Metameric Proclivity

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Abstract

Often, an ink must be formulated to match a CIELAB value without any required spectral match. In this case, the ink formulator (whether in the ink company, or in the printer's ink kitchen) has a number of possible combinations of pigments from which to choose. Without a spectrum to match against, it is not possible to select one formulation that minimizes the metamerism. Measuring a metameric index requires a target spectrum.

If two objects match under one illuminant, then the metameric index is the color difference between them under a second illuminant. It is a measure of how one object changes color with respect to another object. Instead of print buyers specifying an upper limit for the metameric index under specified illuminants, they may make the deceptively simple request that the color of the ink formulation change as little as possible under different illuminants.

Naively, one might set the rule that the best formulation will be the one where the color difference between the ink under D50 and under D65 will be minimized. One of the conclusions of this paper is that this would be ineffective. If an object changes CIELAB values under a second illuminant, the majority of the false color inconstancy is likely attributable to a flaw in CIELAB.

A color space called ConeLab was recently introduced [Seymour, 2020] which corrects this flaw. ConeLab mimics CIELAB, but it is based on the spectral response of the cones rather than the xyz tristimulus functions. This paper demonstrates that ConeLab shows considerably less change in color values with an illuminant change. This suggests a different way of answering the root question: Which ink formulation has the smallest change in apparent color? Which formulation is least likely to cause metameric issues?

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Issues with CIELAB

Why compute CIELAB from tristimulus values, and not cone functions?

It is worth noting that the xyz tristimulus functions that were standardized in 1931 are not an approximation of the spectral response of the cones, nor were they ever intended to be. As can be seen in Figures 1 and 2, the cones have considerably more overlap between L and M than the overlap between x and y. More obviously, the x tristimulus function is bimodal, whereas L, M, and S are all unimodal.

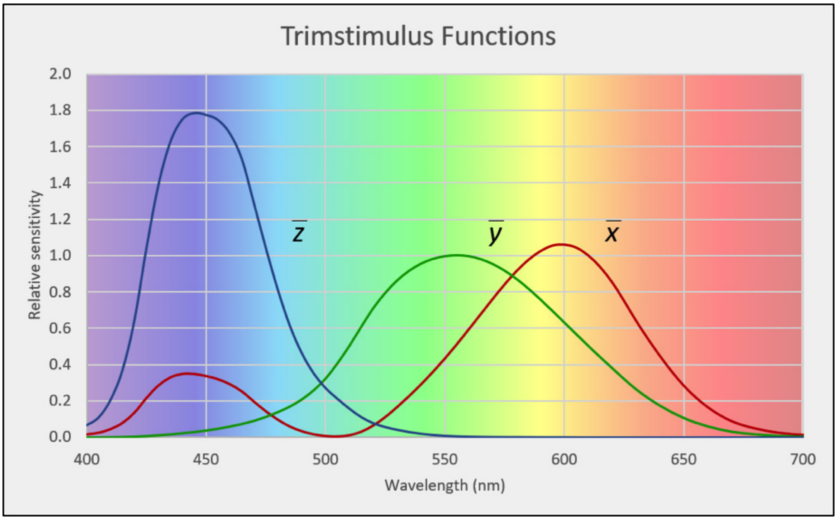

***Figure 1** – The 1931 tristimulus functions*

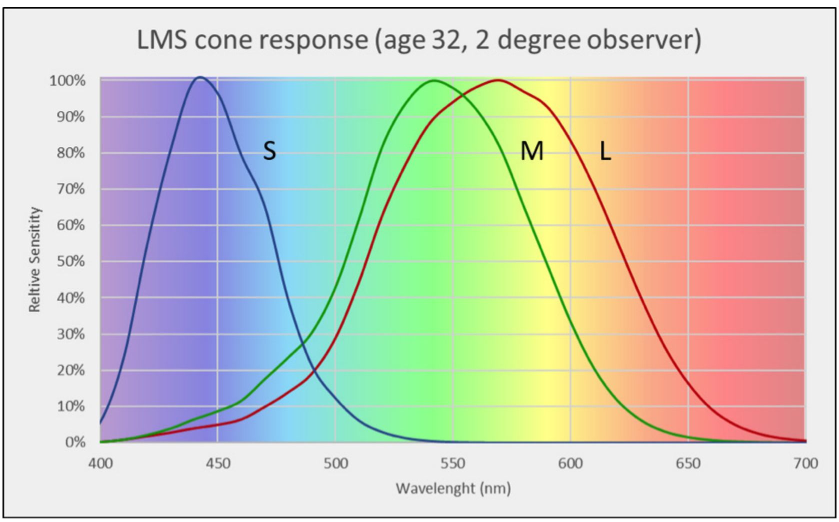

***Figure 2** – The cone functions (from Fairchild, website)*

Before 1931, color measurement was often done with a color matching device. In this device, the user would adjust the intensity of three lights (typically red, green, and blue) until a suitable match was found. The three settings became the color measurement. This method for color measurement was cumbersome, but more importantly, the technique did not have good repeatability amongst users.

Spectrophotometers were available at the time, but what was lacking was a standardized way to convert from a spectrum to a set of color matching values that would emulate those of a color matching device. But which device to emulate? Which version of the primaries ("red", "green", and "blue") would be used in a hypothetical color matching device?

This task of creating color matching functions was undertaken by two British researchers in 1929 and 1930: John Guild (of the British National Physics Laboratory) and W. David Wright (of the University of London). Between them, a total of 17 men were recruited. Guild used monochromatic illuminants at 435.8 nm (blue), 546.1 nm (green), and 700 nm (red) as his primaries. Wright used 460 nm, 530 nm, and 650 nm. After correcting for their use of different primaries, the two data sets were largely in agreement, and were averaged.

The averaged color matching functions can be found in Appendix B of CIE 15:2004, and are called $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$. If you multiply these three functions by a spectrum, and add the products, you will get numbers in reasonable agreement with those of Guild's color matching device.

But they are not the tristimulus values that we use today to compute CIELAB. The CIE had many choices for illuminants in this hypothetical color matching device. The committee drew up a set of criteria that an ideal set of primaries must meet. The 1931 tristimulus functions were chosen, at least in part, because they simplified the arithmetic required for computation. [Seymour, 2020]

A recent analysis [Fairman et al., 1997] came to the conclusion that "likely none of these formulating principles would be adopted if the system were formulated from a fresh start today." There is no compelling theoretical reason to base a color system on the xyz tristimulus functions as opposed to any other color matching function.
Elliot Adams theorized a color space in 1923 that was based on the cone functions [Adams 1923], but when he came to an implementation in 1942, he used the tristimulus functions [Adams, 1942]. From comparing the two papers and especially the neural diagrams, it is likely that Adams conflated the two.

Adams' 1942 paper is important because there is a direct line of progeny from this 1942 set of computations and the computation that we call CIELAB [Seymour 2020].

The "wrong von Kries transform"

The formulas for computation of L*, a*, and b* are as follows

$$L^* = 116f(Y/Y_n) - 16 \quad (1)$$

$$a^* = 500(f(X/X_n) - f(Y/Y_n)) \quad (2)$$

$$b^* = 200(f(Y/Y_n) - f(Z/Z_n)) \quad (3)$$

where f is

$$f(q) = q^{1/3} \quad \text{if} \quad q > (24/116)^3 \quad (4)$$

$$f(q) = (841/108)q + 16/116 \quad \text{if} \quad q \leq (24/116)^3 \quad (5)$$

The normalization of X, Y, and Z by dividing by X_n, Y_n, and Z_n ensures that pure white (100% reflectance at all wavelengths) will always be L*a*b* of 100, 0, 0. To some extent, it also emulates the chromatic adaptation that occurs in the human visual system. This chromatic adaptation is the processes that occur in the human visual system that make an apple always appear red, despite huge changes in the intensity and spectral composition of the illuminants.

But dividing by X_n, Y_n, and Z_n is problematic. Normalization using the spectral responses of the cones rather than the tristimulus functions would more closely match what actually happens in the human visual system. First, at least some portion of the adaptation of the human visual system occurs in the cones themselves. Second, the *XYZ* values are synthetic quantities that were created for the purpose of convenient computation. They don't actually exist in the human visual system.

Consider how CIELAB responds to a change where the illumination is augmented by light around 450 nm. Normalization will adapt the scaling on *Z* to accommodate, but will also significantly adapt *X*. In this manner, adding 450 nm illumination will erroneously have the same effect on *X* as adding light at 650 nm. This happens despite the fact that the change in illumination will largely affect only the *S* cones.

Johann von Kries first postulated in 1878 that the cones had separate gain control mechanisms. This idea was used by Herbert Ives in 1912 to describe a means for normalization to take into account changes of illumination. Such transforms are referred to as von Kries transforms. (This name is in accordance with Stigler's Law of Eponymy, which states that all scientific laws are named after the wrong person. Stigler's Law was first stated by Thomas Merton.)

Thus, the normalization in CIELAB is a "wrong von Kries transform" [Terstiege 1972, Fairchild 2005]. One would expect that this error in how CIELAB emulates the human visual system could cause some difficulties when the illuminant is changed.

CIE 15.2:2004 does not specifically warn against using CIELAB for this purpose, but neither is it recommended. It does make it clear that CIELAB is intended to be used for comparison of colors only for illuminants that are "not too different from that of average daylight." Section 8.2.3 (Notes on CIE 1976 uniform colour spaces) states:

> *Note 5: These spaces [CIELAB and CIELUV] are intended to apply to comparisons of differences between object colours of the same size and shape, viewed in identical white to middle-grey surroundings, by an observer photopically adapted to a field of chromaticity not too different from that of average daylight.*

What if CIELAB were built on LMS as first suggested by Adams in 1923, and not on XYZ values as in Adams 1942?

Color constancy and CIELAB

Color constancy is the fact that objects usually do not appreciably change color as illumination changes. If I look at a basket of fruit under indoor, incandescent lighting, the apple looks red, the orange looks orange, the banana yellow, and the lime green. If I take the basket outside in bright daylight, I do not perceive any change in the colors of the objects. This happens despite the fact that the overall amount of light has changed by orders of magnitude and the balance of the red versus blue light has changed by a factor of ten. There is a clear evolutionary advantage for human beings who are capable of distinguishing between a lemon (for iced tea) and a lime (for a margarita) whether the drinks are being made in the kitchen or on the patio. Several parts of the human visual system are responsible for this feat of engineering, not the least of which is the auto-gain functionality of the cones.

One requirement for a robust color space is that it preserve color constancy. One would hope that an object would have relatively constant coordinates in color space as the illumination is changed.

The Bradford transform on CIELAB

The Bradford transform can be used to estimate how colors can change under different illuminants. It is generally used to convert XYZ values from one illuminant to another. Use of the Bradford transform on CIELAB is not generally recommended, but the extension of Bradford is simple.

Here are the steps:

1. The L*a*b* values of the object under the source illuminant are converted into white-referenced tristimulus values under the source illuminant, $X(src)/(X_n(src))$, $Y(src)/(Y_n(src))$, and $Z(src)/(Z_n(src))$. This is done by inverting the CIELAB formulas.

2. The white-referenced tristimulus values of the object under the source illuminant, $X(src)/(X_n(src))$, $Y(src)/(Y_n(src))$, and $Z(src)/(Z_n(src))$, are converted to absolute tristimulus responses of the object under the source illuminant, $X(src)$, $Y(src)$, and $Z(src)$.

$$X(src) = X_n(src)(X(src)/X_n(src)) \tag{6}$$

$$Y(src) = Y_n(src)(Y(src)/(Y_n(src)) \tag{7}$$

$$Z(src) = Z_n(src)(Z(src)/(Z_n(src)) \tag{8}$$

3. The absolute tristimulus responses of the object under the source illuminant, $X(src)$, $Y(src)$, and $Z(src)$, are converted into absolute cone responses of the object under the source illuminant, using the Bradford transform.

$$\begin{bmatrix} L(src) \\ M(src) \\ S(src) \end{bmatrix} = \begin{bmatrix} 0.8951 & 0.2664 & -0.1614 \\ -0.7502 & 1.7135 & 0.0367 \\ 0.0389 & -0.0685 & 1.0296 \end{bmatrix} \begin{bmatrix} X(src) \\ Y(src) \\ Z(src) \end{bmatrix} \tag{9}$$

4. The absolute cone responses of the object under the source illuminant are converted into absolute cone responses of the object under the destination illuminant.

$$L(dst) = \left(\frac{L_n(dst)}{L_n(src)}\right) L(src) \tag{10}$$

$$M(dst) = \left(\frac{M_n(dst)}{M_n(src)}\right) M(src) \tag{11}$$

$$S(dst) = \left(\frac{S_n(dst)}{S_n(src)}\right) S(src) \tag{12}$$

5. The inverse of the Bradford transform is used to convert cone responses of the object under the destination illuminant to tristimulus responses of the object under the destination illuminant.

$$\begin{bmatrix} X(dst) \\ Y(dst) \\ Z(dst) \end{bmatrix} = \begin{bmatrix} 0.9869929 & -0.1470543 & 0.1599627 \\ 0.4323053 & 0.5183603 & 0.0492912 \\ -0.0085287 & 0.0400428 & 0.9684867 \end{bmatrix} \begin{bmatrix} L(dst) \\ M(dst) \\ S(dst) \end{bmatrix} \tag{13}$$

6. The absolute tristimulus values of the object under the destination illuminant, $X(src)$, $Y(src)$, and $Z(src)$ are converted to white-referenced tristimulus responses of the object under the destination illuminant, $X(dst)/(X_n(dst)$, $Y(dst)/(Y_n(dst)$, and $Z(dst)/(Z_n(dst)$ by dividing by $X_n(dst)$, $Y_n(dst)$, and $Z_n(dst)$ respectively.

7. The tristimulus responses of the object under the destination illuminant are then converted to CIELAB using the standard equations.

Figure 3 shows the predicted change in CIELAB *a**b** values for a grid of points in the L^*=50 plane. A grid spacing of ten was used in *a** and in *b**. The CIELAB values were converted from D50 illumination to D65 (both with 2° observer), using the Bradford transform, and plotted as the somewhat distorted grid in pink.

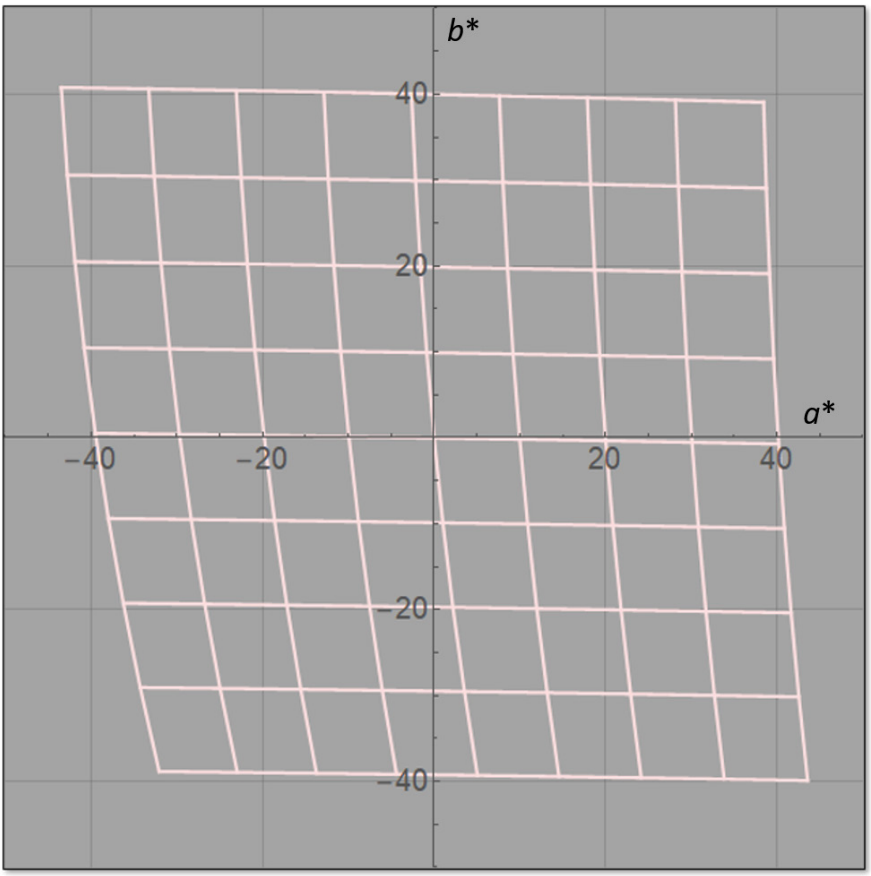

***Figure 3** – a*b* plot of color change predicted by Bradford transform (D50 to D65)*

As can be seen, colors along the a^* axis were relatively unchanged, but colors along the b^* axis have rotated counterclockwise. The largest color shift is nearly 10 ΔE_{ab} for the a^*b^* value of (-40, -40). At least according to the Bradford transform, there is a systematic shift in color when making the relatively minor shift from D50 to D65.

There are several explanations for this shift.

1. This could be true color inconstancy. For example, the cyan color {50, -40, -40} may actually appear to shift in color to the blue when the illumination changes.

2. This could be an artefact of the wrong von Kries transform embedded in the calculations of CIELAB.

3. This could be an artefact of the Bradford transform, which is only an approximation of reality. The Bradford transform does not look at what is going on spectrally.

Color inconstancy with a metameric database of print

There are limitations to color constancy. Most notably, the existence of metamerism demonstrates that perfect color constancy is impossible. If two objects match under one illuminant, but not under another, this shows that at least one of them has changed in apparent color.

A metameric database was created that can be used to test explanation #3. A collection of characterization data was collected from seven different print modalities. The individual data sets were interrogated to find printable colors that were close to grid points covering all of CIELAB. The spectra were mathematically adjusted, using principal component analysis to have the exact color of the grid point.

The grid points were in steps of 5 in L^*, in a^*, and in b^*. The final database consisted of roughly 38,000 entries, each of which is a metamer to at least one other entry. Figure 4 shows a set of metameric septuplets. Each of the seven spectra has a D50/2 CIELAB value of {50, 40, 40} and could hypothetically be printed with a certain printing press and press condition.

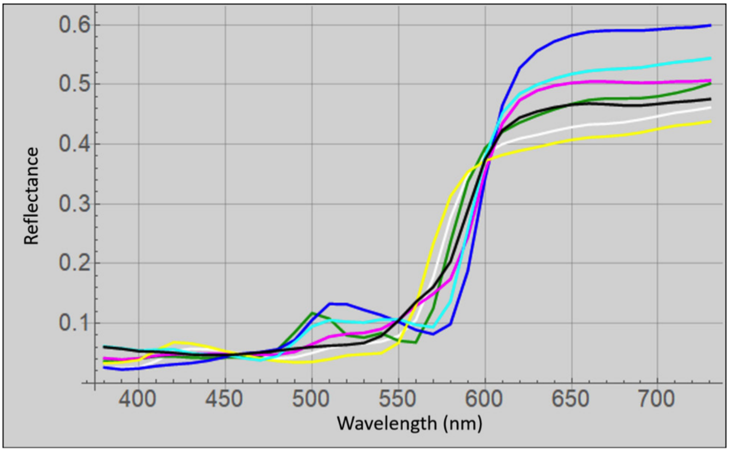

Figure 4 - *Metameric septuplets - L*a*b* of (50, 40, 40)*

Having actual spectra, the CIELAB values can be directly computed, rather than just estimated with the Bradford transform.

Figure 5 shows the change in a^*b^*, for the spectra in Figure 4, when the illumination goes from D50 to D65. The start of each arrow is {40, 40}, since they are all metamers. The end of the arrows is the color coordinates of that spectra under D65.

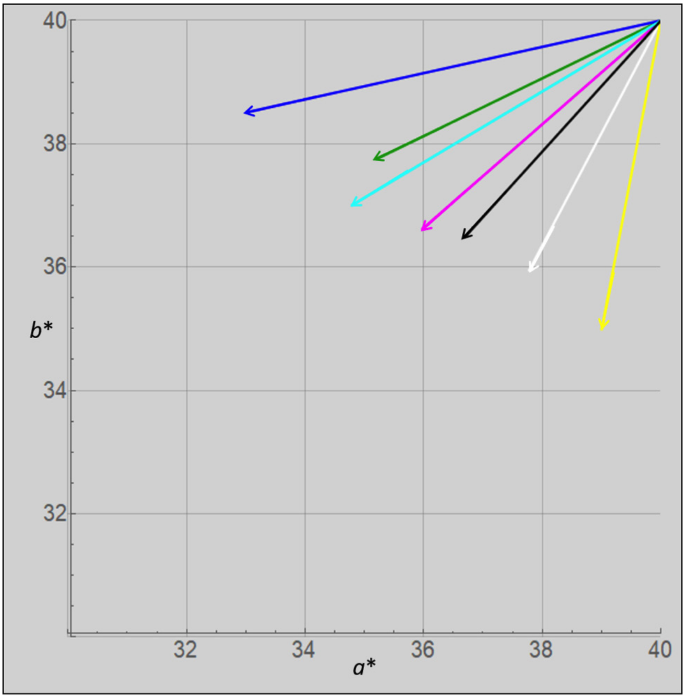

Figure 5 *– Change in a*b* coordinates of seven metamers under illuminant change*

Figure 6 breaks the direction down into two parts. First, it is clear that there is some spread (left side) that could be attributed to metamerism. But there is also a general drift for all the color shifts – all seven metamers moved down and to the left. The chance that four randomly oriented vectors would point within a 90° angle is about one in 4,000.

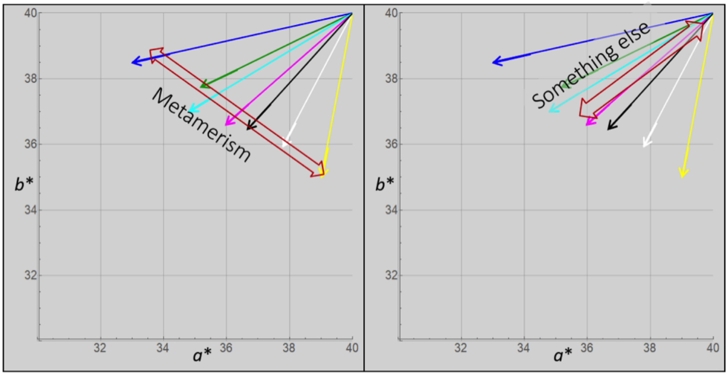

***Figure 6** – Two sources of color change*

Figure 7 looks at all the color shifts in the first quadrant. There is a distinct pattern. Furthermore, the metameric color shifts are shown overlaying the shifts predicted by the Bradford transform. They generally agree, lending further credence to the notion that there is a pattern to the shifts.

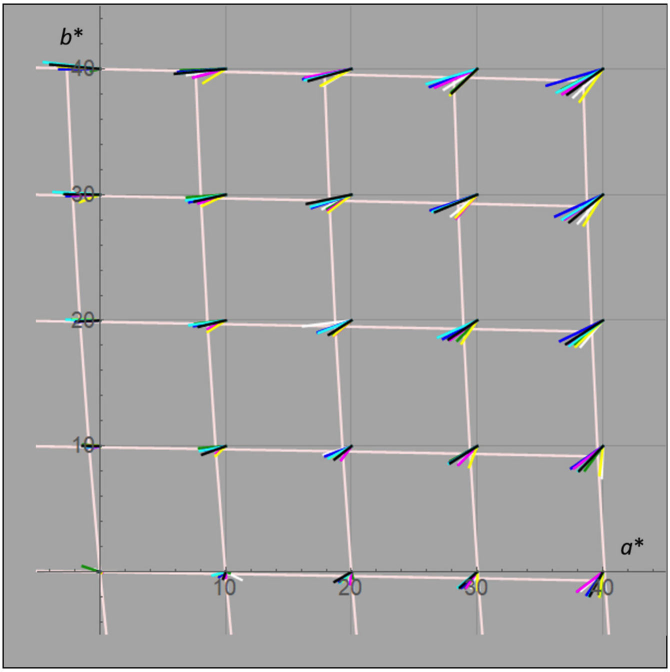

Figure 7

Finally, Figure 8 backs out to see all four quadrants. From this it is clear that in addition to the shifts between the metamers, there is a systematic shift that is largely in accord with the shift predicted by the Bradford transform.

This tells us that, while there may be some issues with the Bradford transform as a way to predict color shift, the issues with the Bradford transform are not causing the systematic shift in CIELAB values. The same shifts are seen when one computes CIELAB values from spectra.

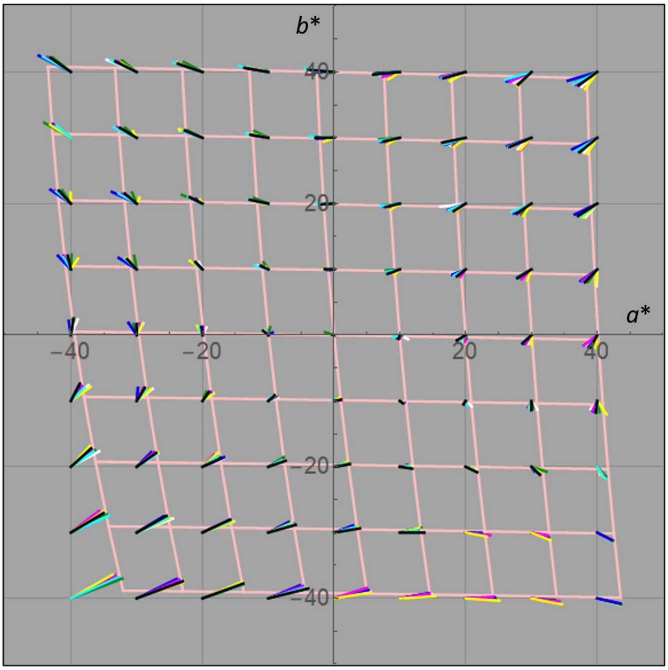

Figure 8

Reviewing the possible explanations for the shift of CIELAB values with different illuminants, we have crossed one off the list.

1. This could be true color inconstancy. For example, the cyan color {50, -40, -40} may actually appear to shift in color to the blue when the illumination changes.

2. This could be an artefact of the wrong von Kries transform embedded in the calculations of CIELAB.

3. This could be an artefact of the Bradford transform, which is only an approximation of reality. The Bradford transform does not look at what is going on spectrally.

First conclusion: The color shift in CIELAB values is not entirely due to metameric effects. The larger portion of it is either an artefact of wrong von Kries transform, or “true” inconstancy.

Color constancy with a color space based on LMS

To finally decide whether the shifts in CIELAB values are due to true color inconstancy or due to the wrong von Kries transform in CIELAB, we make use of a color space that does not have an ingrained wrong von Kries transform. A color space that was dubbed ConeLab was introduced in a previous paper by this author [Seymour, 2020]. The color space is very similar to CIELAB, but the calculations are based on LMS rather than XYZ. The color space is scaled to be close to CIELAB.

Description of ConeLab

The following equations are for the analogs of a^* and b^*, with k_a and k_b being arbitrary constants, and $g(x)$ being a nonlinear function that simulates the nonlinearity of the human visual system.

$$L_c = 25(0.43g(L/L_n) + 0.52g(M/M_n) + 0.05g(S/S_n)) \quad (14)$$

$$a_c = k_a(g(L/L_n) - g(M/M_n)) \quad (15)$$

$$b_c = k_b(g(M/M_n) - g(S/S_n)) \quad (16)$$

The nonlinear function $g(x)$ was chosen based on one developed by Seymour [2015]. This function is considerably simpler than the nonlinear function in CIELAB, and agrees well with the considerably more complicated scaling of ΔL^* in CIEDE2000. The range for L_c is not 0 to 100 as with L^*, but from 0 to 76.113. This scaling brings it into closer alignment with CIEDE2000.

$$g(x) = \log_e(20x + 1) \quad (17)$$

The constants k_a=70 and k_b=50 were selected to provide a color space of roughly the same size as a^*b^* at L^*=50 in the range of $-40 \leq a^*, b^* \leq 40$.

Seymour [2015] included the constant 25 in the formula for g. This constant was removed, since it applies only to the scaling of lightness, and would be absorbed into k_a and k_b anyway. The other constants in Equation 14 (0.43, 0.52, and 0.05) are rounded versions of the Bradford matrix transform from LMS to XYZ. The entries in this matrix are 0.43205, 0.51836, and 0.0492912, which is to say

$$Y = 0.43205L + 0.51836M + 0.0492912S \quad (18)$$

Equation 14 is a subtle departure from the corresponding equation for L^* in CIELAB. L^* is computed by applying the nonlinear function to the weighted sum of the three cone channels. In contrast, for L_c the nonlinear function is applied separately to each of the cone channels before adding the components together. The appropriate place to apply the nonlinearity depends upon which more closely emulates the human visual system. That decision is beyond the scope of this paper.

The Bradford matrix may be used to estimate the values of L/L_n, M/M_n , and S/S_n from the normalized XYZ values.

$$\begin{bmatrix} L/L_n \\ M/M_n \\ S/S_n \end{bmatrix} = \begin{bmatrix} 0.8951 & 0.2664 & -0.1614 \\ -0.7502 & 1.7135 & 0.0367 \\ 0.0389 & -0.0685 & 1.0296 \end{bmatrix} \begin{bmatrix} X/X_n \\ Y/Y_n \\ Z/Z_n \end{bmatrix} \quad (19)$$

Figure 9 shows the relationship between D50/2 colors in CIELAB and the same colors in ConeLab. The same grid points used in Figure 3 were converted to ConeLab. The points represented a grid in steps of 5 in a^* and b^*, all at the level of L^*=50. The resulting points were plotted in ConeLab coordinates.

The original results are on the left. For clarity of exposition, the plot on the right shows the same data, but rotated by 19 degrees counterclockwise, according to the following formula.

$$\begin{bmatrix} L^1{}_c \\ a^1{}_c \\ b^1{}_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(19) & -\sin(19) \\ 0 & \sin(19) & \cos(19) \end{bmatrix} \begin{bmatrix} L^* \\ a^* \\ b^* \end{bmatrix} \quad (20)$$

Note: The positive a* axis of CIELAB does not point toward true red as one might expect, but rather about 25° counterclockwise of true red. Thus, ConeLab is closer to the ideal of having red as a primary axis.

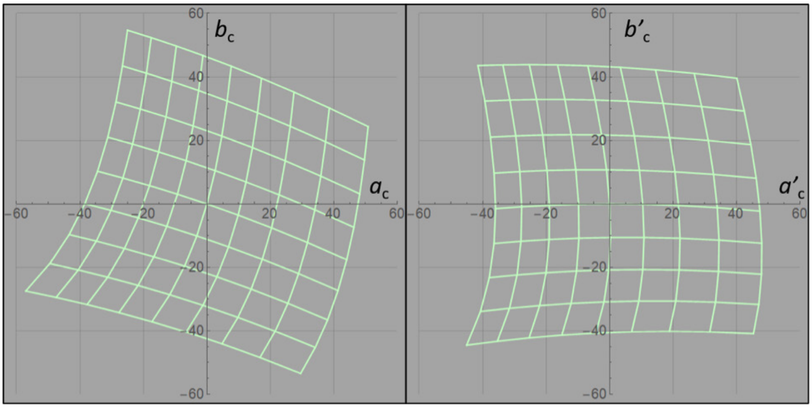

Figure 9

The graph at the right shows that $k_a = 70$ and $k_b = 50$ do a moderate job of scaling ConeLab to CIELAB. This is important for the subsequent analysis, because it enables us to roughly compare color shifts due to illumination on a level playing field.

When spectra are available, ConeLab should be computed directly from the reflectance spectrum of the sample, $r(w)$ under the designated illuminant, $I(w)$.

$$L = \sum_w r(w) I(w) l(w) \tag{21}$$

$$M = \sum_w r(w) I(w) m(w) \tag{22}$$

$$S = \sum_w r(w) I(w) s(w) \tag{23}$$

For the purposes of this paper, the Bradford matrix is used to compute the cone functions $l(w)$, $m(w)$, and $s(w)$.

$$l(w) = 0.8951x(w) + 0.2664y(w) - 0.1614z(w) \tag{24}$$

$$m(w) = -0.7502x(w) + 1.7135y(w) + 0.0367z(w) \tag{25}$$

$$s(w) = 0.0389x(w) - 0.0685y(w) + 1.0296z(w) \tag{26}$$

Use of ConeLab on the metameric database

Are the color inconstancies shown in Figure 8 largely due to the artefact of the wrong von Kries transform embedded in the calculations of CIELAB? ConeLab, which does not embody the wrong von Kries transform, can be used to test this with the aid of the metameric database.

The ConeLab values of each of the spectra in the metameric database were computed using Equations 21 – 23 to determine LMS, and Equations 14 – 17 to determine $L_c a_c b_c$. The ConeLab values were computed under D50 and then again under D65.

The color shifts (from D50 to D65) of the metamers with $L^* = 50$ are shown below. At the left are the color shifts in $a^* b^*$ (as in Figure 8). At the right are the color shifts in acbc. Qualitatively, it appears that the color shift in ConeLab is smaller. For most colors, there is still a consistent shift, but it would appear that metamerism accounts for the majority of the color shift.

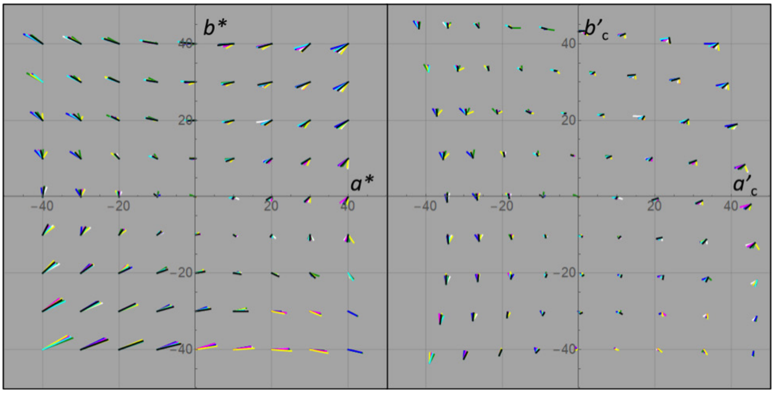

Figure 10

The ΔEab color difference was computed for each color shift (from D50 to D65) in the metameric database. The subjective conclusion from Figure 10 is supported by comparing the medians. For CIELAB, the median color shift is 3.21 ΔEab, and for ConeLab, it is 1.24 ΔEab. Figure 11 compares the cumulative probability functions of the two collections of 38,322 color shifts.

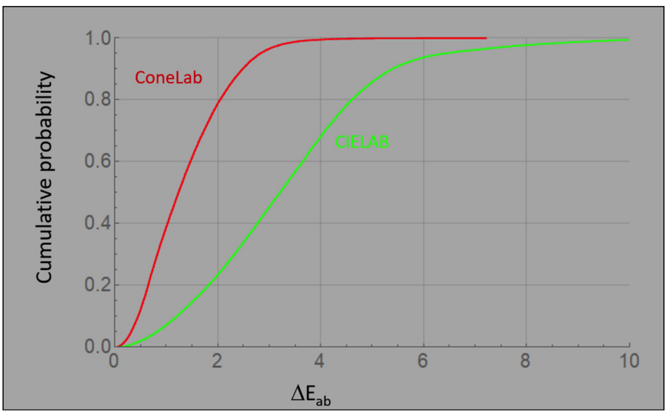

Figure 11

The conclusion is that the majority of the change of CIELAB values between D50 and D65 is not due to metamerism, or to a systematic change in our perception of color. The majority of the change is an artefact of the normalization by the tristimulus values that is embedded into CIELAB.

Application – metameric proclivity

Rationale

When selecting a brand color afresh, it is beneficial to choose a color that doesn't easily *metamerize*. This is a term that is akin to "plays well with others" – it depends upon the other spectra that it is called upon to "play well with". There is an inherent problem. How can you assess how well a child plays well with others when he/she is the only one in the room?

It has been said that a spectrum that is smoother is less likely to cause problems with metamerism. This is a reasonable statement, but to date, no study has been done that proposes an objective measure of spectral smoothness and that tests the hypothesis.

Figure 12 is the upper left portion of Figure 10, showing the color shift of a set of metameric septuplets. Which formulation should be chosen? The blue and the yellow lines are clearly outsiders. Choosing either one will maximize metamerism issues when compared side-by-side with one of the other choices. The optimal choice might be the black or magenta line, since they fall between the rest.

That assessment could be turned into a mathematical statement, but that hides the fact that the optimal choice depends on the collection of spectra that it is compared against. If this is built into a formulation package, the obvious set to check against is the set of all formulations that can be made within the set of available pigments. To take the previous analogy a step further, this is like asking whether a given formulation plays well with other formulations that live in the neighborhood. It doesn't answer whether the formulation plays well with formulations from different countries – that is, with different sets of pigments.

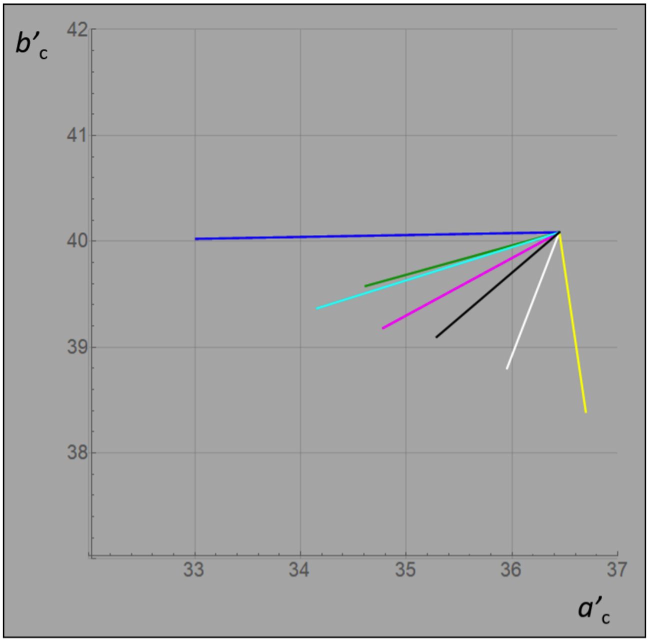

Figure 12

Consider another way to look at this plot. The blue, cyan, and yellow lines are the three longest, which is to say, these represent the three formulations that had the largest color shift when the illumination was changed. It could be argued that this is, in itself, an undesirable trait. Even where metamerism is not an issue, the idea of a brand color changing its perceptual color under different lighting is unpleasant. The size of the color shift in ConeLab could be a useful parameter to qualify the acceptability of a formulation. By this measure, the black line represents the "best" formulation, since there is the smallest color shift.

There is of course, an assumption in this. The untested assumption is that the color shift in ConeLab correlates with our perception of that color shift. While this is reasonable, it is entirely possible that all of the color shifts in this example are below the limit of perceptibility. If a person looked at the blue formulation under D50, and then again under D65, would he/she notice a change in color after adaptation?

Under the assumption that a shift in ConeLab does correlate with perceptual color shift, then a measure of ConeLab shift is a useful parameter when selecting a formulation. Formulations with a large shift in ConeLab should be avoided.

If there were not a systematic color shift, which is to say, if the lines in Figure 12 went off equally likely in all directions, then the shift in ConeLab would be a direct measure of how likely a formulation is to metamerize. If the centroid of the color shift is zero, then the formulation that is closest to all is the one that strays the least from home. So, to some extent, selecting a formulation with a small color shift in ConeLab will tend to reduce metamerism.

With a variety of assumptions, a small ConeLab color shift of a formulation in ConeLab is indicative of a color that is perceived as being stable under that illumination change and of a formulation that is less likely to metamerize against other formulations.

Computation of metameric proclivity, preliminary equations

The lms functions are determined. These may be determined by applying the Bradford transform to the xyz tristimulus functions.

$$l(w) = 0.8951x(w) + 0.2664y(w) - 0.1614z(w) \tag{27}$$

$$m(w) = -0.7502x(w) + 1.7135y(w) + 0.0367z(w) \tag{28}$$

$$s(w) = 0.0389x(w) - 0.0685y(w) + 1.0296z(w) \tag{29}$$

The LMS values are computed from the reflectance spectrum of the sample, r(w) under the first designated illuminant, $I_1(w)$.

$$L_1 = \sum_w r(w)\, I_1(w)\, l(w) \quad (30)$$

$$M_1 = \sum_w r(w)\, I_1(w)\, m(w) \quad (31)$$

$$S_1 = \sum_w r(w)\, I_1(w)\, s(w) \quad (32)$$

ConeLab values are computed from the first set of LMS values.

$$L_c(1) = 25(0.43\mathrm{g}(L_1/\mathrm{L_n}) + 0.52\mathrm{g}(M_1/\mathrm{M}_n) + 0.05\mathrm{g}(S_1/\mathrm{S}_n)) \quad (33)$$

$$a_c(1) = 70(\mathrm{g}(L_1/\mathrm{L}_n) - \mathrm{g}(M_1/\mathrm{M}_n)) \quad (34)$$

$$b_c(1) = 50(\mathrm{g}(M_1/\mathrm{M}_n) - \mathrm{g}(S_1/\mathrm{S}_n)) \quad (35)$$

with

$$g(x)=\log_e(20x + 1) \quad (36)$$

The LMS values are computed from the reflectance spectrum of the sample, $r(w)$ under the second designated illuminant, $I_2(w)$.

$$L_2 = \sum_w r(w)\, I_2(w)\, l(w) \quad (37)$$

$$M_2 = \sum_w r(w)\, I_2(w)\, m(w) \quad (38)$$

$$S_2 = \sum_w r(w)\, I_2(w)\, s(w) \quad (39)$$

ConeLab values are computed from the second set of LMS values.

$$L_c(2) = 25(0.43\mathrm{g}(L_2/\mathrm{L_n}) + 0.52\mathrm{g}(M_2/\mathrm{M}_n) + 0.05\mathrm{g}(S_2/\mathrm{S}_n)) \quad (40)$$

$$a_c(2) = 70(\mathrm{g}(L_2/\mathrm{L}_n) - \mathrm{g}(M_2/\mathrm{M}_n)) \quad (41)$$

$$b_c(2) = 50(\mathrm{g}(M_2/\mathrm{M}_n) - \mathrm{g}(S_2/\mathrm{S}_n)) \quad (42)$$

The color difference between the two sets of ConeLab values is determined. This is the metameric proclivity.

$$M_p = \sqrt{(L_c(1) - L_c(2))^2 + (a_c(1) - a_c(2))^2 + (b_c(1) - b_c(2))^2} \quad (43)$$

Potential improvements

Use of the LMS functions from CIE Publication 170-1:2006 would improve the computations.

Future work may show that the computation of L_c (Equations 33 and 37) could be improved by applying the nonlinear function to the sum of the three values rather than individually to the values.

The constants 70 (Equations 34 and 41) and 50 (Equations 35 and 42) were chosen strictly so that ConeLab results could be compared to CIELAB results. It is likely that another set of constants could improve the perceptual linearity of ConeLab.

Equation 43 is the Euclidean distance formula that was also used as the 1976 CIELAB color difference formula ΔE_{ab}. One could argue that CIEDE2000 might be a more appropriate equation to use, since ultimately metameric proclivity hopes to assess perceptual differences.

On the other hand, it should be stressed that applying the CIEDE2000 formulas to ConeLab values does not result in ΔE_{00} values. None of the research that went into validating CIEDE2000 validates the use of these formulas on ConeLab.

In particular, the use of the nonlinear function *g* instead of *f* is an improvement on the perceptual linearity of ConeLab over CIELAB, at least in terms of differences in the lightness axis. Applying the CIEDE2000 corrections to differences in lightness would be self-defeating. Whether the use of *g* improves the perceptual linearity of the neutral axis is yet to be seen.

Further, a simple rescaling of the b^* axis would improve the perceptual linearity of CIELAB. Reducing the constant 50 in equations 35 and 42 would be an improvement. This will be pursued in a future paper.

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