A New Ink-trap Formula for Newsprint

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When printing color on newsprint the ink-trap calculations often produce numbers that are much lower than expected. However, the color overprints look better than the numbers would indicate. This paper presents an analysis of the GATF ink-trap formula which accounts for the lower trap values and provides a modified formula based on more realistic assumptions about reflection densities.

Before introducing the modified formula, let's take a look at the standard GATF ink-trap calculation currently in use. There are three important factors to look at to understand the situation; the definition of ink-trap, any assumption(s) about reflection density, and the method of ink-trap calculation.

The definition of ink-trap most commonly in use is as follows:

$$\{1\} \qquad \text{TRAP} = 100 \bullet \frac{\text{OBSERVED INK-LAYER THICKNESS}}{\text{EXPECTED INK-LAYER THICKNESS}}$$

This definition indicates that the purpose of an ink-trap measurement is to tell us about the relative amount of ink transferred to paper. It is NOT an indicator of light absorbed, light reflected, or color appearance of the printed patches.

A fundamental assumption underlying the standard calculation of ink-trap is that reflection density is directly proportional to ink-layer thickness. This assumption can be expressed equivalently by the statement that reflection densities are additive:

$$\{2\} \qquad D_{yz} = D_y + D_z \qquad \text{(Additivity)}$$

where D_y and D_z are any two component densities, and D_{yz} is the resultant density.

This assumption allows us to link "ink-layer thickness" to "reflection density" which is easier to measure. As a result, we can convert our definition into a formula for calculation.

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The standard method of calculation is as follows:

$$\{3\} \qquad \text{TRAP} = 100 \bullet \frac{D_{210} - D_{10}}{D_{20} - D_0}$$

where D_{210} = density of two-color patch on paper
D_{20} = density of 2nd ink on paper
D_{10} = density of 1st ink on paper
D_0 = density of paper

Nobody should be surprised to hear that equation {2} isn't really true, although the discrepancies are often slight. This is especially true if the densities are small and the printable density range is long. BUT, if the printable density range is short AND the densities involved are moderate or high, additivity failure is significant.

OK, so what? What does this have to do with ink-trap? The problem is this. If we assume that additivity holds, then we also assume that any reduction in total density must be caused by a failure to transfer a full measure of ink. This means that whenever additivity failure occurs, the ink-trap percentages suffer the consequences. What we need to do is to find a more reasonable assumption about how densities add, and then derive a corresponding formula for calculating ink-trap. Notice, we are NOT changing the definition of ink-trap. We are simply attempting to refine the assumptions and approximations which surround the calculation of ink-trap from available data.

Fortunately, it is easy to find a replacement for the additivity assumption. Yule [1] presented such a density relationship twenty years ago. Suppose we modify our assumption about additivity as follows:

$$\{4\} \qquad D_{yz} = D_y + D_z - \frac{D_y \bullet D_z}{D_m} \qquad \text{(Sub-additivity)}$$

where D_m = maximum printable density for the given substrate.

For example, if D_m were estimated at 1.5 (for newsprint, say) and the two inks had individual densities of 1.0 and 0.8, then the expected density of the two-color overprint would be 1.27 (by equation {4}) instead of 1.8 (by equation {2}). While it can be argued that 1.5 is or is not the maximum printable density on newsprint, I would hope that anyone would accept 1.27 instead of 1.8 as a better estimate of the total density in this case.

On the basis of equation {4}, we can derive a modified formula for computing an ink-trap percentage. The details of this derivation are presented in the appendix, but the resultant formula is as follows:

$$\{5\} \qquad \text{TRAP} = 100 \bullet \frac{\log\left(1 + \dfrac{D_{210} - D_{10}}{D_m - D_{210}}\right)}{\log\left(1 + \dfrac{D_{20} - D_0}{D_m - D_{20}}\right)}$$

This formula is clearly more complicated than equation {3}, but it should be because it corresponds to a more complicated (and more realistic) assumption about density additivity.

Let's take a look at a couple of examples. Suppose our laydown order is CMYK and we estimate D_m to be 1.5 as in the earlier newsprint example. Consider first the trapping of yellow over cyan:

EXAMPLE #1	Blue Density	
D_{210} = Green	0.90	
D_{20} = Yellow	0.85	
D_{10} = Cyan	0.34	
D_0 = Paper	0.23	
Standard trap calculation		90%
Newsprint trap calculation	=	98%

The difference is not that great. This, in part, is caused by the relatively low density of the cyan. Consider now the trapping of the yellow over magenta:

EXAMPLE #2	Blue Density	
D_{210} = Red	1.12	
D_{20} = Yellow	0.85	
D_{10} = Magenta	0.89	
D_0 = Paper	0.23	
Standard trap calculation		37%
Newsprint trap calculation	=	71%

Here the difference is much more striking. Also, notice how high the density of the magenta is. If you are assuming that densities are additive, you would be expecting the red density ideally to be 1.74 (the sum of 0.85 and 0.89). Naturally, the actual density of 1.12 falls considerably short of this value and the trap value of 37 percent indicates this fact. On the other hand, if you are assuming sub-additivity (as in equation {4}), the total density for the red patch comes out at 1.24 with perfect trap. In this case, the actual density of 1.12 still falls short, as indicated by the newsprint trap value of 71 percent, but with much less severity than before.

It is interesting to notice that the key difference between examples 1 and 2 is the density of the first ink down. It is the high blue density of the magenta (0.89) which brings the issue of sub-additivity into play. From this we can see that other ink problems, such as contamination, can have an adverse effect on our calculation of ink-trap unless we take sub-additivity into account. After all, the color of an ink should have nothing to do with our measurement of how much of that ink is transferred.

We now turn our attention to an interesting property of the newsprint trap formula which has to do with letting the value of D_m grow without bound:

as D_m gets larger and larger . . .

$$\frac{Dy \bullet Dz}{D_m} \quad \text{gets closer and closer to zero.}$$

When this happens, equations {2} and {4} become more and more similar until finally there is no difference. This means that when the maximum printable density becomes very large (say a density greater than 5.0) the issue of sub-additivity ceases to exist. It can also be shown that:

as D_m gets larger and larger . . .

$$\frac{\log\left(1 + \dfrac{D_{210} - D_{10}}{D_m - D_{210}}\right)}{\log\left(1 + \dfrac{D_{20} - D_0}{D_m - D_{20}}\right)} \quad \text{gets closer and closer}$$

$$\text{to} \quad \frac{D_{210} - D_{10}}{D_{20} - D_0}$$

Thus, as the additivity assumption is restored, the newsprint formula reduces to the standard formula.

An interesting situation now presents itself. Using the standard ink-trap formula is EQUIVALENT to using the newsprint ink-trap formula with the claim that our maximum printable density is greater than 5. In a sense, by using the standard formula, we are ALREADY using the newsprint trap formula, it's just that we have an unrealistic estimate for D_m. So, if one thinks that his maximum printable density is not greater than 5 and that reflection densities do exhibit additivity failure, then one should consider using the newsprint trap formula with more reasonable estimates for D_m. One would think that reasonable values for D_m might range between 1.5 and 2.5 or so depending on the quality of the inks and the paper stock. The most appropriate value for D_m is clearly an arguable point and undoubtedly depends on a number of factors which need experimental investigation.

In conclusion, we have seen that density additivity is a key assumption which underlies the standard ink-trap formula. Furthermore, under that assumption, any sub-additivity is incorrectly reported as poor ink tranfer. Thus, in situations with significant sub-additivity such as printing color on newsprint or perhaps when ink contamination occurs, the standard trap values are much lower than they should be. By modifying the additivity assumption to include some sub-additivity, we have derived a modified trap formula (called the newsprint formula) which takes the maximum printable density into account. When additivity failure is significant, the newsprint formula gives more realistic trap percentages, and when additivity failure is not significant, the two trap formulas agree.

Literature Cited:

[1] Yule, J., *Principles of Color Reproduction,* John Wiley & Sons, Inc., 1967, pp.216-232

APPENDIX

The following derivation is based solely on equation (8.06) in Yule [1]. Accordingly, whenever there is a maximum reflection density D_m, one can not simply add up component densities to find the overall density. Rather, one must "add" densities based on how close the component densities come to the maximum. For the combination of just two densities, Yule's equation simplifies to:

$$\{6\} \qquad D_{yz} = D_y + D_z - \frac{D_y \bullet D_z}{D_m}$$

which was already presented earlier as equation {4}.

Notice that the amount being subtracted from the initial sum increases as the component densities increase relative to D_m. Thus, if D_y were already equal to D_m, the resultant value of D_{yz} would also be D_m regardless of the value of D_z.

We begin by introducing the term "intrinsic density" which we define as the density or density component associated with some specific material used in forming an image. For example, the intrinsic density of an ink-layer is the density component associated with just the ink itself and not the paper it is printed on. While such densities can't be directly measured, they provide convenient intermediate quantities to be calculated in the following derivation.

We will now use our own equation {6} to do two things. First we will relate the thickness of an ink-layer to its intrinsic density. Secondly, we will compute the intrinsic density of the second ink both on paper and on the first ink. Finally, we will derive the new ink-trap formula for newsprint.

Let D(x) designate the intrinsic density function of ink-layer thickness, i.e. D(x) is the intrinsic density of an ink-layer x units thick. Because D(x) is an intrinsic density function we know that $D(0) = 0$, which simply means that an ink-layer zero units thick has zero intrinsic density. Let x be a given ink-layer thickness and let h be a very small thickness compared to x. We can express the intrinsic density of the two layers together by {6} as follows:

$$D(x+h) = D(x) + D(h) - \frac{D(x) \bullet D(h)}{D_m}$$

Now, we have

$$D(x+h) - D(x) = \left(1 - \frac{D(x)}{D_m}\right) \bullet D(h)$$

and dividing by h, we get

$$\frac{D(x+h) - D(x)}{h} = \left(1 - \frac{D(x)}{D_m}\right) \bullet \frac{D(h)}{h}$$

Now, recalling that $D(0) = 0$, we replace $D(h)$ by $D(h) - D(0)$ and let h go to zero. Thus, we get the following differential equation:

$$D'(x) = \left(1 - \frac{D(x)}{D_m}\right) \bullet D'(0)$$

where $D'(x)$ denotes the first derivative of $D(x)$.

We can regroup these terms as follows:

$$\frac{D'(x)}{D_m - D(x)} = \frac{D'(0)}{D_m}$$

Integrating with respect to x now produces:

$$-\ln\left(D_m - D(x)\right) = \frac{D'(0)}{D_m} \bullet x + C$$

A quick substitution of $x = 0$ yields $C = -\ln(D_m)$, thus

$$\{7\} \qquad x = \frac{D_m}{D'(0)} \bullet \ln\left(\frac{D_m}{D_m - D(x)}\right) \text{ and}$$

$$\text{conversely,} \quad D(x) = D_m \bullet \left(1 - \exp\left(-\frac{x \bullet D'(0)}{D_m}\right)\right)$$

By {7}, we have achieved our first goal; to relate an ink-layer thickness to its intrinsic density. When we apply {7} to ink-trap calculations we will see that the unknown constant $D'(0)$ cancels out and need never be determined.

We now turn our attention to ink-trap. As before, we shall use {1} as our definition for ink-trap. We also need to extend our earlier notation as follows:

let D_0 = measured density of paper
D_{10} = measured density of ink 1 on paper
D_{20} = measured density of ink 2 on paper
D_{210} = measured density of ink 2 over ink 1 on paper

D_2 = intrinsic density of ink 2
x_2 = ink-layer thickness of ink 2

and D_m = maximum printable reflection density

In order to compute ink-layer thickness, we must first compute ink-layer densities. The intrinsic density we will need is D_2 and the two cases we will consider are:

Case 1 : When ink 2 is printed over ink 1.
Case 2 : When ink 2 is printed directly on paper.

CASE 1 : First apply {6} as follows

$$D_{210} = D_2 + D_{10} - \frac{D_2 \bullet D_{10}}{D_m}$$

Thus,

$$D_{210} - D_{10} = \left(1 - \frac{D_{10}}{D_m}\right) \bullet D_2$$

and finally,

$$D_2 = \frac{D_m \bullet (D_{210} - D_{10})}{D_m - D_{10}}$$

Substituting into {7} and simplifying, we get

$$\{8\} \qquad x_2 = \frac{D_m}{D'(0)} \bullet \ln\left(1 + \frac{D_{210} - D_{10}}{D_m - D_{210}}\right) \qquad \text{[Case 1]}$$

CASE 2 : An identical derivation involving D_2, D_{20}, and D_0 gives us

$$\{9\} \qquad x_2 = \frac{D_m}{D'(0)} \bullet \ln\left(1 + \frac{D_{20} - D_0}{D_m - D_{20}}\right) \qquad \text{[Case 2]}$$

Now we can express ink-trap as a ratio of the ink-layer thickness which are given in {8} and {9}. So,

$$\text{TRAP} = 100 \bullet \frac{x_2 \text{ [Case 1]}}{x_2 \text{ [Case 2]}}$$

which reduces to

$$\{10\} \qquad \text{TRAP} = 100 \bullet \frac{\ln\left(1 + \dfrac{D_{210} - D_{10}}{D_m - D_{210}}\right)}{\ln\left(1 + \dfrac{D_{20} - D_0}{D_m - D_{20}}\right)}$$

This equation is identical to {5} except that here we have natural logarithms (base e) and in {5} we have common logarithms (base 10). It is a convenient property of logarithms that when taking ratios of them, the quotient is independent of the choice of base. We are therefore free to choose the base as long as we use the same base for both numerator and denominator. As a result, equation {5} is expressed with common logarithms as they are more widely used in the graphic arts literature.

Our final observation is that as D_m becomes larger and larger without bound, the sub-additive term

$$\frac{D_y \bullet D_z}{D_m} \text{ becomes smaller and smaller}$$

Thus, the effects of sub-additivity go to zero and we have additivity of densities holding. The calculus student can apply l'Hospital's rule to {10} letting D_m go to infinity and show that, in the limit, equation {10} produces

$$\text{TRAP} = 100 \bullet \frac{D_{210} - D_{10}}{D_{20} - D_0}$$

which is identical to the ink-trap formula {3}, under the additivity assumption. From this perspective, the standard GATF ink-trap formula is a special case of the more general newsprint ink-trap formula presented here.