A METHOD AND MATHEMATICS FOR STUDYING WEB FEED-UP AND TENSIONS IN MULTI-STAGE OFFSET AND GRAVURE PRESSES

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Abstract: This paper is about web feed-up and interstage tensions in offset and gravure printing presses. In it will be shown a method and the development of simple algebraic equations that may be used to calculate and study steady-state web feed-up and tensions under various web and press conditions. The equations are applicable to single stage and multiple stage presses.

In the experimental work and the development of the mathematics these factors of special interest have evolved:

First, there is an inherent slippage of the web in the printing nip of presses and this slippage is a function of the tension unbalance across the nip.

Second, within the nip, high tension stresses are transmitted to the web which cause a velocity stratification within it and result in the establishment of an effective neutral plane in the web with respect to the impression cylinder.

Third, the print length and interstage tensions in a press are dependent upon the tension balances across the printing nips, the elasticity and caliper of the web, the slippage of the web in the nips, and the effective neutral plane of the web on the impression cylinders.

Data Source and Characteristics

The data that forms the basis of the mathematics was taken from a study of cylinder-to-cylinder nips and their ability to measure web lengths with a degree of accuracy high enough to control the web length produced by business

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forms presses. In those presses an error of plus or minus twenty thousandths of an inch in ten feet is significant. This is a tolerance of one part in six thousand. The study was conducted and the data taken with the care that those kinds of tolerances require.

The study showed that the web feed-up of all cylinder-to-cylinder nips, whatever their configuration, appear to be governed by the same physical properties and, because of the many variables involved with them, are not suitable for the high degree of accuracy required for the business forms application. A more accurate way was found to solve the problem that gave rise to the study.

The data obtained from the study revealed a new insight into web feed-up through cylinder-to-cylinder nips and the capability of translating that data into general equations that are thought to be applicable to the feed-up of webs through gravure and offset printing presses.

The data was produced on a simulated printing couple at low speeds with cylinders and web one inch wide. While it has not been possible as yet to confirm the equation constants with data from real press units, the analyses and results produced by the equations are remarkably close to those that have been arrived at by experience and other experimental work. Therefore, it is believed that there is fundamental validity within them.

In the discussion that follows, reference is usually made to offset printing nips. The discussion is equally applicable to gravure printing nips because both consist of an elastic cylinder bearing against an inelastic cylinder with the web between. When the impression cylinder of an offset printing unit is referred to it will be understood that the reference applies also to the gravure cylinder of a gravure printing unit.

In addition it must be particularly noted and kept in mind that references to web feed-up are in terms of the off-press, unstretched web length produced at the delivery end of the press and after all of the press and process influences have had their effect on it. This is referred to as the "free web length".

Figure 1 shows a schematic of the simulated printing couple and the raw data that was obtained from it.

The couple consisted of two cylinders of 4.952 inches diameter, gear connected, without bearers, but engaged with a twelve pound force. The blanket was a piece of Vulcan 714 cemented to the blanket cylinder and not stretched. Since the cylinders were of equal diameter, they simulate a blanket and impression cylinder without packing. Three weights of paper web were run through this couple. Their calipers were .0022, .0044, and .0077 inches. All were business forms grades of paper.

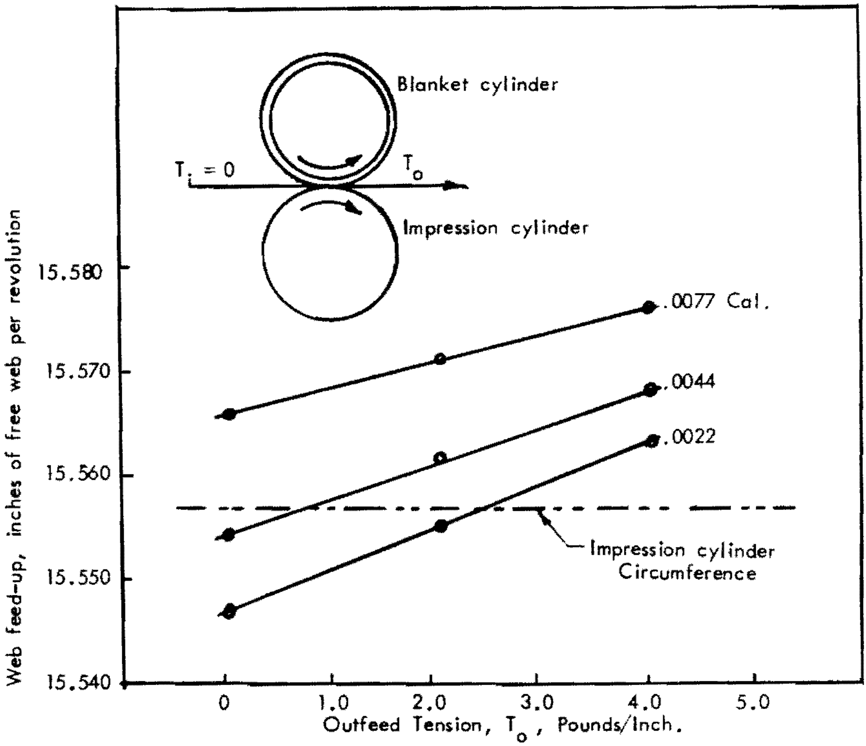


Figure 1. Plotted web feed-up data taken from the experimental roller-to-roller nip.

The tension unbalance between infeed and outfeed was set at three different values, zero, two, and four pounds per inch for each grade of paper run. The tension unbalance was accomplished by holding the infeed tension at zero and adjusting the outfeed tension to the three values.

The raw data curves of Figure 1 show the feed up of web per revolution of the cylinders for the three weights of paper with the tension unbalance varying from zero to four pounds. It shows that as the unbalance increases, in this case the outfeed tensions, the web feed-up increases. The curve for the .0022" paper for example shows that by increasing the outfeed tension from zero to four pounds, the feed up per cylinder revolution increases by .016". This means that repeat length through a couple of this kind can be changed by adjusting the infeed-outfeed tension relationship. That is, the repeat length can be increased by increasing the outfeed tension or decreasing the infeed tension. Conversely, it can be decreased by decreasing the outfeed tension or increasing the infeed tension.

Mechanically this effect is a creepage or slippage of the web in the nip and it is an important factor in the feed-up process and in establishing a stable tension distribution through a press. While this effect has escaped the notice of other authors, hundreds of business forms press operators use it every day to get the correct throw length from their press runs by adjusting infeed and outfeed tension values The only reference to this slippage and an acknowledgement of it that I have found is in a patent on interstage register control issued to W. F. Huck (1948) in which he uses the effect to accomplish the registration function.

The second important and interesting characteristic of the raw data is that web feed up per revolution at zero tension unbalance and thin webs can be less than the impression cylinder circumference. This is shown by the lower two thirds of the curve for the .0022 thick web in Figure 1. At zero tension unbalance the web feed-up per revolution is .010 less than the impression cylinder circumference. The impression cylinder circumference is indicated by the broken line.

A third characteristic is that feed-up increases with an increase in web thickness.

A fourth important characteristic the raw data reveals is that the feed-up versus tension unbalance relationship is perfectly linear for each caliper of paper. A change of slope and intercept points occurs with a change in caliper of the web. As will be shown, the linear relationships are a great advantage in setting up the mathematics for the system. However, the method does not depend upon this linearity.

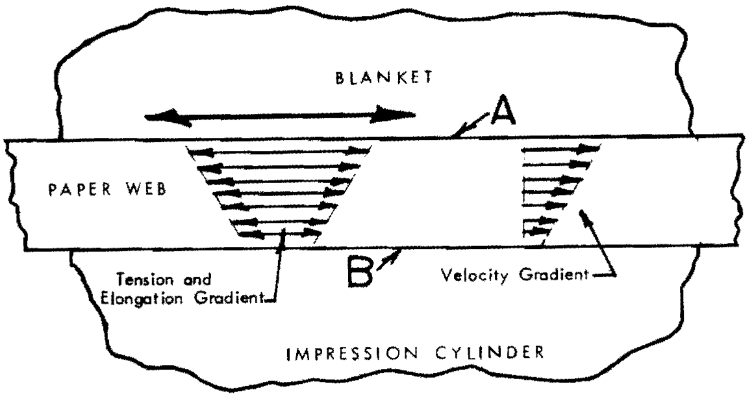


Figure 2A

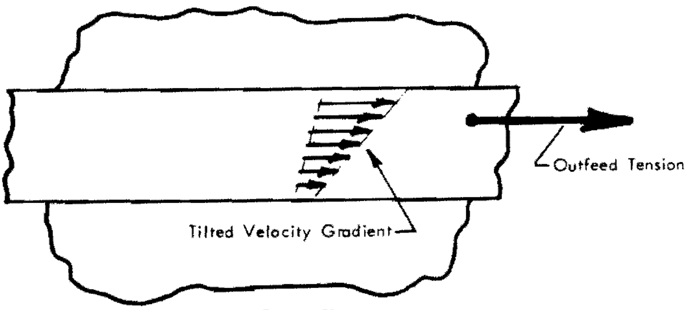


Figure 2B

Figure 2. Greatly enlarged cross section of a roller-to-roller nip showing the tension and velocity gradients within the web.

Development of Mathematical Relationships

The first step in converting the raw data into generally applicable and useful relationships was to find a definable relationship between the velocity of the paper through the nip of the stage and the velocity of the impression cylinder. The relationship must be to the impression cylinder because it has an inelastic surface and thus most stable.

The relatively elastic surface of the blanket cylinder has a variable velocity through the nip. This has been described in detail by several researchers. (Sandor, 1961; Brink, 1963) It has also been shown experimentally (George, 1964) that the paper web surface engaging a gravure cylinder, moves at the same velocity as the cylinder surface. Therefore, since the feed-up per cylinder revolution can be less than the cylinder circumference as noted above, there must be an effective elongation of the web within the nip and an effective neutral plane within the paper thickness along which the paper can be considered to move through the nip.

Figure 2A is a much enlarged cross section of a part of the nip between a blanket cylinder and impression cylinder. The blanket cylinder and blanket are the upper part of the sketch, the impression cylinder is the lower part, and the paper web is sandwiched between them.

The blanket is the most elastic part of this sandwich. The paper web is less so while the impression cylinder is inelastic.

As the blanket cylinder, paper web and impression cylinder roll into engagement at the nip, the elastic web engages an elastic medium on its top surface at A and a non-elastic medium on its oppposite surface at B. As the nip pressure increases, the elastic blanket develops tangetial forces at its interface with the web. The web surface at A, being in positive engagement with the elastic blanket, stretches with it and is thus subjected to tension stresses in its upper surface that are additive to its infeed tension. On the other hand, at B, its lower surface is in positive contact with an inelastic surface with which it moves and subject to no increase in tension. This introduces a tension gradient across the web which is highest at the blanket interface and lowest at the impression roller interface. Since the web is elastic, this tension gradient is translated into a stretch gradient across the web, as indicated in Figure 2A. Since the web is stretched according to this gradient, it must also have a velocity gradient of the same proportions such as is also diagrammed in Figure 2A.

Normally a web running in free contact with a cylinder will have a velocity relative to the roller surface which is dependent upon the location of the neutral plane of bending of the paper web. In another set of experiments and measurements, that neutral plane has been found to be at 60% of the web thickness from the cylinder surface. In a nip, on the other hand, the velocity gradient introduced by the nip stresses causes the effective neutral plane to be shifted closer to the impression cylinder at B.

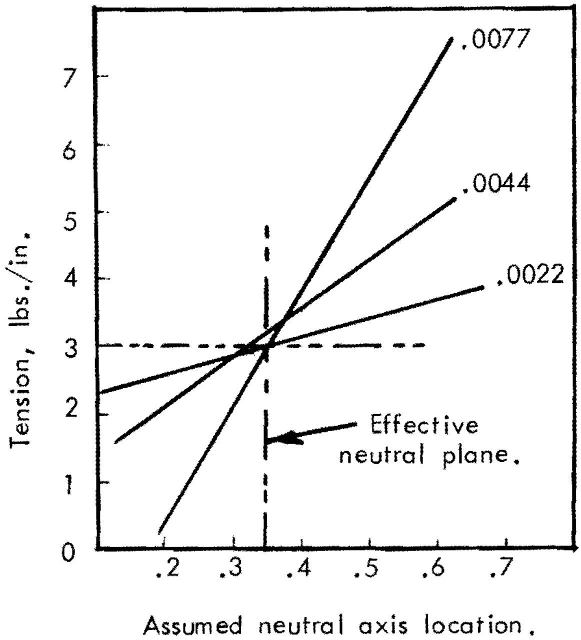


Figure 3. Graphical solution of the location of the effective neutral plane of the web in the roller-to-roller nip.

To find this neutral plane it was assumed that the difference in tension stresses on opposite sides of the web in the nip would be the same for any caliper of paper, thick or thin. Thus, the stress gradient and in turn the velocity gradient across paper of any caliper will have the same terminal values. At an effective neutral plane location to be determined, the velocity will be such that the feed-up of webs of various calipers will fit the raw feed-up data of Figure 1.

A series of calculations for each web caliper were made from the known data of the impression cylinder circumference, the feed-up per revolution, the paper stretch modulus and a series of assumed values for the effective neutral plane location. This gave curves of hypothetical average web tension in the nip versus the assumed neutral plane location. Curves of the results for each caliper paper were plotted as shown in Figure 3. These curves intersect within a narrow area around the three pounds per inch tension line which gives an approximate location of the neutral plane as 35% of the web thickness from the impression cylinder surface.

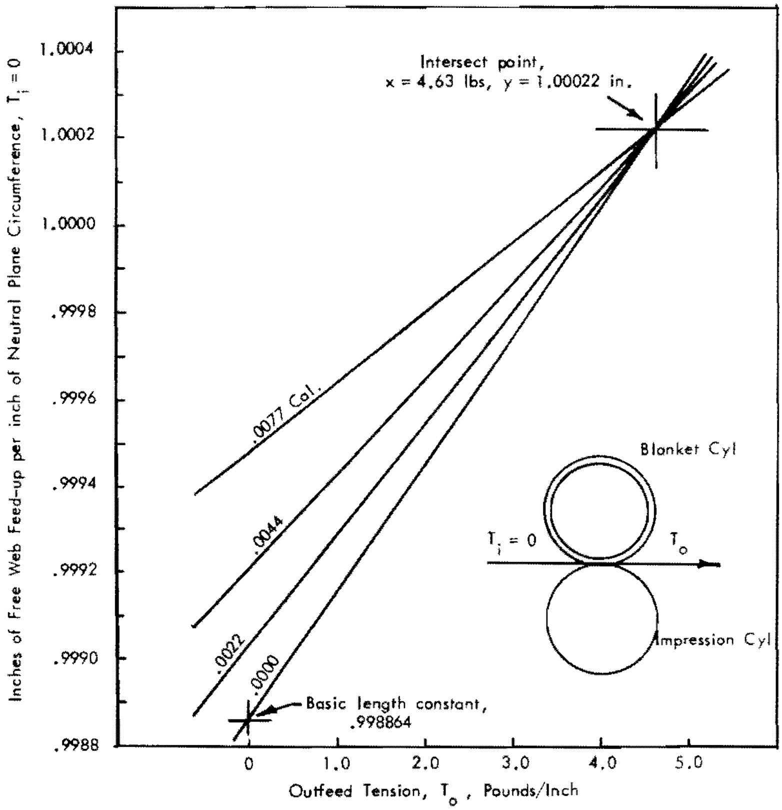


Figure 4. Plots of restructured web feed-up data in terms of the effective neutral plane of the web in the roller-to-roller nip.

The indication of this intersection is that the average web tension in the nip was three pounds per linear inch, that the web as it traversed the nip was stretched by that tension, and that the web moves in that stretched condition through the nip at a radius that is equal to the impression cylinder radius plus 35% of the thickness of the web.

The velocity gradient across the cross section of the web may give a clue to the mechanism of the "creep" factor of a web through a nip. It is possible that the web tension at the infeed or outfeed of a nip can "tilt" the velocity gradient to decrease or increase the effective velocity of the web through the nip as Figure 2B suggests. It is one possible explanation.

The data taken as previously noted was with a loading of the blanket cylinder against the impression cylinder of twelve pounds per inch. Previous test runs showed that feed-up data was not materially different with loadings of 3, 6, and 9 pounds. From this it is believed that the loading variable that might be encountered in a real printing couple may be small if not negligible in effect.

Using the previously arrived at value of the neutral plane, the raw data was converted to data in terms of inches of free web feed-up per inch of neutral plane circumference versus tension unbalance. This data is plotted in Figure 4.

Several features of the curves and data of Figure 4 are notable. First, they have a common intersection point at a web tension unbalance of 4.63 pounds and a feed-up value of 1.00022 inches. Second, the gradient of feed-up values is extremely small, in the one-ten thousandth of an inch per inch order. Third, a theoretical curve of zero web caliper has been added. This latter curve establishes that a web of zero thickness and zero tension unbalance across the couple would feed up .998864 inches of web for each inch of neutral plane circumference of the impression cylinder and not one full inch as might be suspected. This number, .998864, will become a basic constant in the equations to be developed.

The intersection point of these curves of Figure 4 is a phenomena that, as yet, has no explanation. A parallel effect has been noticed in other experiments involving web feed up through roller nips and so it is believed that this intersection point is real.

In any event, the intersection point gives a very functional advantage in deriving equations for the feed up of any caliper paper under the conditions of any tension unbalance. It allows the use of the intersect point as a translated origin of rectangular coordinates and the definition of the slope of a line through that origin in terms of paper caliper, and the slope, in turn, to define the web feed-up versus tension unbalance relationship of the caliper of the web under consideration.

Going forward from the above, the slopes of the feed-up curves for the three web calipers were calculated and plotted. The slope versus caliper relationship was found to be linear and from this the equation for slope in terms of any caliper of web was found to be:

$$m = (.00029287 - .017278 \text{ Cal.}) \quad (1)$$

in which: m = Slope of the curve of tension unbalance versus web feed-up for any caliper of web.

Cal. = Caliper of the web under consideration.

With equation 1 of the slope and translation of the axis of Figure 4 to the point x = 4.63, y = 1.00022, the equation for the curve of the feed-up of a web of any caliper in terms of the tension unbalance, $(T_o - T_i)$, can be written:

$$F = m\,[(T_o - T_i) - 4.63] + 1.00022 \quad (2)$$

where: F = Web feed-up, inches per inch of neutral plane circumference under the condition that $T_i = 0$.

T_i = Infeed web tension.

T_o = Outfeed web tension.

m = Slope per equation 1.

Expanding and combining terms of equation 2 gives:

$$F = [m\,(T_o - T_i) + .079997 \text{ Cal.} + .998864] \quad (3)$$

As in equation 1, F is in inches under the condition that the infeed tension, T_i, is zero. For tension at the infeed greater than zero, equation 3 is multiplied by the term $(1 - T_i S)$ to account for the stretch of the web due to the infeed tension, T_i, and the elastic modulus of the web, S, in inches of elongation per inch of web per pound of tension. Further, to arrive at the feed-up in terms of one press revolution, equation 3 is multiplied by the neutral plane circumference as previously described. Making the multiplications and combining constants gives:

$$F_f = C_1 \, [mT_o - T_i \, (m + C_2S) + C_2] \tag{4}$$

where: F_f = Free web feed up per press revolution, inches.

C_1 = Neutral plane circumference of web on impression cylinder, inches, with the neutral plane of the paper as .35 times its caliper.

C_2 = (.079997 Cal. + .998864)

Cal. = Caliper of web, inches.

T_i = Infeed tension, pounds per inch.

T_o = Outfeed tension, pounds per inch.

S = Elastic modulus of the web, inches of elongation per inch of web per pound of tension.

m = (.00029287 - .017278 Cal)

Equation 4 gives the feed-up of the web F_f in terms of inches of free web, that is, off press, unstretched web length for each revolution of the impression cylinder. The first factor, C_1, is the effective neutral plane circumference of the web on the impression cylinder in the nip. It should be noted that in most offset presses the impression cylinder radius is greater than the bearer radius.

The second factor, the bracketed expression, consists of three terms: The first, mT_o, is proportional to and concerned with the outfeed tension T_o of the web; the second, $T_i \, (m + C_2S)$ is proportional to the infeed tension and is fittingly negative since its effect is subtractive form the outfeed tension, T_o.

This second term, $T_i \, (m + C_2S)$, also includes the elastic modulus, S, of the web as a part of the coefficient of T_i. The elastic modulus is a part of the infeed coefficient because the nip feeds-up or feeds-out whatever is fed in. A part of what is fed in is any elongation of the web due to stretching under the influence of the infeed web tension T_i. This second factor may also be modified by other effects encountered by the web which shrink or stretch it such as moisture loss or absorption and which have an effect on its elastic modulus.

The third term, C_2, is a constant of basic feed-up which is modified by the other two and which contains the basic constant noted above. All three terms are functions of the caliper of the web being considered. Note that if the infeed and outfeed tensions are zero, the feed-up is a function of the basic constant, C_2, only.

Applications of the Mathematics

While this equation was developed to study web feed-up and tension in web presses, a digression will show its application in two respects to sheet-fed offset presses.

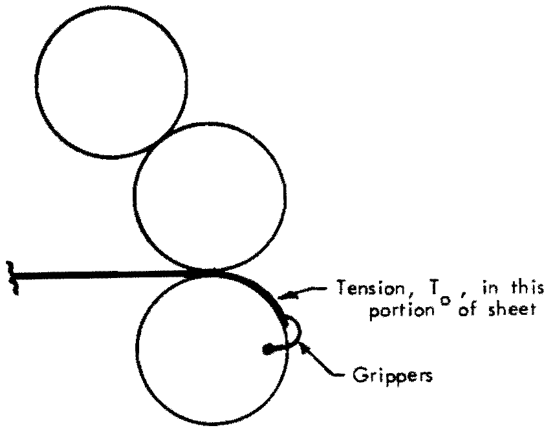
Sites and Kuehn (1953) and many later authors have studied and written about the nature of true rolling of the soft blanket cylinder against the hard impression cylinder and the technique of packing the latter cylinders to obtain true rolling contact between them.

Sites and Kuehn, in particular, noticed that when this was done, the print length was increased beyond the plate length. Sites explained this as being the result of the stretching of the plate and compensated for it by decreasing the circumferential dimension of the image on the offset plate.

However, an exercise with Equation 4 will give another factor to account for the increase of the image length. That is, the effect of web creep through a nip under the influence of tension.

Figure 5 shows schematicaly a sheet being pulled through the printing nip of a sheet fed press. The grippers hold the sheet to the periphery of the impression cylinder and cause the sheet to move through the printing nip at the velocity of the neutral plane circumference of the sheet on the cylinder. In this case the neutral plane circumference, not being in the nip, is based on the normal neutral plane of the paper.

As it happens, the velocity of the neutral plane is greater than that which the printing nip will allow and therefore a tension is introduced into the web between the nip and the grippers. This is the outfeed tension T_o of a printing couple, one of the three unknown terms of Equation 4.



$$F_f = C_1 (mT_o + C_2) \qquad (5)$$

$$T_o = \frac{1}{S} (1 - C_2 - m) - 1 \qquad (6)$$

Figure 5. Application of the basic equation 4 to a sheet-fed offset press and the equation for its outfeed tension.

In the case of a sheet fed press, the term T_i, the infeed tension, is zero because the tail end of the sheet going through the printing nip has no restraint and therefore no infeed tension. The equation for feed-up of a sheet fed press then becomes Equation 5 of Figure 5. The second term in the brackets of Equation 4 being a function of T_i, is zero and has dropped out to give Equation 5.

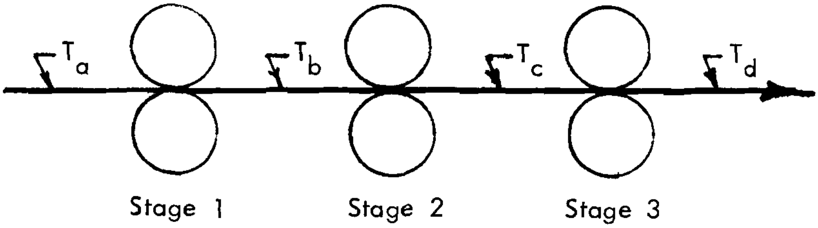
In Equation 5 we have the means of determining the outfeed tension T_o, that tension between the nip and the grippers. The logic of its derivation is this:

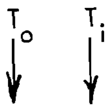
The nip allows a feed-up of the sheet according to Equation 5 which is a function of T_o. The grippers pulling the sheet are moving at the peripherial rate of the neutral plane which is greater than the feed-up through the nip. The difference in these two rates must be made up by the stretch of the sheet between the nip and the grippers. That is, the feed-up rate of the nip plus the incremental increase in length due to the tension applied equals the rate of travel of the grippers. When these two rates are equated and solved for the only unknown, T_o, we obtain Equation 6 for T_o. A value for T_o can now be found for any condition of the sheet-fed printing couple. The equations predict tensions of between one-half and one and one-half pounds per linear inch. These values seem reasonable, but cannot be confirmed due to the absence of experimental data in this area.

When values of feed-up versus packing are explored with these two equations it is found that the values of packing required for true rolling give oversized print lengths as found by Sites and Kuehn. In the case of a sheet-fed press with 20 inch bearers and with an impression cylinder diameter .020 inches larger as recommended by them, the print length from a size to size blanket image as calculated by Equation 5 is predicted as .070 inches over size for a full cylinder circumference or about .050 for a 45 inch print image that might come off of that press. This is an interesting corroboration of Sites' observation but it gives an added explanation of it. Incidentally, Equation 2 predicts that a size for size image of the plate will be printed when the plate and impression cylinder diameters are .006 inches over bearers.

Sites and Kuehn also cite as one of the properties of true rolling the ability of a sheet-fed press to print uniform print lengths over a wide range of paper calipers. While Equation 5 does not predict exact print lengths over a wide range of calipers it does predict that over a range of 1.5 mils to 10 mils the print length will vary less than one tenth of one percent or about .047 inches in 63 inches. That is another strong element of correlation of their observations.

A final note on sheet fed offset presses. The math of equations 5 and 6 points up the features that have made the sheet-fed offset printing process the paragon of quality and precision. Those features are: First, a steady and uniform draw of the sheet through the printing nip with a positive tension; second, the absence of the disrupting influence of tension in the tail of the sheet; and third, the insensitivity of the process to slight changes in the caliper of the sheet.





$$\text{Stage 1:} \quad F_f = C_{11} \left[mT_b - T_a (m + C_2 S) + C_2 \right] \tag{7}$$

$$\text{Stage 2:} \quad F_f = C_{12} \left[mT_c - T_b (m + C_2 S) + C_2 \right] \tag{8}$$

$$\text{Stage 3:} \quad F_f = C_{13} \left[mT_d - T_c (m + C_2 S) + C_2 \right] \tag{9}$$

Where: C_{11}, C_{12}, C_{13} = Neutral plane circumferences of impression cylinders of stages 1, 2, and 3 respectively.

T_a, T_b, T_c, and T_d = Infeed, interstage and outfeed web tensions respectively per diagram.

Other constants are the same as those for equation 4.

Figure 6. Application of the basic equation 4 to a three stage offset or gravure press.

Now to web presses. Equation 4 was developed specifically for the web press condition and more specificially for the multi-stage web press where the interstage web tensions, repeat length and other factors are of interest.

Figure 6 is a schematic of a three stage web-offset press and the three basic equations 7, 8, and 9 which are the expressions for the steady state free web feed-up through each stage. A similar set of equations can be written for a press of any number of stages. For each additional stage an additional unknown is added so that the number of unknowns always is equal to the number of stages.

In Figure 6 T_a, T_b, T_c and T_d are the web tensions, T_a being the infeed tension, T_b and T_c being the two interstage web tensions and T_c being the outfeed web tension in pounds per linear inch. T_a and T_c, the infeed and outfeed tensions are presumably known or values can be assigned to them.

F_f is the web feed-up or velocity per press revolution in terms of its free or unstretched length. Since the same indentical free length of web must pass each stage on each revolution, F_f for each of the three equations is equal although its absolute value is dependent on the other factors and variables of the equations.

The velocity of the web in and out of each stage is different by the amount of elongation or shrinkage of the web due to different tensions and other effects it encounters at different locations through the press. For example, in a print unit having a higher tension in the web going into it than leaving it, the velocity of the web going into it will be greater than the velocity of the web leaving it. The differences are very small but significant.

The constants C_{11}, C_{12}, and C_{13} are the effective neutral plane circumferences of the web on the three impression cylinders. If all cylinders are of the same diameter, then these circumference constants will be equal. If not, then they will be unequal and the effect of unequal cylinder diameters will show up in the results and the effect of unequal diameters can be studied. The other constant, C_2, is as described before.

In each stage the free web feed-up is controlled by two variable factors: First, its infeed tension which gives it a web of variable elongation to work on; and second, the tension unbalance between its infeed and outfeed which generates a degree of creep or slippage of the web through its nip.

Notice that the outfeed tension of one stage becomes the infeed tension of the following stage and vice-versa. This makes the web feed-up of all of the stages interconnected and interdependent. A change in web tension at any point in the run of the web through the press affects the tension balance and feed-up of all stages both upstream and downstream of the place where the change may have occurred. The one invarient in the system is that the steady state feed-up of the web in terms of its free length per press revolution is identical through each stage. The whole press will seek a state of stability in which the feed-up of each stage produces this equality.

In the process of establishing that state of stability and equality, the press infeed tension, T_a, which is set at or assigned a particular value, sets the criteria of web fed into stage one. The press outfeed tension, T_b, which is likewise set at or assigned a particular value, sets the criteria for the web fed out of stage three. The interstage tensions, T_b and T_c, then adjust to the particular values that give identical feed-up through each stage. Finally, the exact dimension of the free web feed-up is determined by the stable state thus established.

The process of establishing stability can be visualized more easily by examining the reaction of one stage to a disruption. Assume that the stage is operating stably with a particular infeed tension, outfeed tension and slippage in its nip. Now something occurs to cause an increase in the feed-up through its nip. Its reaction is to draw more web from the preceeding stage, increasing its infeed tension in that direction, and to deliver more web to its neighbor downstream, decreasing its web tension in that direction. The tension unbalance across the nip is thus changed in the direction to decrease the feed-up of the stage. In addition, the incremental increase in upstream web tension caused a corresponding elongation of the web going into the stage and a corresponding decrease in free web feed-up through the stage. Thus two factors have reacted to restore the stable feed-up condition.

All of the factors involved in the establishment of the steady state stability of the web feed-up of a three stage press are contained in equations 7, 8, and 9. The three equations are then solved simultaneously for the three unknown variables: the free web feed-up per press revolution, F_f, and the two interstage tension values T_b and T_c.

In addition to the effects that may be studied that have been mentioned, the effects of interstage web shrinkage from dryers and expansion from moisture pick-up may also be studied. This can be done by adding a factor within the brackets of Equation 4 representing the incremental change in length due to those effects. If those effects also result in a change in the elastic modulus, S, of the web as they quite often do, then the coefficient S of T_i must also be changed. Other modifications of the equations by which other factors in the progress of a web through a press may be studied are limited only by the ingenuity of the user.

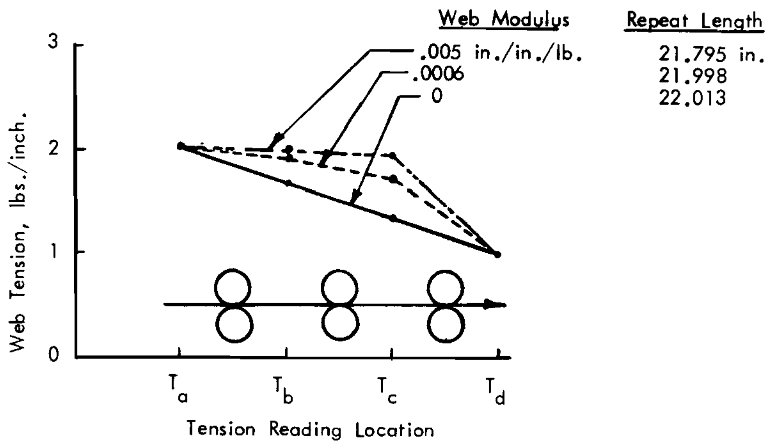


Figure 7. Web tension distribution in a three stage press with webs having three different moduli of elasticity.

Figures 7, through 10 show the results of some studies using the equations of the effects of several variables on web tension and print lengths through a three stage press. Shown in each figure is the web tension level in, out, and between stages.

Figure 7 shows the effect of a change in the elastic modulus of a web from a very inelastic value of zero inches per inch per pound to a very elastic value of .005 inches per inch per pound of web tension. It shows that a change of modulus will change the print length and that the print length will decrease as the modulus increases. In this study the in-and-out tensions T_a and T_d were purposely made unequal to show the change in web tension pattern with a change in modulus. The very stiff web, that with zero modulus, shows a uniform decrease in tension from stage to stage while the very soft web, that

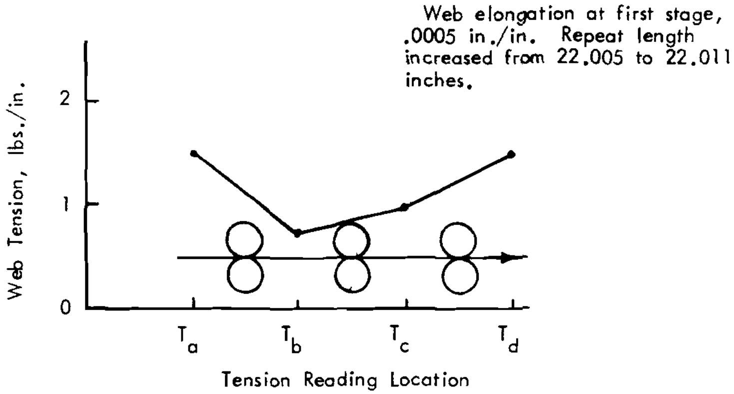


Figure 8. Web tension distribution in a three stage press with moisture pick-up in the first stage.

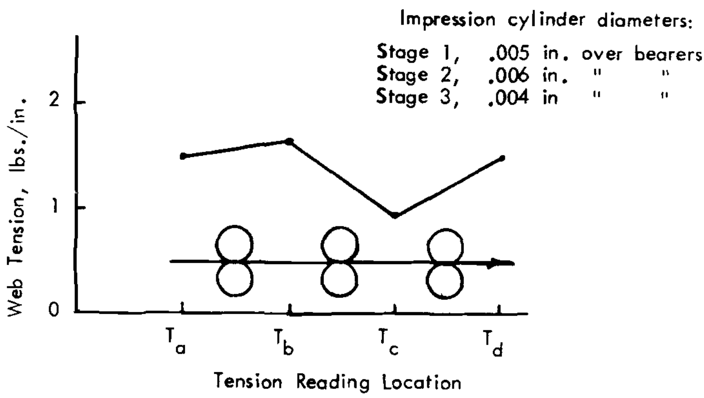


Figure 9. Web tension distribution in a three stage press with unequal impression or gravure cylinder diameters.

with a modulus of .005 in./in./lb., most of the decrease occurs between the last two stages. The curve of the web having a modulus of .0006 in./in./lb. is a typical curve.

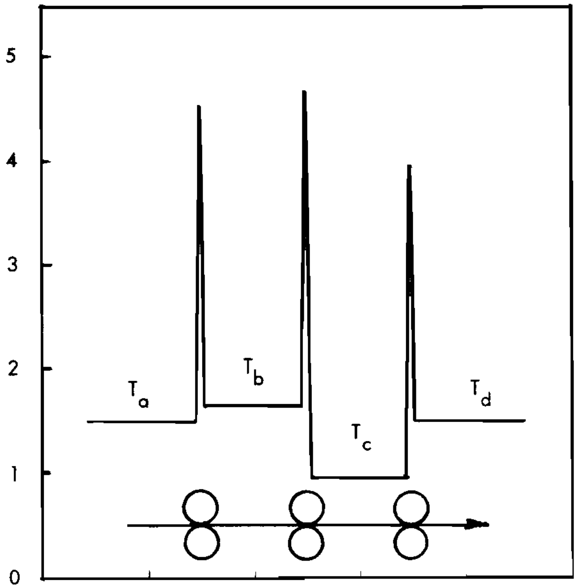


Figure 10. The data of Figure 9 presented to show interstage tension levels and tension spikes in the printing nips.

Figure 8 shows the effect of moisture absorption in stage 1 of a three stage press in an amount to increase the web length .0005 inches per inch. This introduces a decided decrease in tension at the outfeed of the first stage and a gradual recovery of tension in the two succeeding stages. The math shows that excessive moisture pick-up in one stage can cause the outfeed tension to go to zero and a slack web result. It also shows that the print length is increased as a result of the moisture pick-up.

Figure 9 shows the effect on interstage tensions of unequal impression cylinder diameters in the three press stages. It shows that differences of only .001 inch can cause substantial differences in interstage tensions.

Figure 10 shows the data of Figure 9 in a different mode of presentation, one which depicts the web tension spikes in the nips. This provides a more realistic visualization of the tension changes imposed on the web in its traverse of the press.

Many other applications of the principles and equations are possible. For example, in the operation of business forms presses, the equations predict the difficulty and high web tensions required to maintain print length on heavy papers such as card stock on presses set up for light papers. They also indicate several methods that might be employed to decrease those difficulties.

Further, the math suggests that for high quality exact register printing, it may be just as important to accurately control the outfeed tension of a press as the infeed tension.

Another example relates to some experiences of the author's with gravure presses in the early sixties. At that time one school of thought believed that successive gravure cylinders should have progressively greater diameters to insure against a loss of web tension between stages. A study using the equations will show, not only why that strategy was unnecessary, but also why that strategy was not detrimental.

Summary

The mathematics that have been presented here explain the mechanics of web flow through a single printing nip and through multi-stage presses. They give steady-state solutions of web feed-up and tensions through a press.

This is where the work of the author has ended for the time being. There is much more that must be done in two areas: That of verification of the equation constants, and that of expanding the mathematics to cover the transient conditions that a web disturbance creates. In the latter area, it is planned to rewrite the equations as differentials with respect to time and to set up a series of simultaneous differential equations covering multi-stage presses. They will then be the dynamic counterpart of the steady state equations and as such they will provide a tool in the study of register control systems and their response to the various disturbances that affect register and print length.

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