COLOR MATCHING FUNCTIONS FOR OBSERVERS OF ARIBTRARILY SIZED TARGETS*

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ABSTRACT

CIE has adopted two basic color matching systems - the first, published in 1931, for observers viewing a 2 degree target - and the supplementary system, published in 1964, for observers viewing a target of 10 degrees subtense. When the target subtends a viewing angle between (or, for that matter, outside) these limits, we are faced with the questions of what color matching function to use and to what degree of approximation are we dealing with?

While we know there are small differences between the 2 and 10 degree systems (and, in many cases, can use either with relative impunity), it is not really understood how these functions behave as they approach infinitesimally small targets. A primary purpose of this study, then, is to suggest an extrapolation of these functions to this limit.

A power function of the spectral tristimulus values which has as its limit an exponential function as the viewing angle approaches zero was used for the extrapolation model. The choice of this model is discussed.

It was discovered that the 2 degree system should be adequate for color matching problems of targets less than 2 degrees subtense.

INTRODUCTION

The original CIE color matching system, worked out in 1931, provided color matching functions for observers of 2 degree targets, equivalent to a dime (diameter 17,5 mm) viewed at arm's length (500 mm). A supplementary system, published in 1964, provides functions for targets of 10 degrees.

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There are some differences between these two systems (thereby justifying the existence of the second). For example, the $\bar{z}$-function is narrower (at half-height) for the 10 degree system, while the $\bar{y}$-function is wider.

See Figure 1 for the color matching functions $\bar{x}$, $\bar{y}$, and $\bar{z}$, for observers of targets subtending 2 degrees.

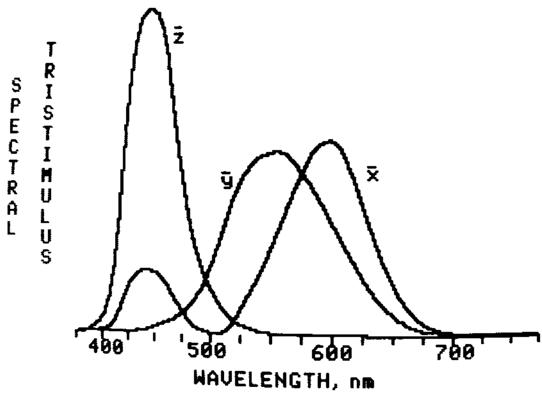

Figure 1. CIE 2-degree Color Matching Functions.

In many applications, even a 2 degree target is too large. For example, if we were interested in measuring the color of a period (diameter 1mm) imbedded in a typewriter key viewed at a distance of 350mm, we would want color matching functions for observers of targets of 0,16 degrees (about 10 minutes of an arc).

An example that applies to colored printing would be that of a 50% halftone dot, with a ruling of 150 lines per inch (6,3 lines per mm - 0,085mm), viewed at the same distance, for a target of 0,014 degrees (50 seconds of an arc).

PHYSIOLOGICAL CONSIDERATIONS

The retina of the eye is believed to contain two different type of receptors --- one sensitive primarily to detail (the rods), while the other (the cones) are sensitive to color. Further, it is generally assumed that cones come in three different color sensitivities, viz, approximitely red, green, and blue.

The mixture of rods and cones is not constant throughout the surface of the retina. In particular, the center of the retina (the fovea centralis) contains a high concentration of cones, and is where most of our color vision occurs.[1]

At the center of the fovea is an area called the foveola, which contains no rods, only cones. In fact, the concentration of cones in the foveola is approximitely 4 times as dense as for the rest of the fovea.

The foveola occupies an area on the retina corresponding to a target of approximitely 1,4 degrees subtense, while the fovea occupies an area corresponding to a target subtending approximitely. 5,2 degrees.[2]

Because about half standard 2 degree observer is on the foveola, and the entire target certainly falls on the fovea, we would expect that targets smaller than this would evoke similar sensations of color.

TRANSFER OF TECHNOLOGY

Consider a patch in a halftone reproduction. This patch may conveniently be assumed to have lineal dimensions equal to half the pitch of the halftone screen, although we will not limit patches of other sizes from consideration. Unless we are looking at a benday tint, the patch under consideration lies on a complex surround.

Bartleson and Breneman [3] have illustrated that an observer of an achromatic target on a complex field experiences a brightness sensation exponentially related to a luminance. If, in the problem at hand, it is reasonable to assume that the observation of an infinitesimal colored target is a microcosm of the problem tackled by Bartleson and Breneman, a possible avenue of approach can be suggested.

The Bartleson and Breneman equation for brightness, when normalized, is:

$$(1) \quad \Lambda(L) = 0{,}0631\, L^{b}\, e^{2{,}76 L^{0{,}0912}}$$

where Λ is brightness, L is luminance (in this case, relative to the maximum luminance on the field), and b is a constant. (This equation uses exponentials of different bases than the original; however, they are equivalent within a normalization constant.) Brightness was until that time considered roughly a power-function transformation on luminance (as with the Munsell value). Bartleson and Breneman pointed out that the brightness of a very small patch on a complex field is the product of a power function and an exponential function of the relative luminance.

This exponential factor, then, holds particular significance as we pass from large targets to small, particularly those with complex surrounds. Consider the factor a simple power-function transformation of luminance must be multiplied by to obtain brightness. Calling this "brightness factor" $\tilde{B}$, we may write:

$$(2) \quad \tilde{B}(L) = e^{2{,}76\,L^{0{,}0912}}$$

In particular, it is interesting that this function is the limit of the function

$$(3) \quad \tilde{B}_v(L) = \left(1 + 2{,}76(\sin v)\,L^{0{,}0912}\right)^{1/\sin v}$$

where the angle the target subtends, v, approaches zero. Recalling that the sine of a small angle is very close to the measure of the angle itself, measured in radians, we may write:

$$(4) \quad \tilde{B}_v(L) = \left(1 + 2{,}76\,v\,L^{0{,}0912}\right)^{1/v}$$

Therefore, we have as a simplification:

$$(5) \quad \lim_{v \to 0} \tilde{B}_v(L) = \lim_{v \to 0} \left(1 + 2{,}76\,v\,L^{0{,}0912}\right)^{1/v}$$

METHODOLOGY

We abstract color matching functions for observers of targets infinitesimally small by studying this limit. Substituting in turn for relative spectral luminance of each of the tristimulii, $\bar{x}$, $\bar{y}$, and $\bar{z}$, we may write for $\bar{x}$:

$$(6) \quad \tilde{B}_v[\bar{x}_v(\lambda)] = \left[1 + 2{,}76\,v\,\bar{x}_o(\lambda)^{0{,}0912}\right]^{1/0{,}0912}$$

where, as before, the v subscript on the x denoting the measure in radians of the angle the target subtends. Naturally, the $\bar{x}_0(\lambda)$ is the spectral tristimulus value for an observer of an infinitesimally small target. Similar expressions are of course derived for the $\bar{y}$ and $\bar{z}$ tristimulii.

Now the inverse of (2) is needed to transform from brightness back into spectral tristimulus. This inverse is:

$$(7)\quad \bar{x} = \left(\frac{\ln \tilde{\beta}}{2.76}\right)^{1/0.0912}$$

This is substituted into (6) to obtain $\bar{x}_v(\lambda)$ as a function of $\bar{x}_0(\lambda)$:

$$(8)\quad \bar{x}_v(\lambda) = \left[\frac{\ln\left(1 + 2.76\, v\, \bar{x}_0(\lambda)^{0.0912}\right)}{2.76\, v}\right]^{1/0.0912}$$

However, because $\bar{x}_v(\lambda)$ is known for at least one size target, and $\bar{x}_0(\lambda)$ is not, we must find the inverse of (8) to obtain the unknown color matching function $\bar{x}_0$. Three-quarters of the 10 degree observer's retinal image falls off the foeva, while nearly all the retinal image of a 2 degree observer is on the foeva. Physiologically, it would make sense, then, to use the spectral tristimulus values for the 2 degree observer to estimate the unknown $\bar{x}_0$.

Inverting (8), and using the 2 degree spectral tristimulus values, we obtain the color matching function for an observer of an infinitesimally small target:

$$(9)\quad \bar{x}_0(\lambda) = \left[\frac{e^{[2.76 \frac{\pi}{90} \bar{x}(\lambda)]^{0.0912}} - 1}{2.76 \frac{\pi}{90}}\right]$$

As was mentioned before, solutions for the color matching functions $\bar{y}_0$ and $\bar{z}_0$ are obtained by the obvious substitution.

RESULTS

The color matching function $\bar{y}_0(\lambda)$ was normalized by a multiplicitive constant so that its maximum, occuring as for the 2 and 10 degree systems at 555 nm, equals 1. The other functions, $\bar{x}_0(\lambda)$ and $\bar{z}_0(\lambda)$, were normalized so their integrals (as computed by Simpson's Rule) equaled the integral of $\bar{y}_0(\lambda)$. These functions are tabulated in Table 1, and are illustrated in Figure 2.

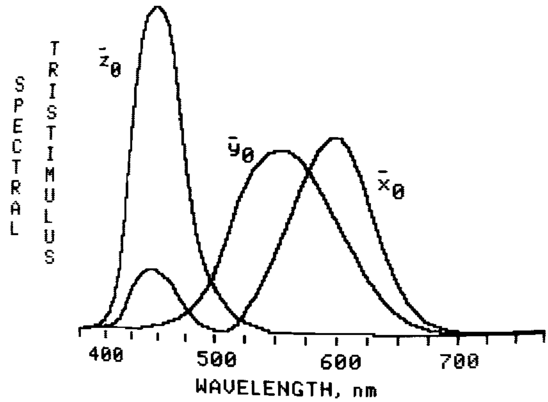

Figure 2. 0-degree Color Matching Functions.

It appeared that the set of color matching functions $\bar{x}_0(\lambda)$, $\bar{y}_0(\lambda)$, $\bar{z}_0(\lambda)$ very closely resembled their respective counterparts in the 2 degree system. The deviations between the two sets are illustrated in Figures 3, 4, and 5. In these illustrations the vertical axis has been magnified 50 times that of the original scale used in Figures 1 and 2.

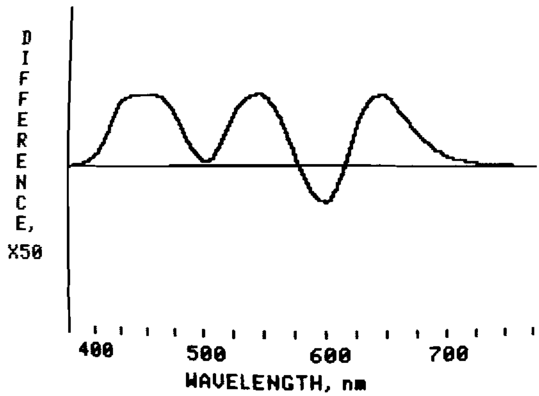

Figure 3. Difference (X50) between $\bar{x}_2$ and $\bar{x}_0$.
(Vertical scale is 50 times as large as in Figures 1 & 2.)

These deviations never seem to exceed 1% (0,01), so we may tentitively conclude that the two systems are very similar. Errors of a similar magnitude would result if one set of color matching functions were used in place of the other set.

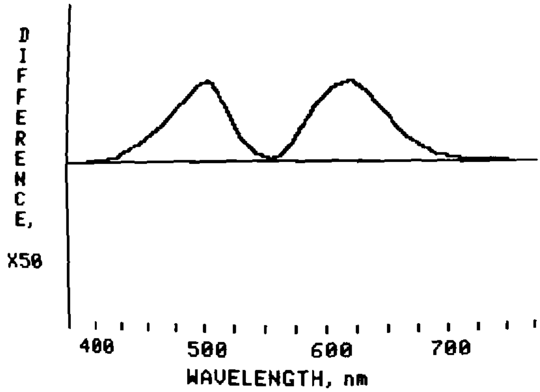

Figure 4. Difference (X50) between $\bar{y}_2$ and $\bar{y}_0$.

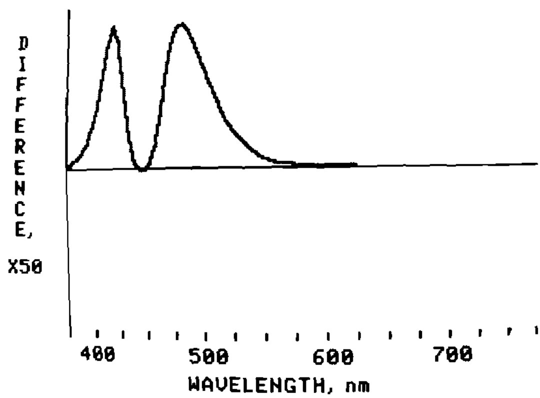

Figure 5. Difference (X50) between $\bar{z}_2$ and $\bar{z}_0$.

This result is further supported by the physiological considerations mentioned before, which hypothesized that color matching functions for observers of targets falling on the foeva would be similar to the color matching functions for the 2 degree system.

CONCLUSIONS

Because no experimental work was actually performed, no hard conclusions can be drawn from the work presented here. The purpose of this paper was to suggest the possible behaivor of the color matching functions as the angle of target subtense becomes arbitrarily small. The following caveat should go without saying: These functions represent merely a suggestion of the limiting case.

Very succinctly, the author would not expect color matching functions experimentally derived for arbitrarilly small targets to differ much from those for the 2 degree system. Similarly, it would be expected that color matching functions for observers larger than 10 degrees could be approximated by the color matching functions $\bar{x}_{10}(\lambda)$, $\bar{y}_{10}(\lambda)$, and $\bar{z}_{10}(\lambda)$, because only 1/4 of the retinal image of a 10 degree target falls on the foeva.

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   As a secondary source, see:
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3. Bartleson, C J, and E J Breneman, "Brightness Perception in Complex Fields," Journal of the Optical Society of America, 57 (1967) : 7 : 953 - 56.

Table 1. Suggested Color Matching Functions $\bar{x}_0(\lambda)$, $\bar{y}_0(\lambda)$, and $\bar{z}_0(\lambda)$ for observers of infinitesimally small targets.

NM	$\overline{XO}$	$\overline{YO}$	$\overline{ZO}$
380	0,0011	0,0000	0,0052
385	0,0018	0,0001	0,0085
390	0,0034	0,0001	0,0166
395	0,0063	0,0001	0,0306
400	0,0121	0,0003	0,0587
405	0,0200	0,0005	0,0971
410	0,0383	0,0009	0,1876
415	0,0698	0,0017	0,3445
420	0,1235	0,0032	0,6143
425	0,2012	0,0060	1,0116
430	0,2691	0,0097	1,3684
435	0,3134	0,0142	1,6158
440	0,3332	0,0197	1,7459
445	0,3330	0,0257	1,7832
450	0,3211	0,0331	1,7722
455	0,3037	0,0422	1,7427
460	0,2760	0,0531	1,6641
465	0,2368	0,0660	1,5166
470	0,1823	0,0819	1,2669
475	0,1308	0,1022	1,0144
480	0,0866	0,1272	0,7821
485	0,0516	0,1562	0,5850
490	0,0278	0,1936	0,4360
495	0,0124	0,2429	0,3270
500	0,0040	0,3064	0,2489
505	0,0019	0,3904	0,1922
510	0,0078	0,4868	0,1415
515	0,0252	0,5938	0,0985
520	0,0565	0,6982	0,0680
525	0,0999	0,7842	0,0492
530	0,1533	0,8556	0,0359
535	0,2119	0,9108	0,0250
540	0,2756	0,9516	0,0168
545	0,3446	0,9791	0,0109
550	0,4187	0,9945	0,0070
555	0,4985	1,0000	0,0045
560	0,5827	0,9945	0,0031
565	0,6690	0,9774	0,0021
570	0,7558	0,9495	0,0016
575	0,8395	0,9113	0,0014
580	0,9168	0,8639	0,0013
585	0,9822	0,8081	0,0011
590	1,0325	0,7467	0,0008
595	1,0646	0,6827	0,0008
600	1,0705	0,6171	0,0006
605	1,0529	0,5515	0,0004
610	1,0075	0,4868	0,0002
615	0,9400	0,4244	0,0001
620	0,8520	0,3641	0,0001
625	0,7446	0,3044	0,0001
630	0,6319	0,2492	0,0000
635	0,5288	0,2023	0,0000
640	0,4333	0,1617	0,0000
645	0,3457	0,1265	0,0000
650	0,2687	0,0969	0,0000
655	0,2050	0,0731	0,0000
660	0,1528	0,0541	0,0000
665	0,1109	0,0391	0,0000
670	0,0789	0,0277	0,0000
675	0,0567	0,0199	0,0000
680	0,0413	0,0144	0,0000
685	0,0286	0,0100	0,0000
690	0,0195	0,0068	0,0000
695	0,0134	0,0047	0,0000
700	0,0096	0,0033	0,0000
705	0,0067	0,0023	0,0000
710	0,0048	0,0017	0,0000
715	0,0033	0,0012	0,0000
720	0,0023	0,0008	0,0000
725	0,0016	0,0005	0,0000
730	0,0011	0,0004	0,0000
735	0,0008	0,0003	0,0000
740	0,0005	0,0002	0,0000
745	0,0004	0,0001	0,0000
750	0,0002	0,0001	0,0000
755	0,0002	0,0001	0,0000
760	0,0002	0,0001	0,0000
765	0,0001	0,0000	0,0000
770	0,0001	0,0000	0,0000
775	0,0000	0,0000	0,0000
TOTALS:	20,9112	20,9119	20,9071