for linear lightness*

Notes:
The equations listed above may not be the same equation as was originally published. Where possible, the equations have been scaled so that $0 \leq Y \leq 1.0$. Some of the equations (Priest et al., Munsell et al., Foss et al., and Richter et al.) have been multiplied by ten so that the output ranges as of L*, $\leq L * \leq 100$. It is unclear how to scale various other equations (Saunderson et al., Ladd et al., Glasser, and Wyszecki) scaled so as to match, so the equations have not been adjusted.

Wyszecki and Stiles (2000, p. 823) incorrectly report the Foss, Nickerson, and Judd formula as $0.25 + \log_{10}(Y)$. The original paper did not give the equation, just gave a table of 26 values for Y that are equally spaced. There are no numbers associated with each step, only "aa", "bb", etc. The equation above is my own derivation based on the table.

The Newhall, Nickerson, and Judd formula determines the Y value (reflectance) as a function of V, the perceptual quantity. Thus, a numerical solution is necessary. Moon and Spencer provided an approximation to this. Their original equation had a leading coefficient of 1.4. This was changed to 100 to give the proper scaling.

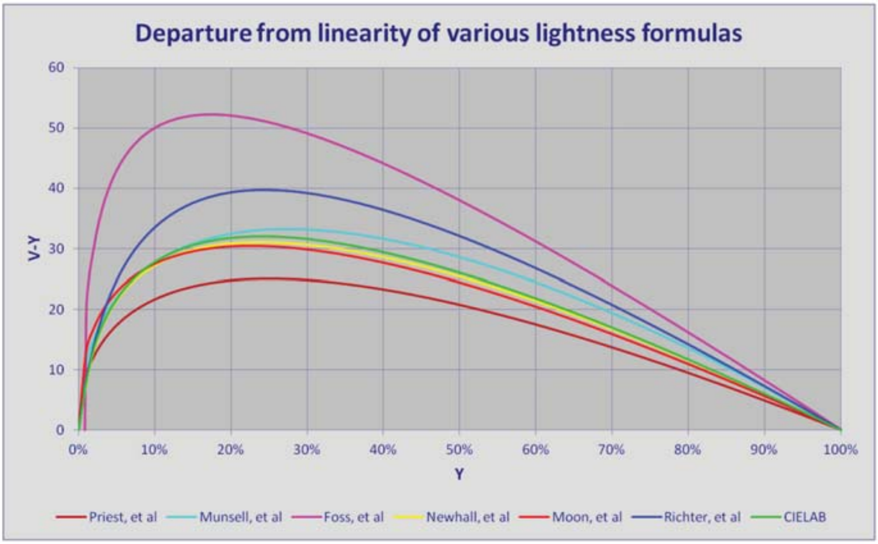

***Figure 1** – Comparison of various formulas for linearity*

For more information, see Norwich 1997, Wyszecki and Stiles 2000 (p. 823), Hunt 1991, (p. 155), Hunter, Chapter 9, Moon et al. 1943, and Saunderson et al., 1944, and CIE 2001.

CIELAB and beyond

A number of the formulas in the previous section follow the form of Stevens' power law, giving a clear progression to CIELAB. The CIELAB formula for L* is a modification of Stevens' power law:

$$L^* = 116f(Y) - 16, \tag{3}$$

where

$$f(Y) = (Y)^{1/3} \text{ , if } Y > \left(\frac{24}{116}\right)^3 \text{ and}$$

$$f(Y) = \left(\frac{841}{108}\right)(Y) + \frac{16}{116}, \text{ if } Y \leq \left(\frac{24}{116}\right)^3$$

Modifications to CIELAB were made almost immediately. One line of improvements began in 1976 by McLaren. Rather than devise a color space that outperformed the CIELAB formula, McLaren initiated the dubious practice of correcting the color differences determined from CIELAB based on the location in color space.

In addition to McLaren's formula, there have been at least six other McLaren-style correction formulas proposed:

ΔE_{JCP79} – McDonald (1979),
ΔE_{CMC} – McDonald, Clark, and Rigg (1986),
ΔE_{BFD} – Luo and Rigg (1987),
ΔE_{94} – CIE (1994),
ΔE_{LCD} – Kim (1997),
ΔE_{00} – CIE (2000).

The last one on the list, also known as CIEDE2000, has become an official standard, despite the fact that the official definition of it comprises 19 equations (including the equations to compute L*a*b* from XYZ), and includes 25 free parameters, most of which have no physical meaning. This level of complexity cannot be supported by the available data.

One disadvantage of the McLaren-style color difference modifications is that there is not an associated color space. Another basic problem with many of these formulas is that they are complicated formulas.

Another long list of researchers have developed color spaces:

Labmg – Colli, Gremmo, and Moniga (1989)
ATD – Guth (1994)
DCI-95 – Rohner and Rich, 1995)
LLAB – Luo, Lo, and Kuo (1995)
No name given – Tremeau and Laget (1995)
CIECAM97 – CIE standard (1997)
RLAB – Fairchild (1998)
IPT – Ebner and Fairchild (1998)
L*'a*'b*' – Thomsen (1999)
DIN99 – DIN standard 6176 (2000)
CIECAM02 – CIE standard (2002)
QTD – Granger (2008)
$L^E a^E b^E$ – Berns (2008)
LAB2000 – Lissner and Urban (2010)

Many of these are computations based on first calculating L*, a*, and b*, for example Labmg, DCI-95, LLAB, L*'a*'b*', DIN 99, and $L^Ea^Eb^E$. This adds an unnecessary complication to the formulas. If the CIELAB calculations don't work well, why build on top of them, rather than replace them?

The sheer length of the list of proposed uniform color spaces is demonstration that the problem has yet to be adequately addressed.

Principle of Parsimony

The number of free parameters that go into the formula is one measure of how complicated a formula is. As more parameters are incorporated, the danger of overfitting data increases.

For the purposes of this paper, we will consider only the parameters that go into the calculation of our perception of lightness, which is to say, the equivalents to the L* scale.

The L* equation includes three free parameters: one parameter defines the point where the function f turns linear, and the function L* has the two additional parameters: 116 and 16. Thus, the L* equation has a parsimony level of three.

(The equation for *f*(x) includes the numbers 24/116, 841/108, and 16/116. It could be argued that there are three free parameters. I would argue that the only free parameter was the decision to linearize the function from the point where the derivative would be continuous between that point and (0,0). Thus, I say that the function for L* has a parsimony level of three.)

The L** and L99 equations each add two parameters to those of L*, so that the parsimony level of these are both five.

Although ΔE_{CMC}, ΔE_{94}, and ΔE_{00} are not uniform color spaces, we can still evaluate them in terms of the total number of parameters that are included in computing the lightness component of the color difference. All of these color difference equations include a scaling of ΔL* by the value S_L.

For ΔE_{CMC}, the computation of S_L involves three additional parameters beyond the three involved with L*. The CMC equation for S_L actually has four numbers: there are two parameters in the ratio (0.049075 and 0.01765), a cutoff point of L* = 16, and a value of 0.511 for S_L for values less than 16. Since 0.511 is the value of S_L where L* = 16, I don't consider this an additional parameter, but rather a parameter derived from the others.

The ΔE_{94} equation is quite parsimonious, since the equation for the scaling of ΔL^* is $S_L = 1$. I do not count this as an additional parameter.

The calculation of S_L for ΔE_{00} includes three numbers. While there are three numbers in the scaling function S_L, I would argue that the form of the equation itself further increases the parsimony, since it requires taking the ratio of to algebraic phrases which themselves are of difference forms. The numerator is a polynomial in L*, and the denominator is the square root of a polynomial in L*.

Color difference equation	**# of parameters for ΔL**
ΔE_{ab}	3
ΔE_{CMC}	6
ΔE_{94}	3
ΔE_{00}	6
Color space	
LABmg	6
L** (Rohner and Rich)	5
ΔE_{99} (DIN 99)	5

Table 2 *– Parsimony of lightness component of various color difference formulas*

Stated in these terms, the goal of this project is to provide an equation for the lightness axis of a uniform color space, , which is more parsimonious than any of the equations in Table 2, and which provides a suitable approximation to changes in L* as computed in ΔE_{00}.

Experimental procedure

In the first step, a numeric approximation to L_{00} was tabulated. A sequence of L* values along the neutral axis was determined such that each step in the sequence is 1.0 ΔE_{00} from the previous. The sequence starts with L* = 0.000, 1.734, 3.442, 5.124, and so on. These values were used to create the L00 values, which will be as perceptually linear as ΔE_{00}. L* = 0 will correspond to $L_{00} = 0$, L* = 1.734 will correspond to $L_{00} = 1.0$, and so on. This sequence of L* values was translated to Y values. Figure 2 shows the relationship between Y (along the x axis) and L_{00} (along the y axis).

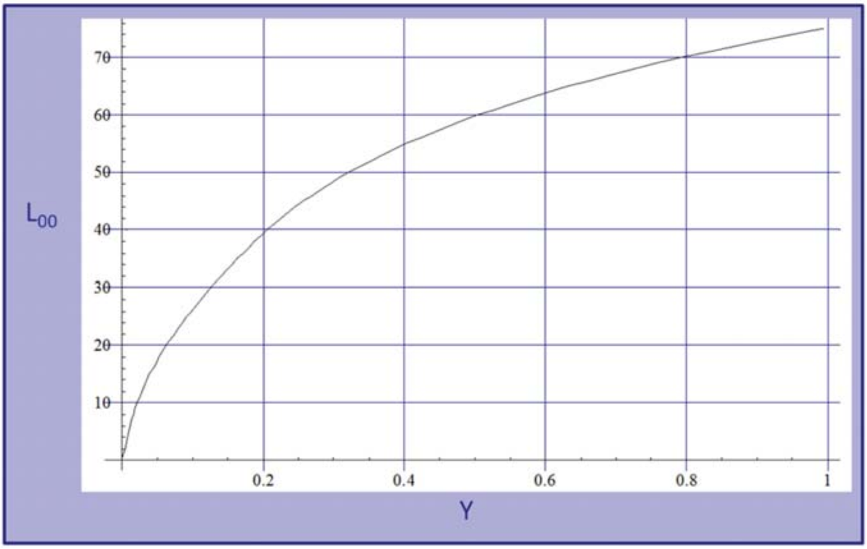

***Figure 2** – values of L00 as a function of Y tristimulus value*

Note that L_{00} values go up to about 76, which is to say, according to ΔE_{00} there are 76 levels of gray. This is about midway between the number of gray values in CIELAB (100) and the number in the title of the book (50).

Five different candidate functions were fit to the function in Figure 2. Table 3 shows the functions, the RMS error of the fit, and the parameters used. (For convenience, the scaling of Y by Y_n has been omitted in the equations.)

	Function	RMS	Parameters
Offset logarithm	$L_{00} = a \log_e(bY + 1)$	0.64 ΔE_{00}	a = 24.7, b = 20
Pade	$L_{00} = a \frac{Y}{b + Y}$	1.70 ΔE_{00}	a = 96.2, b = 0.281
Polynomial	$L_{00} = aY + bY^2 + cY^3 + dY^4$	1.69 ΔE_{00}	a = 328, b = 793, c = 943, d = -407
Exponential	$L_{00} = a(1 - e^{-bY})$	3.16 ΔE_{00}	a = 77.4, b = 3.5
decay			
Power	$L_{00} = aY^b$	3.19 ΔE_{00}	a = 75.2, b = 0.434

Table 3 – Results of various regressions

From this table, the offset logarithm function clearly provides the best fit to the data. The fact that it has a level of parsimony of 2 (less than that of L*) is a big bonus. The fact that it is based on a formula proposed by Delbouef 142 years ago is also interesting.

Figure 3 shows a comparison between color difference values computed with ΔE_{00} (line) and those computed with L_{00} (dots).

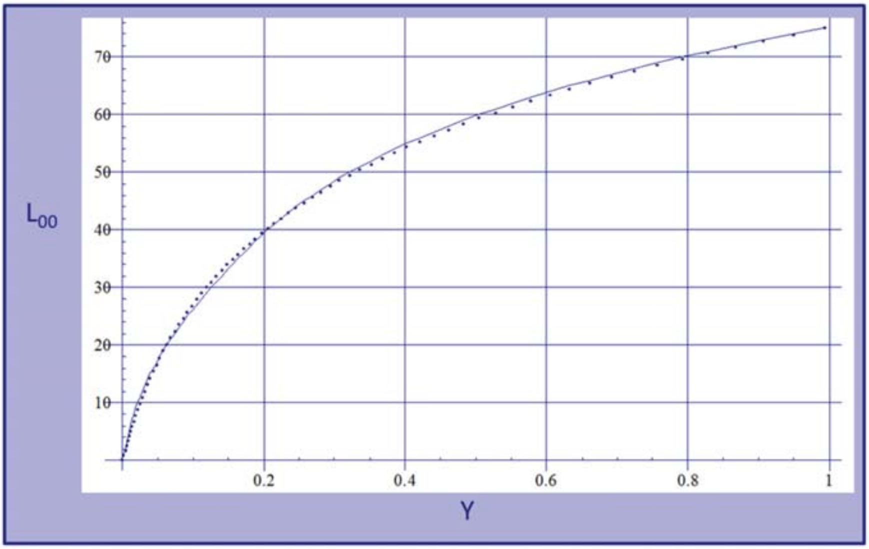

Figure 3 *– the log function (dots) as compared with the L00 function*

Figure 4 shows another comparison between color differences as computed in L_{00} and in ΔE_{00}. The y axis of this plot is the plot is the ratio of color difference values as computed by the two equations.

As can be seen, a color difference as computed by L_{00} is generally between 0.9 ΔE_{00} and 1.1 ΔE_{00}.

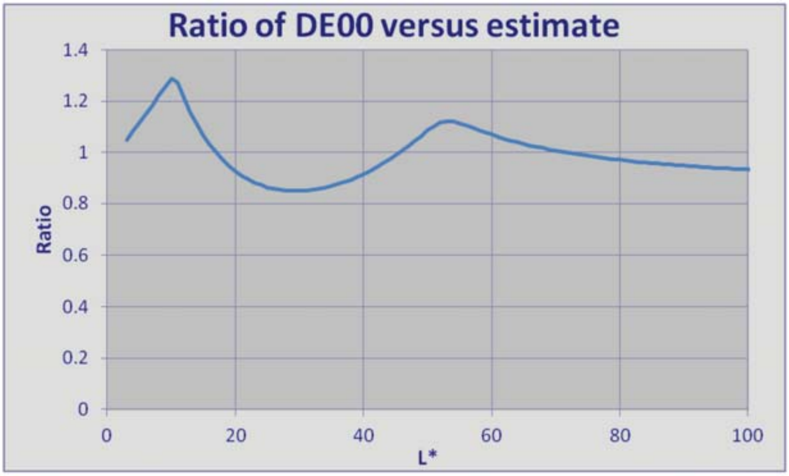

Figure 4 *– Comparison of the two formulas for color difference*

The blip below L* = 15 in Figure 4 deserves some discussion. At first it may seem a bit of a shortcoming of the offset logarithm formula that there is an error of about 29% around L* = 7. If the ultimate goal is to find a formula that approximates ΔE_{00}, then this would indeed be a shortcoming.

But is ΔE_{00} reliable in this region? A bit of history might be helpful. The research that led to CIELAB was directed at finding a Stevens' law equation that agreed well with Munsell data. But it was found that a straight power law could not approximate the data at the dark end. It was eventually discovered that shifting this formula downward (by subtracting 16 and then rescaling) brought the equation to within good agreement with the available data.

This unfortunately led to an anomaly at the very darkest end. At values of Y below 0.00262, the modified Stevens law gave a negative result. This was not of huge practical concern. That value represents an optical density of 2.58D, which is rare in the real world, and perceptually very close to being pure black. A suggestion was made by Pauli that the curve below Y=0.008856 be changed to a straight linear curve. His suggestion became standardized into the CIELAB formula.

It is extremely unlikely that this change from a cube-root and straight line response is physically manifested in the human visual system, and the use of two functions here is completely unsupported by data. So, CIELAB is suspect in areas near reflectance of 1%.

The switchover between linear and cube root-base functions occurs at L*=8, which is where the offset logarithm function has the most difficulty agreeing with ΔE_{00}.

The most reasonable conclusion is that the failure of L00 to match the ΔE_{00} function below L*=8 is not a failure of L00, but rather traces back to a deficiency of CIELAB.

The recommended formula

The formula arrived at through regression is quite simply

$$L_{00} = 24.7 \log_e(20Y + 1) \quad (4)$$

Note that the scaling on CIELAB L* is somewhat arbitrary. Are there truly 100 steps? Similarly, the scaling on ΔE_{00} is arbitrary. Like many other color difference formulas, it was not scaled to match "just noticeable differences". It was scaled so as to match ΔEab at L*=50.

For simplicity, Equation 4 can be approximated with integers as shown below. This is the recommended formula.

$$L_{00} = 25 \log_e(20Y + 1) \quad (5)$$

Extension to a*b*

This is a work-in-progress, since it only addresses one of the three axes of a color space. Variants of Equation 5 have been applied directly to the computation of analogs of a* and b*, but currently has not shown much improvement in estimating ΔE_{00} values.

Extension to a color appearance model

The computation of ΔE_{00} includes numerous obscure parameters that have no relationship with anything that is directly measureable in the real world. It would be a fool's errand to decide how to change the parameters to reflect differing viewing conditions.

One benefit of Equation 5, is that the parameters have a physical meaning. This opens the possibility that this equation can be used to extend L_{00} to other viewing conditions.

In the previous discussion of the Weber-Fechner law, it was pointed out that a purely logarithmic response to light is untenable, particularly at the dark end due to the fact that there will be some response (dark current and stray light) in the absence of true stimulus. This is reflected in the value "1" in Equation 5.

Note that "1" is not a free parameter. This value must be unity to ensure that Y=0 corresponds to $L_{00} = 0$.

The value of 20 in Equation 5 can be interpreted as the signal to noise ratio of the visual system. This suggests that this number can be adjusted so as to account for differences in signal to noise ratio, which includes ambient light. It is well known that the intensity of the background can affect our ability to discern differences in lightness. Thus, this parameter could be used to model our perception with different background levels.

The value of 25 in Equation 5 similarly has an easily explained meaning. This relates directly to the number of steps in the lightness scale. It is known that the number of just noticeable differences depends on the level of illumination. The leading coefficient in Equation 5 could be adjusted so as to reflect this.

Conclusions

The lightness axis of a new color space, L_{00}, has been introduced. This new equation is simpler than the formula for the corresponding part of ΔE_{00}, and is even simpler than the formula for L* itself. Yet, it agrees to within 10% with ΔE_{00} over most of the range, and disagrees in areas where there is little support for ΔE_{00}.

Due to its simplicity, the parameters can be readily related to physical quantities so that this formula has promise for the underpinning of a color appearance model.

Further research will be devoted toward extending this formula to the computation of a_{00} and b_{00}.

References

CIE, "Improvement to Industrial Colour Difference Evaluation", CIE Technical Report 142:2001

Colli, A., E. Gremmo, and P. Moniga, "Proposta per un nuovo spazio di colore (LABmg) lineare rispetto alle tolleranze CMC", Unificazione e Certificazione, anno XXXIII, n. 4, 33-42, (1989) (from "LABmg: New color space linear to the CMC tolerances", an English translation by P. Chausser.)

DIN 6176: "Farbmetrische Bestimmung von Farbabständen bei Köroerfarbennach der DIN99-Formel", Berlin: DIN Deutsche Institut für Normung e.V. 2000.

Hunt, R. W. G., Measuring Color, 2nd Edition, Ellis Horwood, 1991

Hunter, Richard S., The Measurement of Appearance, John Wiley and Sons, 1975

Lissner, Ingmar and Philipp Urban, "How perceptually linear can a hue linear color space be?", 18th Color Imaging Conference, 2010

Moon, Parry; Spencer, Domina Eberle (May 1943). "Metric based on the composite color stimulus". JOSA 33 (5): 270–277

Munsell, A.E.O.; Sloan, L.L.; Godlove, I.H. (November 1933). "Neutral value scales. I. Munsell neutral value scale". JOSA 23 (11): 394–411

Munsell, A.E.O.; Sloan, L.L.; Godlove, I.H. (December 1933). "Neutral value scales. II. A comparison of results and equations describing value scales". JOSA 23 (12): 419–425

Newhall, Sidney, Dorothy Nickerson, Deane Judd, "Final Report on the O.S.A. Subcommittee on the Spacing of the Munsell Colors", JOSA Vol 33, No 7, July 1943

Norwich, Kenneth, and Willy Wong, "Unification of psychophysical phenomena: The complete form of Fechner's law", Perception & Psychophysics, 1997, 59 (6), 929-940

Pauli, H., Proposed extension of the CIE recommendation on uniform color spaces, color difference equations, and metric color terms, 1976 J. Opt. Soc. Am. 66, pps. 866-867

Priest, Irwin G., Gibson, K.S., McNicholas, H.J., "An examination of the Munsell color system. I: Spectral and total reflection and the Munsell scale of Value". Technical paper 167 (3). United States Bureau of Standards. p. 27, September 1920

Rohner E, Rich DC. "Eine angenähert gleichförmige Metrik für industrielle Farbtoleranzen von Körperfarben", Die Farbe 1996; 42:207–220

Saunderson, Jason L.; Milner, B.I. (March 1944). "Further study of ω space". JOSA 34 (3): 167–173

Stevens, S. S., "On the psychophysical law", Psychological Review, Vol 64(3), May 1957, 153-181

P. Urban, D. Schleicher, M. R. Rosen, and R. S. Berns. "Embedding non-Euclidean color spaces into Euclidean color spaces with minimal isometric disagreement", Journal of the Optical Society of America A, 24(6):1516–1528, 2007.

Wyszecki, Guenther and W. W. Stiles, Color Science, 2nd Edition Wiley 2000