A Case for a Linear Color Difference Metric

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Abstract

At TAGA 2008 (Granger,2008), a paper on color differences showed that by adding the influence of the chromatic channels of color vision to the perception of brightness, a set of simple equations could predict color differences as well as the more complex equations of the CIE. This paper further develops the concept where the color space is limited to the sRGB or the ITU-R BT 709-5 color gamut (ISO).

The sRGB or the ITU-R BT 709-5 color standards dominate color reproduction in the communication and reproduction of images. The colorimetric vector space with a defined D65 white point has had successful commercial application to cameras, printers and television. A majority of images are being recorded with these standards. Pointer (Pointer, 1980) defined a gamut of real world colors that is larger than the sRGB color gamut, but success of the sRGB system in the commercial world suggests that on a statistical basis the extreme colors of Pointer's gamut are seldom seen in most of our images. The current paper limits its study of space linearity to the sRGB gamut since the majority of imagery is produced using this standard.

The study of linear transformations over the smaller gamut limit has led to a very simple set of equations that transform RGB values in the sRGB color space. The new QTD color space is shown to be more uniform than either CIE Luv or CIE Lab. This transformation also has the properties of being well correlated to the hue discrimination of the Munsell color space and of being able to predict substrate brightness or whiteness.

Introduction

The measurement and the specification of color have been well established since the 1930's. The use of the XYZ tristimulus values to define color matches is also well known. The problem with the XYZ tristimulus values is that they occupy a large volume of color space that is not used by modern display and reproduction devices.

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This problem persists with the introduction of the CIE-Luv and CIE-Lab Uniform Color Spaces. Both spaces extend well beyond the useful range required to describe the color operation of modern displays and printers. This can be avoided by limiting the support of color to the region defined by the ubiquitous sRGB or ITU-R BT 709-5 standards. The QTD Uniform Color Space proposed at TAGA (Granger, 2008) shows that a much simpler set of equations can describe color differences as well as the CIE standards. This paper further modifies the QTD over the sRGB color space to produce an even simpler uniform color space.

The CIE 1931 Standard Colorimetric Observer produced three independent color mixing functions that provide " ideal observer" matches using linear combinations of the color mixing functions, illumination and reflectance for two or more color samples. Therefore, small differences in two colors under the conditions of colorimetry should also be linear. The following paper illustrates that a simpler linear model of the Granger, 2008 paper produces a better transformation of the MacAdam ellipse data than CIE Luv or CIE Lab. The QTD limited color space is well correlated with the hue discrimination of the Munsell color system. The brightness vector, Q, is shown to be a better predictor of brightness.

The QTD Color Space

The first vector, Q, of the space is the brightness component. The term brightness is emphasized because Q is not the standard CIE-Y lightness used in all the other color spaces. This term corrects for the induced increase in brightness contributed by the chromatic channels of human vision. The original measurements for luminance were made using flicker photometry which eliminated the influence of the chromatic channels on the luminance measurement. This gave an excellent measure of luminance but not the brightness that is observed when the chromatic channels of color vision are involved in the perception of color. This effect is being used by most TV manufacturers to increase the apparent brightness of their monitors by setting the white point to a very blue D 10K white point. Measurement by a luminance meter would not predict the observed increase in brightness.

For the curious, this effect can be readily observed by using Photoshop. Set up two squares and fill one with a level of 255 blue. Observe the L* value in the info pallet. Then fill the second square with a gray that has the same L* value as the blue square. You will immediately be able to see that the brightness of the two squares at the same L* level are not equal. The gray square is considerably dimmer. Using Q as a measure of brightness, the gray square will equal that of the blue square.

The second vector, T, is associated with the unique hues, red and green, and the third vector, D, with the unique hues, yellow and blue. These two vectors support our perceptions of hue and colorfulness. T and D are described as color opponents. Therefore, the T vector can signal either red or green but not both and D can signal either yellow or blue but not both. The ratio metric actions of T and D relate to the perception of hue and the magnitude of T and D are related to the colorfulness or saturation of a color sample.

The QTD Color Space Definition

The development of the QTD color space is based on the ITU-R BT.709-3 standard, where the RGB primaries are defined as shown below.

	X	Y	Z
R	0.64	0.33	0.03
G	0.30	0.60	0.10
B	0.15	0.06	0.79

The development of the CIE DE2000 color difference metric is based on judgments made at the chromaticity coordinates shown in Figure 1. Most of the data gathered is contained within the sRGB gamut. This led to the possibility of developing a simpler set of color difference equations based on the sRGB primaries.

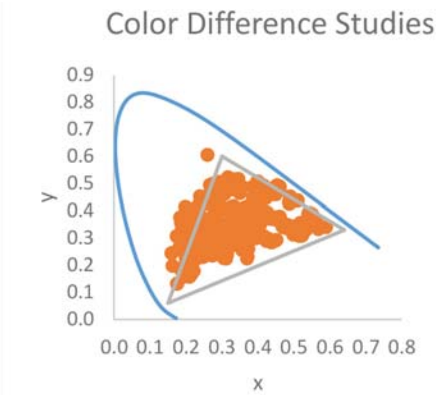

Figure 1 *Color Difference Studies*

S.L. Guth (Guth, 1973) proposed a vision model based on the idea of color opponents. He suggested a vector, A, to model the achromatic response of human vision. A second vector, T, to describe the Red-Green vision response. Finally, a vector, D, was described to model the Yellow-Blue vision response. The T vision channel produces a sensation of either Red or Green but not both and in the same manner the D channel produces either a Yellow or Blue response but not both. The next sections develop an approximation to Guth's model using the sRGB primaries as the basis vectors for A, T and D.

The T-D Vector Definition

The T and D color vectors are approximated by using the sRGB primaries as shown in Equations 1 and 2,

$$T = R - G \qquad (1)$$

$$D = (R + G) / 2 - B \qquad (2)$$

where R, G and B are the primaries of the sRGB gamut. These channels form the chromatic diagram shown in Figure 2.

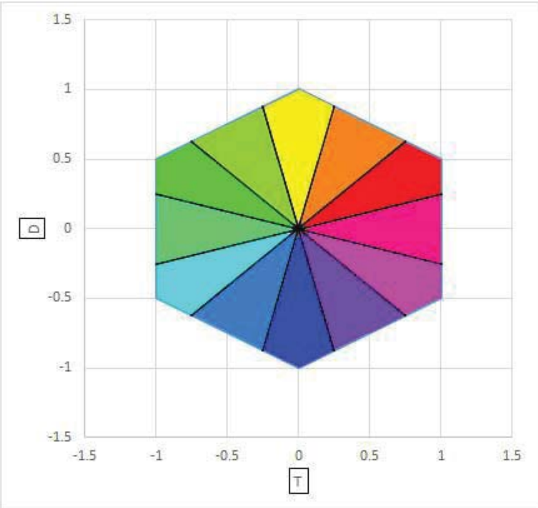

Figure 2 *The T-D Chromatic Plane*

This is the familiar color wheel that is seen in art books. The axis of the T and D vectors as defined by Equations 1 and 2 are alined with the unique hues of color vision (Kuehni, 2004). The T axis is aligned with the Red (510c) and Green (510) unique hues and the D axis passes thru Yellow (570) and Blue (465) unique hues.

Hue angles in the T-D dimensions are computed using the ratio of T and D. The ratio is computed by using,

if (abs(T) >abs(D)) Hue = S1* abs(D) / abs(T) + S2 Else (3)
Hue = S1* abs(T) / abs(D) + S2

where the constants S1 and S2 scale the hue angle have the same scaling as the Munsell color scale. Table 1 gives the values of S1 and S2 uses to convert T and D to hue angle.

Greater	T	D	D	-T	-T	-D	-D	T
Lesser	D	T	-T	D	-D	-T	T	-D
S1	12.5	-12.5	12.5	-12.5	12.5	-12.5	12.5	-125
S2	0	25	25	50	50	75	75	100

***Table 1** Scaling Values for Hue Angle Computation*

The correlation of QTD hue and Munsell Hue (Munsell Color system,1929) are shown on Figure 3.

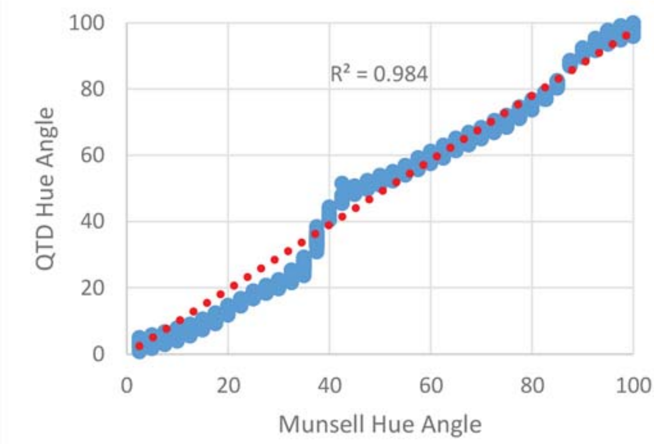

***Figure 3** QTD Hue vs. Munsell Hue*

The T and D vectors using the support of the sRGB primaries have been shown to produce a chromatic space that is both well correlated with the hue scaling of Munsell. The T-D plane froms a color wheel that is familiar to artist. The next section will develop the lightness and brightness aspects of color vision.

The Lightness and Brightness Definition

The achromatic channel, A is given by,

$$A = .2125 * R + .7152 * G + .0722 * B \qquad (4)$$

It should be noted that A is equivalent to CIE luminance,Y. A yields an accurate predition of brightness for an achromatic stimulus. When the stimulus is colorful, the stimulus will appear brighter that an achromatic stimulus having the same A value. This is is referred to as the Helmholtz-Kohlraush effect. The brightness to lightness ratios (Wyszecki and Stiles, 1982a) are plotted in Figure 4.

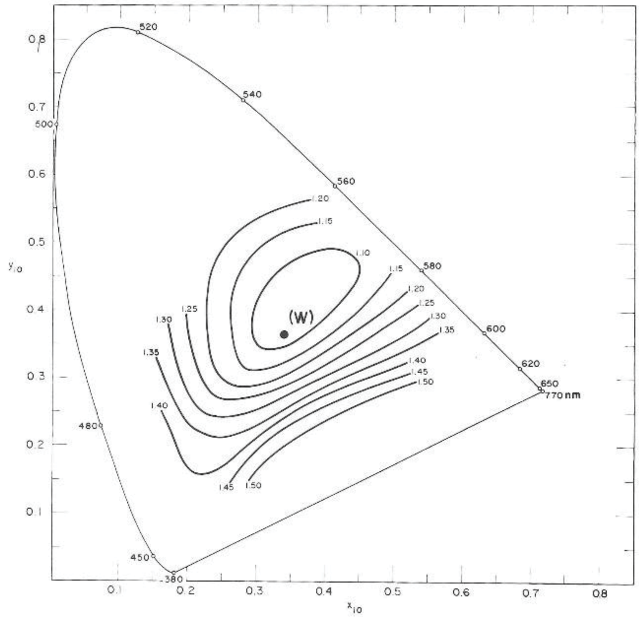

Figure 4 *Brightness- Lightness Ratios (CIE 1964 (x10, y10))*

All the color difference metrics are based on transformations of the CIE XYZ color vectors. None of these account for the intrusion of the chromatic channels in the perception of brightness. A new brightness vector, Q, is postulated that adds in the extra brightness contributed by the chromatic opponent channels of color vision. Q is defined,

$$Q = A + Red + Green + Yellow + Blue \qquad (5)$$

where Red, Green, Yellow and Blue represent the brightness contribution of each of the opponent vison channels. Equation 5 can be rewritten as function of A, T and D as,

$$Q = A + K^* (abs(T) + abs(D)) \qquad (6)$$

where the constant K can be determined using the suggested brightness-lightness ratios shown on Figure 4. A nonlinear regression is used to process 10,000 points in the CIE1931(x, y) plane. Unlike Figure 4, the white point used in the regression is D65. The resulting is a value of K of 0.125. The brightness– lightness contours produced by the regression are shown on Figure 5.

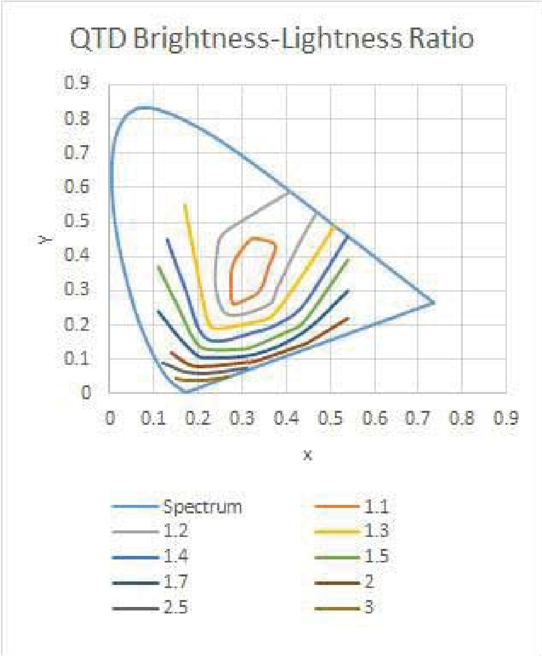

Figure 5 *QTD Brightness-Lightness Contours*

A more complex model for Q could be developed but this study is centered on using the minimum number of constants to form the color space model. Therefore, Equation 7 is the model for brightness used in the rest of this paper.

$$Q = A + 0.125 * (abs(T) + abs(D)) \qquad (7)$$

The CIE 1931 Color-Matching Functions (Wyszecki and Stiles,1982b) were converted to produces equivalent color matching functions for the QTD color system shown in Figure 6.

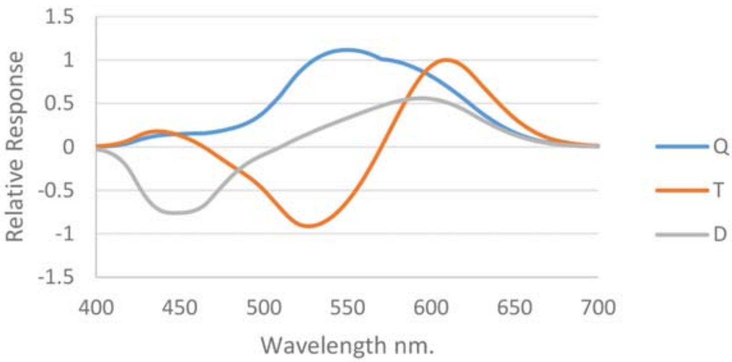

Figure 6 *QTD Color Mixing Functions*

The CIE luminance, Y, as explained above, was developed by flickering the light field to remove the influence of the chromatic channels. This led to the development of the 1931 xyz color mixing functions. Figure 7 shows the increased blue response of the Q vector caused by the contributions of the chromatic vectors. The Q vector is then correlated with the increase in brightness observed in colorful images.

The next section of this paper uses the linear definitions of Q, T and D to build a color difference metric.

Weber's Law- Interval Scaling

Weber's law states that the just-noticeable difference between two stimuli is proportional to the magnitude of the stimuli or an increment is judged relative to the previous amount. Weber's law is stated,

$$\Delta P = k^* \Delta S / S \qquad [8]$$

where S is the magnitude of a stimulus and ΔS is the amount S must be increased by to produce, ΔP, a just noticeable difference [JND] between S and S + ΔS.

Figure 8 shows Weber's law applied to luminance producing a brightness scale. The JND assumed for the luminance scale was 4% of the luminance value. The brightness scale developed using Weber's law is compared to similar scales used by the Munsell , the CIE Luv and CIE Lab color systems. The CIE brightness uses Steven's law where the reaction to a stimulus is assumed to be a power law relation of the form,

$$B = L^{(1/3)} \qquad [9]$$

where B is the brightness sensation produced by a luminance L.

Figure 2 compares the three color systems. The Weber's law system produces a log-linear relation between perceived brightness and luminance. Weber's law and Steven's law are well correlated as compared to the Munsell color system.

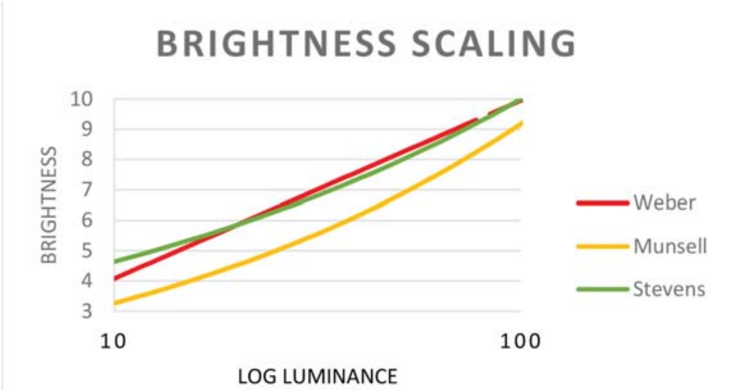

***Figure 7** Brightness Scaling*

Figure 9 plots the JND step for each of the color systems as a function of luminance. This plot shows that the JND steps are not uniform as compared to the Weber's law scaling. The CIE and Munsell systems JND steps are larger at low luminance and smaller at high luminance. If we assume that the brightness response of the human visual system is logarithmic, then Weber's law is the best fit to that assumption.

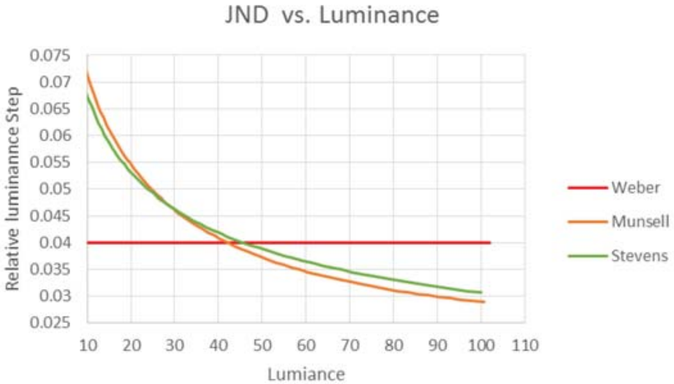

***Figure 8** Color System JND Performance Comparison*

Weber's law using linear luminance differences produces a brightness scaling that correlates well with CIE Lab and CIE Luv color systems. This correlation was achieved by assuming a 4% luminance step that gave a unit JND step. Figure 3 illustrates that this assumption is approximately the average of the JND scaling of the other systems considered in thus study.

Integrating Equation 8 above gives,

$$P = k * \ln(S) + C \qquad (10)$$

where ln is the natural logarithm and the constant of integration, C, is set to S0, the threshold of perception. Below this threshold perception the value of P is 0.0. Using S0 , we can rearrange Equation 10 to produce,

$$P = k * \ln(S / S0) \qquad (11)$$

The relationship given by Equation 11 is known as the Weber-Fechner Law. It shows that the relationships between a linear stimulus and perception of that stimulus is logarithmic.

The perception of the brightness vector, Q, can be treated in a similar fashion. Equation 12 expresses the perception of Q,

$$P = k * \ln(Q / Q0) \qquad (12)$$

Using the definition of Q in Equation 3, the change in perception in Q as a function of A, T and D are as follows.

$$\frac{\partial P}{\partial A} = k * \frac{\partial A}{Q}$$

$$\frac{\partial P}{\partial T} = k * 0.125 * \frac{\partial T}{Q} \qquad (13)$$

$$\frac{\partial P}{\partial D} = k * 0.125 * \frac{\partial D}{Q}$$

These are the rate of change in each of the physiologic dimensions of color space. The three derivatives can be combined in a Euclidean manner to define a color difference metric given in Equation 14,

$$\Delta P = k * \sqrt{} (\Delta A\text{^}2 + (.125*\Delta T)\text{^}2 +(.125*\Delta D)\text{^}2) / Q \qquad (14)$$

The next section of the paper uses the MacAdam Ellipse data (Wyszecki and Stiles,1982c) to compare the performance of ΔP to that of the CIE DE2000 in determining a JND of color difference.

The QTD MacAdam Ellipse Test

David MacAdam's color discrimination ellipse data is used to compare the performance of the QTD space to the CIE standards. Ellipse data in the region of the ITU-R BT.709-5 standard is used. Twenty of the original 25 ellipse centers fall within that range. The data for the 20 ellipses is listed in Table 1 below.

Figure 10 is a plot of the ellipse data in CIE 1931 (x,y) coordinates. The sRGB gamut is shown to illustrate the selection of ellipse data used in the study. The study assumed a value of 0.18 for A which corresponds to a CIE L value of 50.

The analysis of ΔP determined that value of k in Equation 14 has a value of 65.5 to produce a JND value of 1.0 for the ellipses shown on Figure 10.

QTD , CIE Luv (1976), CIE Lab (1976) and CIE DE2000 are compared for their ability to predict the JND observed by MacAdam. A perfect transform of the ellipse would result in a JND=1 for all points on the family of ellipses. The sample data set is 40 points around each ellipse. This is done to assure that the errors produced by each transformation would be accurately reproduced.

	x center	y center	10^3*a	10^3*b	Angle
1	0.16	0.057	0.85	0.35	62.5
2	0.187	0.118	2.2	0.55	77
3	0.253	0.125	2.5	0.5	55.5
4	0.258	0.45	5	2	92
5	0.28	0.385	4	1.5	75.5
6	0.38	0.498	4.4	1.2	70
7	0.228	0.25	3.1	0.9	72
8	0.305	0.323	2.3	0.9	58
9	0.385	0.393	3.8	1.6	65.5
10	0.472	0.399	3.2	1.4	51
11	0.527	0.35	2.6	1.3	20
12	0.475	0.3	2.9	1.1	28.5
13	0.51	0.236	2.4	1.2	29.5
14	0.596	0.283	2.6	1.3	13
15	0.344	0.284	2.3	0.9	60
16	0.39	0.237	2.5	1	47
17	0.441	0.198	2.8	0.95	34.5
18	0.278	0.223	2.4	0.55	57.5
19	0.3	0.163	2.9	0.6	54
20	0.365	0.153	3.6	0.95	40

Table 1 *MacAdam Ellipses (Observer PGN)*

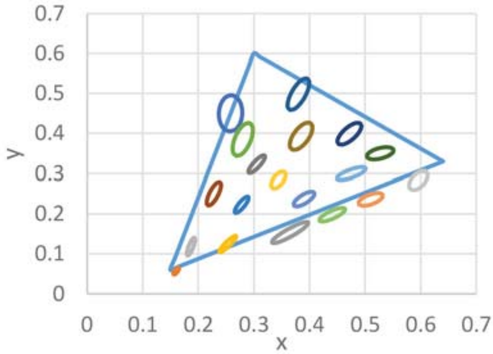

Figure 9 *MacAdam Ellipses in a sRGB Gamut*

The first test was to determine how well each metric was able to convert the ellipses on Figure 9 to circles of unit JND. The ratio of maximum JND to minimum JND was used to measure how well each color difference method transformed the 20 ellipses. Figure 10 plots the eccentricity for each ellipse and for the four color difference metric. A perfect transformation would yield a max to min ratio of 1.0. Figure 10 shows that none of the metrics were able to achieve this goal.

The average eccentricity of CIE Luv, CIE Lab, CIE DE2000 and QTD respectively are 2.56, 3.75, 2.94 and 1.95. As can be seen on Figure 10, QTD perform better than the rest. A surprise, CIE DE2000 performed the poorest.

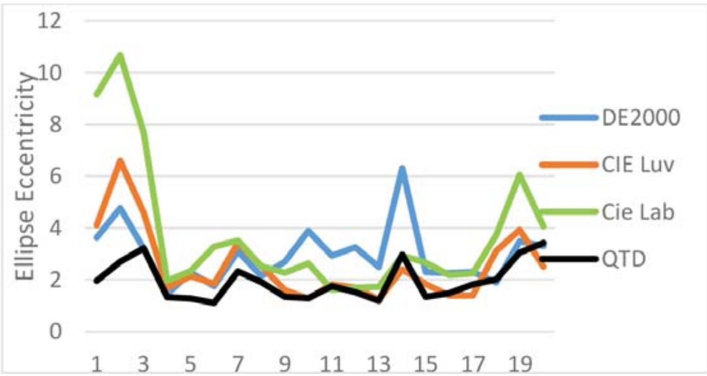

Figure 10 *Ellipse Eccentricity*

The second test was to determine how well each metric was able predict the JND for the ellipses on Figure 9.The maximum JND and minimum JND were used to computed the average JND each metric produced for the 20 ellipses.

Figure 11 plots the average JND for the four color difference metrics. A perfect transformation would yield an average JND of 1.0. Figure 11 shows how well the metrics were able to achieve this goal.

The average JND of CIE Luv, CIE Lab, CIE DE2000 and QTD respectively are 1.08, .90, .524 and 1.0. As can be seen on Figure 10, QTD, CIE Luv and CIE lab perform in a near equal fashion. A surprise, CIE DE2000 was the one outlier giving a prediction that is one half that of the other three.

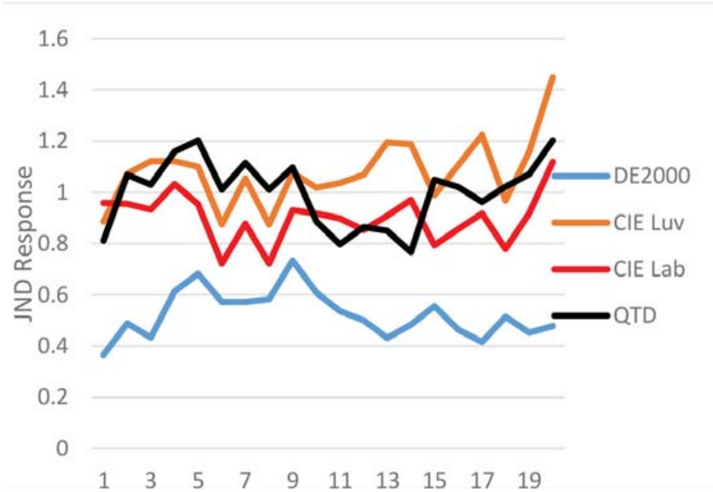

***Figure 11** JND Performance*

Conclusions

The QTD color space produces the first transformation that considers the intrusion of the opponent color channels on the perception of brightness. Q does an excellent job of predicting brightness and therefore could be a standard for applications such as displays and paper substrates.

The T-D chromatic plane is shown to be well correlated to the Munsell hue plane. The plot of T and D vector limits form an artist's color wheel. The T and D axes are oriented to pass thru the unique hues of color vision.

The QTD color space produces a color difference metric that is a linear function of the Q, T and D vectors. The QTD color difference metric yields a better transformation of the MacAdam ellipse data than CIE Luv (1976), CIE Lab (1976) and CIE DE2000.

As surprising result of the study is that the CIE DE2000 is a very poor predictor of color differences. Its results are about half those produced by the other color metrics.

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