A Corrected Kubelka-Munk Model for Color Prediction of Pre-colored Fiber Blends

Chun-ao Wei and Xiaoxia Wan

Keywords: pre-colored fiber blends, color prediction, Kubelka-Munk theory,
Saunderson correction, linearity of K/S

Abstract

The goal of this work is to propose a corrected single-constant Kubelka-Munk model for color prediction of pre-colored fiber blends. The K/S for the medium of pre-colored fiber blends does not hold good linearity with the proportion c, causing inaccurate color prediction of the single-constant model. Aiming at achieving good linearity of K/S, a new correction model for the measured reflectance has been established based on the inverse function of Sanderson correction. Cotton fibers blending samples were prepared to assess the color prediction accuracy. The average color difference of the corrected singleconstant Kubelka-Munk model was 0.82 CIEDE2000 unit, which was significantly better than that of the original model (~6.35). The results indicate the proposed model is much more suitable for color prediction of pre-colored fiber blends.

I. Introduction

Blending pre-colored fibers to obtain a wide variety of colors is an important coloration method in the textile industry. Generally, the blends of a limited number of pre-colored fibers (or primaries) can obtain any desired color within the color gamut by formulating recipes. In this field, one of the most important problems is to seek the appropriate recipe for a given color standard. Therefore, it is fundamental to establish color prediction models to describe the corresponding relationship between the recipes and the blended colors.

Wuhan University

The most popular color prediction models now in use for pre-colored fiber blends are based on the Kubelka-Munk (KM) theory[1, 2] and Duncan's additivity theorem[3]. For an infinitely thick, opaque layer the KM equation can be written as,

$$\frac{K_\lambda}{S_\lambda} = \frac{(1-R_\lambda)^2}{2R_\lambda} \tag{1}$$

Where R is the reflectance of the layer; K and S are absorption and scattering coefficients, respectively; λ indicates a specific wavelength. According to the Duncan's additivity theorem[3], the absorption and scattering coefficients for a mixture (K_{mix} and S_{mix}) can be expressed as linear combinations of those for the individual components (K_i and S_i) and their proportions (c_i), which is the socalled two-constant KM model,

$$\begin{aligned} K_{mix,\lambda} &= c_1 K_{1,\lambda} + c_2 K_{2,\lambda} + \ldots + c_N K_{N,\lambda} \\ S_{mix,\lambda} &= c_1 S_{1,\lambda} + c_2 S_{2,\lambda} + \ldots + c_N S_{N,\lambda} \end{aligned} \tag{2}$$

Burlone suggested the application of the two-constant model to the pre-colored fiber blends[4-6]. In his study, nylon fiber blending samples were used to test the color prediction accuracy. The average color difference of the two-constant model was 1.6 CIELAB units, which was significantly better than that of the Stearns-Noechel model (~2.4) and the Friele model (~2.7)[5]. Therefore, Burlone believed that the two-constant model could be useful in color prediction of pre-colored fiber blends. However, sample preparations and mathematical calculations are considerably complex in implementing the two-constant model since a large number of parameters (K and S for each primary within a blend) are required to be determined[7]. For simplicity, researchers argued that in the case of precolored fibers, most of the scatter was introduced by the fiber surface rather than dyes or pigments inside the fiber[5, 8]. In other words, the scattering coefficient S of pre-colored fibers depends on the fiber itself, regardless of the internal dyes or pigments. In this case, the two optical coefficients K and S can be simplified to a single coefficient K/S, which is the so-called single-constant model,

$$(K/S)_{mix,\lambda} = c_1(K/S)_{1,\lambda} + c_2(K/S)_{2,\lambda} + \ldots + c_N(K/S)_{N,\lambda} \tag{3}$$

In terms of practical application, the single-constant model is very attractive because only one optical coefficient (K/S) is needed to characterize each primary and the coefficient can be directly derived from a measurement of reflectance according to Eq. (1). As shown in Eq. (3), the linearity between the K/S of primaries and their respective proportions in a mixture is the basis of the single-constant model to be established. In other words, the better the linearity is, the more accurate the model is. Unfortunately, it has been proved that the K/S for the medium of pre-colored fiber blends does not hold good linearity with the proportion [5]. As a result, the single-constant KM model yielded poor accuracy in color prediction of pre-colored fiber blends.

The aim of this study is to correct the single-constant KM model for use in the color prediction of pre-colored fiber blends. To achieve high accuracy of the model, a new correction method of the measured reflectance has been established by improving the linearity between *K*/*S* of primaries and their proportions. In the present study, cotton fiber blending samples were prepared. A set of samples with known spectral reflectance and recipes were used to test the color prediction accuracy of the corrected single-constant KM model. And the results were compared with those obtained by the original model.

II. Materials

In the present study, five primary cotton fibers were adopted to prepare blending samples, including four pre-colored fibers dyed by reactive dyes and the raw undyed fibers. The cotton fibers were with the linear density of 1.67 dtex and the average length of 37mm. The reflectance of these five primary fibers is shown in Fig. 1, in which primary #1 is the raw undyed fibers.

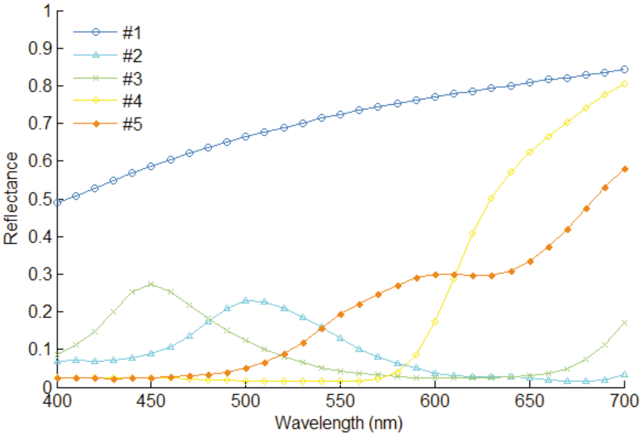

Figure 1. *Reflectance of the five primary fibers used in the present study*

To obtain reliable and repeatable color measurement data, the samples were prepared in the form of knitted fabrics with the property of uniformity and stabilization. At first, the primary fibers were fed into a small-sample carding machine three times to obtain homogeneous mixtures as much as possible. Then the mixtures were spun into yarns with the count of 29.2 tex and twist coefficient of 450 atex by open-end spinning. The yarns were eventually knitted into single jersey fabrics with 24 threads /inch. In addition, the samples to be measured were folded in thick-enough layers to avoid the translucent effect and no glass cover or sample holder was attached to the samples.

Color measurement of the samples were carried out by X-rite Ci7800 spectrophotometer with the optical geometry of d/8 system. The measured values were recorded as 31-dimensional spectral reflectance data at intervals of 10nm in the visible wavelength range (400-700nm), and the specular component of the reflectance was excluded. To minimize the measurement error as much as possible, the largest aperture of 25mm was selected to randomly measure nine times at different locations and directions over the whole surface of samples. Then the average of the nine measurements was taken as true color data of the samples.

As described above, a total of 60 pre-colored cotton fiber blending samples were prepared and measured, as shown in Fig. 2. The samples were divided into two groups. The first group included 16 samples, for which the four pre-colored fibers (#2~#5) were respectively blended with the raw fibers #1 at the proportion of 20%:80%, 40%:60%, 60%:40% and 80%:20%, as shown in Fig. 2(a). They were used to assess the linearity of *K/S* in the medium of pre-colored fiber blends and establish the correction method of measured reflectance. The second group included 44 randomly designed samples, including 23 two-primary, 10 three-primary and 11 four-primary blending samples, which were used to test the color prediction accuracy of the proposed model, as shown in Fig. 2(b).

***Figure 2.** Pre-colored fiber blending samples prepared for the present study*
(a) Samples for establishing the correction method of measured reflectance
(b) Samples for testing color prediction accuracy

III. Experimental

As mentioned above, the linearity between the *K/S* of primaries and their respective proportions in a mixture is the basis of the single-constant KM model to be established. In this section, the linearity of *K/S* in the medium of pre-colored fiber blends has been assessed. And based on the test results, a correction method of the measured reflectance has been established by improving the linearity of *K/S*.

A. Linearity of K/S in the medium of pre-colored fiber blends

Assuming that two primary fibers are mixed together at the proportion of c: 1-c and the *K/S* holds good linearity with the proportion c, then the single-constant KM model can be written as,

$$(K/S)_{\mathrm{m},\lambda} = c(K/S)_{q,\lambda} + (1-c)(K/S)_{p,\lambda} \tag{4}$$

where subscripts p and q indicate the two different primaries; m indicates the mixture; λ indicates a specific wavelength; (K/S)m, λ, $(K/S)p$, λ, and $(K/S)q$, λ are calculated by Eq. (1) using the actually measured reflectance R. By transposition and collection of terms, Eq. (4) can be rearranged as,

$$c = [(K/S)_{\mathrm{m},\lambda} - (K/S)_{p,\lambda}] / [(K/S)_{q,\lambda} - (K/S)_{p,\lambda}] \tag{5}$$

If $[(K/S)$m, λ-$(K/S)p$, $\lambda]$/ $[(K/S)q$, λ-$(K/S)p$, $\lambda]$ is plotted against c in Cartesian coordinate system, it will be found that a straight line with a slope equal to 1 can be obtained at each wavelength.

To assess the relation, the pre-colored fibers (#2~#5) were blended with the raw fibers #1 at the proportion of 20%:80%, 40%:60%, 60%:40% and 80%:20% to prepare test samples, as mentioned in the Section II. The test data of samples prepared by the different primaries (#2~#5) showed similar results. Thus in this paper, only the data of samples prepared by primary #2 were given below, and the results are shown in Fig. 3(a).

As expected, the desired straight lines are not obtained but are actually replaced by the convex curves, indicating the K/S for the medium of pre-colored fiber blends does not hold good linearity with the proportion c.

In the derivation of the KM equation (Eq. (1)), Kubelka and Munk assume that the index of refraction of the colored film is the same as that of the medium in which the measurements are made, that is, there is no internal surface reflection and multiple scattering in the medium. This assumption is not valid in practice since the measurements are made in air. To correct for this difference, Saunderson proposed a correction method of the measured reflectance [7],

$$R_{Saunderson} = \frac{R_{measured} - k_1}{1 - k_1 - k_2(1 - R_{measured})} \tag{6}$$

where $R_{Saunderson}$ and $R_{measured}$ represent the corrected reflectance by Saunderson correction and the actually measured reflectance, respectively; k_1 and k_2 are two variable parameters, which are usually set to k_1=0.04 and k_2=0.6 [7].

Considering the discontinuity of refractive index existing in the medium of pre-colored fiber blends, we carried out Saunderson correction to the measured reflectance before calculating (K/S)m, λ, $(K/S)p$, λ, and $(K/S)q$, λ, then the linearity of K/S was assessed again as described above. The results are shown in Fig. 3(b). For compare, the average of $[(K/S)$m, λ–$(K/S)p$, $\lambda]$/ $[(K/S)q$, λ–$(K/S)p$, $\lambda]$ at all wavelengths were taken to plot against the proportion c, as shown in Fig. 3(c). Unexpectedly, after Saunderson correction to the measured reflectance, the linearity of K/S has not been improved, but rather more convex.

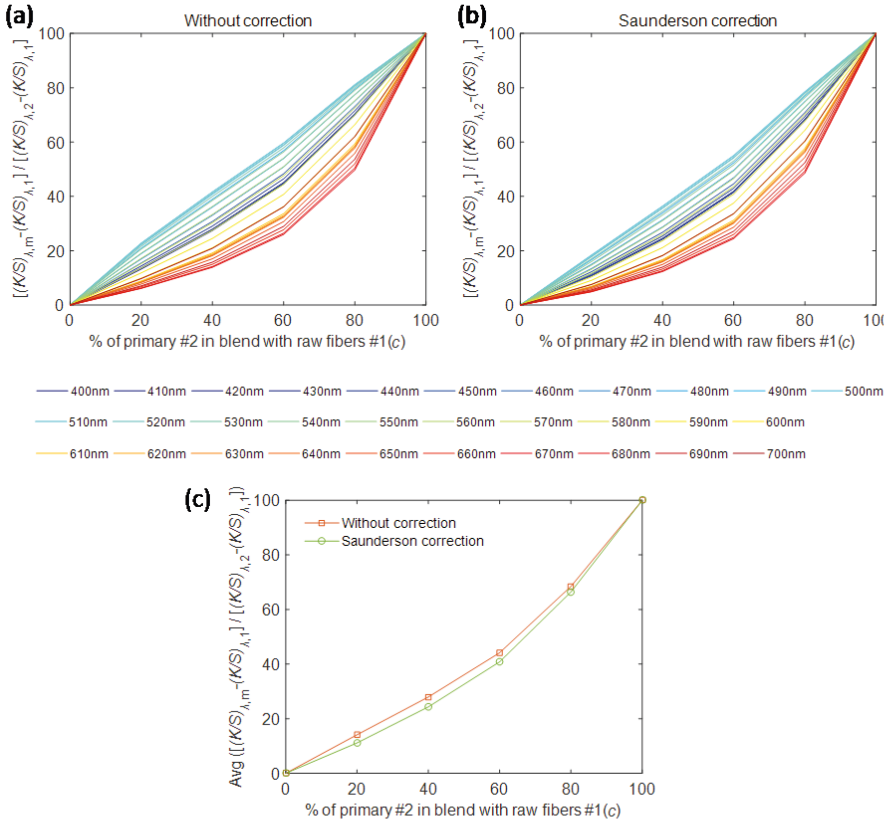

***Figure 3.** Linearity of K/S in the medium of pre-colored fiber blends. (a) Original KM model at each wavelength (b) KM model with Saunderson correction at each wavelength (c) Average of all wavelengths*

B. A new correction method of measured reflectance

The poor linearity or the approximate convexity of the K/S in the medium of pre-colored fiber blends has been proved in the Section III(A). It is interesting to note that, after Saunderson correction to the measured reflectance, the linearity of K/S has not been improved, but rather more convex. In other words, the Saunderson correction has a negative effect on improving the linearity of K/S. Therefore, to achieve good linearity of K/S, an efficient method is to correct the measured reflectance in the opposite direction to the Saunderson correction. Based on this consideration, a new correction method has been established to achieve good linearity of K/S based on the inverse function of Saunderson correction.

The new correction formula can be written as,

$$R_{our} = k_1 + \frac{(1-k_1)(1-k_2)R_{measured}}{(1-k_2R_{measured})} \qquad (7)$$

where k_1 and k_2 are two variable parameters. Given that the purpose of correction is to compensate convexity and then achieve good linearity of *K/S*, the problem to seek k_1 and k_2 values becomes an optimization problem. And the optimal values of k_1 and k_2 should minimize the linear deviation [9],

$$(k_1,k_2) = \arg \min_{(k_1,k_2)\in R^+} \left\{ \sum_{\lambda=1}^{m} \left[\sum_{i=1}^{n} \left(c_i - c_{\lambda,i}\right)^2 / n \right]^{1/2} \right\} / m \qquad (8)$$

where ci indicates the sampling points of the ideal straight line with the slope equal to 1; c_i,λ indicates the sampling points of the actual curve at wavelength λ, and the actual curve is obtained by plotting $[(K/S)_{m,\lambda}-(K/S)_{p,\lambda}]/[(K/S)_{q,\lambda}-(K/S)_{p,\lambda}]$ against c as described in Section III(A), where $(K/S)_{m,\lambda}$, $(K/S)_{p,\lambda}$, and $(K/S)_{q,\lambda}$ are calculated after carrying out the new correction to the measured reflectance; n indicates the number of the sampling points; m indicates the number of the wavelengths. In this study, the Interior Point Method[10] is used to solve this optimization problem, obtaining the optimal values of k_1=0.056 and k_2=0.153.

After the new correction to the measured reflectance, the linearity of K/S has been improved significantly compared with that of the original KM model (Fig. (3)), as shown in Fig. 4. It can be seen that the curves in Fig. 4(a) are very nearly to the straight line with the slope equal to 1, indicating that very good linearity of *K/S* has been achieved by the new correction method. It could be concluded that the new correction method of measured reflectance is reasonable and beneficial.

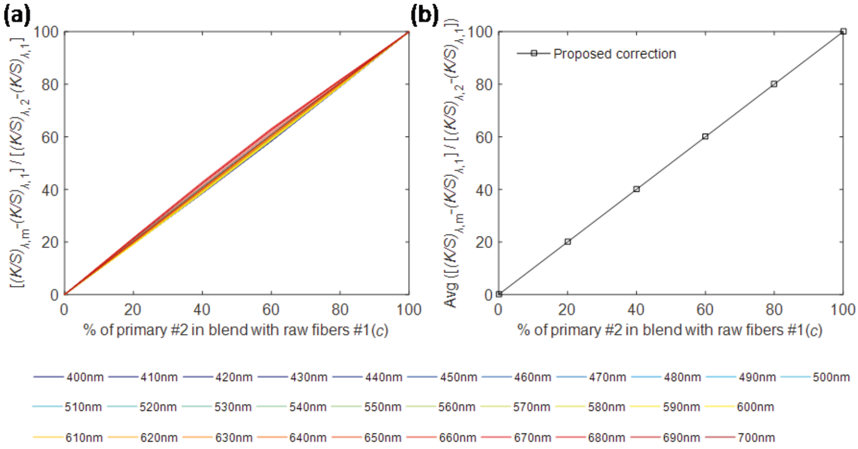

Figure 4. *Linearity of K/S in the medium of pre-colored fiber blends. (a) KM model with our proposed correction method at each wavelength (b) Average of all wavelengths*

IV. Results and Discussions

In this section, 44 samples specified in the Section II were used to test the color prediction accuracy of the corrected single-constant KM model. For the sake of comparison, the original single-constant KM model and the single-constant KM model with Saunderson correction were also tested by the same 44 samples.

Two criterions were adopted to evaluate color prediction accuracy. The RMSE between the measured and predicted reflectance was calculated as the spectral metric. The CIEDE2000 color difference formula [11], which is the most uniform color difference formula so far, was selected as the colorimetric metric. The colorimetric values for the color difference formula was obtained under the CIE standard illuminant D65 and the CIE 1931 standard observer.

The color prediction results of all the 44 test samples are shown in Fig. 5, and TABLE I shows the statistical results. As it can be observed, the color prediction accuracy of our proposed model is significantly superior to that of the original single-constant KM model in terms of both spectral metric RMSE and colorimetric metric DE2000. The average value of RMSE is improved to 0.0027 from 0.0306 while the average color difference DE2000 is improved to 0.82 from 6.35. Especially, the average color difference of the corrected KM model was less than 1, which met the colorimetric demand in the textile industry. It could be appreciated that our proposed model was more suitable for color prediction of pre-colored fiber blends and achieved high color prediction accuracy.

It should also be noted that, the color prediction accuracy of KM model with Saunderson correction is slightly less than that of the original model. This result is predictable, because the Saunderson correction makes the linearity of *K/S* a little worse. This observation further confirms that it is reasonable to improve the color prediction accuracy of the single-constant KM model by improving the linearity of *K/S*.

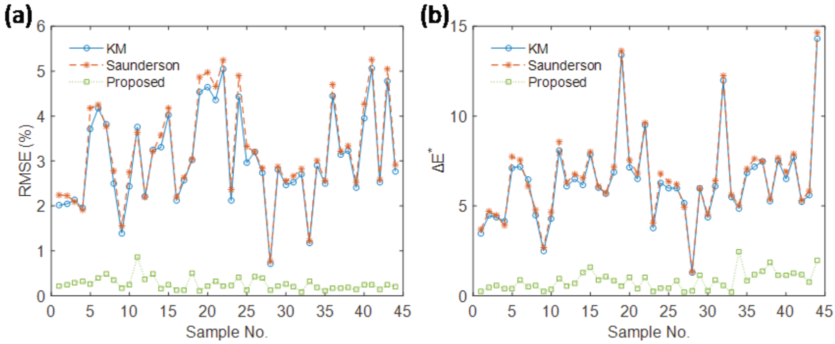

Figure 5. *Color prediction results of 44 test samples.*
(a) The spectral metric RMSE (b) The colorimetric metric ΔE^*

Metric	RMSE			ΔE^*		
Model	Mean	Min	Max	Mean	Min	Max
KM	0.0306	0.0071	0.0506	6.35	1.28	14.31
Saunderson	0.0321	0.0077	0.0526	6.55	1.32	14.63
Proposed	0.0027	0.0009	0.0087	0.82	0.20	2.46

Table 1. *Staticstic of the color prediction results*

V. Conclusions

In this paper, a corrected single-constant KM model was proposed for the use of color prediction of pre-colored fiber blends. Good linearity of *K/S* has been achieved by a new correction method of the measured reflectance. The color prediction accuracy of the corrected model was assessed by 44 pre-colored cotton fiber blending samples. The color prediction accuracy was significantly better than that of the original KM model. From the results, it could be concluded that the proposed correction method of the measured reflectance was reasonable and beneficial, and the corrected single-constant KM model achieved high accuracy of color prediction of pre-colored fiber blends.

Acknowledgment

This work was supported by the National Natural Science Foundation of China [grant numbers 61575147, 61275172].

References

1. P. Kubelka and F. Munk, “An article on optics of paint layers,” *Z Tech Phys, vol. 12*, pp. 593-601, 1931.
2. P. Kubelka, “New contributions to the optics of intensely light-scattering materials,” *JOURNAL OF THE OPTICAL SOCIETY OF AMERICA AOPTICS IMAGE SCIENCE AND VISION, vol. 38*, pp. 448-457, 1948.
3. R. D. Duncan, “The colour of pigment mixtures,” *Proc Phys Soc, vol. 52*, pp. 390-401, 1940.
4. D. A. Burlone, “Formulation of Blends of Pre-colored Nylon Fiber,” *COLOR RESEARCH AND APPLICATION, vol. 8*, pp. 114-120, 1983.
5. D. A. Burlone, “Theoretical and practical aspects of selected fiber-blend color-formulation functions,” *COLOR RESEARCH AND APPLICATION, vol. 9*, pp. 213-219, 1984-12-01 1984.
6. D. A. Burlone, “Effect of Fiber Translucency on the Color of Blends of Precolored Fibers,” *Textile Research Journal, vol. 60*, pp. 162 -167, 1990-03-01 1990.
7. H. Davidson and H. Hemmendinger, “Color prediction using the two-constant turbid-media theory,” *JOURNAL OF THE OPTICAL SOCIETY OF AMERICA A-OPTICS IMAGE SCIENCE AND VISION, vol. 56*, pp. 1102-1109, 1966.

8. J. Cai, D. Han, C. Chen, and S. Chen, “Application of the golden section search algorithm in the nonlinear isoconversional calculations to the determination of the activation energy from nonisothermal kinetic conversion data,” *SOLID STATE SCIENCES, vol. 12*, pp. 829-833, 2010.
9. C. Wei, X. Wan and J. Li, “Color prediction model for pre-colored fiber blends based on modified Stearns-Noechel function,” *DYES AND PIGMENTS, vol. 147*, pp. 544-551, 2017.
10. S. Mehrotra, “ON THE IMPLEMENTATION OF A PRIMAL-DUAL INTERIOR POINT METHOD,” *SIAM JOURNAL ON OPTIMIZATION, vol. 2*, pp. 575-601, 1992.
11. M. R. Luo, G. Cui and B. Rigg, “The development of the CIE 2000 colour-difference formula: CIEDE2000,” *COLOR RESEARCH AND APPLICATION, vol. 26*, pp. 340-350, 2001.