On the Rendition of Unprintable Colors

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Abstract: Exact reproduction of the colors input to a Color Electronic Pre-press System (CEPS) by a scanner is seldom possible, since the gamuts of colors for input and output media rarely match. Input colors which cannot be printed on a given ouput medium must be mapped into printable colors in such a way as to yield pleasing results. This process of compressing the input gamut so as to be compatible with the output gamut has traditionally been performed by scanner operators. In a modern CEPS, however, gamut compression can be carried out automatically, subject to editorial consent by the user. In this paper, we discuss a mathematical formalism for describing gamuts and mapping from one to another, with emphasis on the advantages of using a uniform color space.

Introduction

The primary goal of any color reproduction system is to faithfully represent original (input) images on a given printing (output) device. The difficulty comes in what is meant by a "faithful" reproduction. Traditionally, the input image data has been represented directly in terms of output ink (CMYK) in the scanning process, so the faithfulness of the reproduction is inseparable from the aesthetic taste of the scanner operator. A colorimetric approach is to assign to each of the unique colors in the input a different set of coordinates in a uniform color space (UCS). In this way, the faithfulness of the reproduction may be measured objectively, and is not subject to confusion with the pleasingness of the output image: if the UCS coordinates of the same colors in the input and output images do not match (under suitable viewing conditions), then the reproduction is not "faithful".

The UCS approach is useful in digital CEPS because it facilitates a conceptual separation between the editorial functions provided to the user (which allow him to make the input image pleasing) and the reproduction processes themselves (which should remain as far as possible transparent to the user). That is, the user may concern himself solely with how to make the input image pleasing while the CEPS worries about the details of how to reproduce an image on the desired output device. The colorimetric approach

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outlined above is too naïve for practical purposes, capturing more the notion of "accurate" rather than "faithful" reproduction. This distinction is important, for in the current state of printing technology, no output process can accurately reproduce all of the colors which may be present in an original transparency. And yet we know that such transparencies may be faithfully reproduced.

In this paper we propose that, at least in the realm of digital CEPS, there is a natural definition of "faithful reproduction" which derives from the concept of a uniform color space and its associated color difference formula (i.e., color metric). In essence, a faithful reproduction is achieved by means of a transformation of the input colors into output colors such that:

1.) input neutrals map into output neutrals, and
2.) the color difference between any two input colors is preserved in the corresponding output colors up to a constant scale factor.

We call this transformation the "gamut compression" transformation, since it attempts to map the range of input colors into the range of printable (output) colors so as to preserve as much color difference information as possible.

Choice of Uniform Color Space

Several attempts have been made to represent perceived colors by means of three independent coordinates. Uniform color spaces have coordinates chosen in such a way that the perceptual difference between two colors depends only upon the distance between their coordinates, and not on where in the space the separate colors are located. Thus, one could decide that two yellowish colors are just as perceptually different from each other as two greenish colors because the coordinate distances for each pair of colors are the same. MacAdam (1985, pp. 129-161) argues cogently that three Euclidean coordinates are insufficient to achieve this uniformity. Nevertheless, approximately uniform spaces have been defined by the *Commission Internationale de l'Éclairage* (CIE) and the Optical Society of America (OSA).

The choice of color space is essentially the choice of a language in which to speak about color. Hence, many of the concepts which follow in this paper may be straightforwardly adapted to any uniform color space. For the sake of concreteness, we will use the uniform color space defined in 1976 by the CIE known as CIELUV (CIE 1978). The coordinates of this space are denoted (L^*, u^*, v^*), and are defined in terms of tristimulus values (X, Y, Z) as follows:

$$L^* = 116\,(Y/Y_n)^{1/3} - 16, \qquad \text{for } (Y/Y_n) > 0.008856, \text{ and}$$
$$L^* = 903.3\,(Y/Y_n), \qquad \text{for } (Y/Y_n) \leq 0.008856$$

$$u^* = 13\,(u' - u'_n)\,L^*$$

$$v^* = 13\,(v' - v'_n)\,L^* \tag{1}$$

where

$$u' = \frac{4X}{X + 15Y + 3Z}, \qquad v' = \frac{9Y}{X + 15Y + 3Z}$$

and (u'_n, v'_n) are the (u', v') coordinates (the "chromaticity") of the neutral color. Y_n is the Y tristimulus value (the "luminance") of the brightest color (usually a neutral) being represented in the space. (The tristimulus values themselves are derived from the spectral distribution of light reflected from or transmitted through an object. See, e.g., MacAdam (1985, pp. 9, 71).) The color metric is defined by:

$$\Delta E^* = [(L^*_2 - L^*_1)^2 + (u^*_2 - u^*_1)^2 + (v^*_2 - v^*_1)^2]^{1/2} \tag{2}$$

where (L^*_1, u^*_1, v^*_1) and (L^*_2, u^*_2, v^*_2) are the CIELUV coordinates of the two colors whose difference is being measured. A just-noticeable difference (JND) between two colors results in $\Delta E^* = 1.0$. CIELUV has several desirable properties:

1.) it is a standardized, mathematically tractable space
2.) neutral colors are always represented by $(u^*, v^*) = (0,0)$
3.) L^*, the "psychometric lightness", is based on perceptual rather than physical measures of brightness
4.) L^* is always scaled to range from 0 to 100, and is easily separated from u^* and v^*
5.) the color metric captures the intuitive notion that, at least for small color differences, the eye is most sensitive to lightness changes among dark colors and to chromaticity changes among light colors.

A Representation for CIELUV Data

Only a finite number of discrete colors may be represented in a digital CEPS due to the finite number of bits available for digital memory. A natural scheme for data representation might be to assign a word of memory to each of the three color-space variables, with each increment in the least significant bit of a word corresponding to a fixed increment in the associated color-space variable. If, for example, the CEPS computer deals with 8-bit words, each variable could then take on one of 256 values, giving a total of 16,777,216 representable colors, laid out on a regularly spaced rectangular lattice. In order to make the most of available memory, however, one would like to minimize the possibility that any of these colors are visibly indistinguishable from one another or lie outside the largest gamut of colors that the CEPS will ever encounter.

The CIELUV system automatically adapts itself to the range of available luminances by means of Y_n. For constant chromaticity, there are always only 101 distinguishable values of psychometric lightness. Moreover, there is only one distinguishable chromaticity for the darkest CIELUV color ($L^* = 0$), namely neutral $(u^*,v^*) = (0,0)$. Indeed, the number of distinguishable chromaticities increases with L^*, since each of u' and v' are confined to the range 0.0 to 0.65 (MacAdam, 1985, p. 150, fig. 8.26). The regular rectangular lattice is therefore not a good choice for the representation of CIELUV data. It would be better to simply string together the three 8-bit words of the above example to form a single 24-bit index into a lookup table of CIELUV coordinates. This in effect assigns a sequence number to each of the represented colors.

The discrete colors corresponding to the lookup table entries should be chosen so that nearest neighbors in CIELUV space are equally distinguishable. Since the CIELUV color metric is the Euclidean distance, this means that all line segments connecting adjacent points in the space must have the same length. If this length is less than 1.0, adjacent colors will not be noticeably different. A topology which satisfies this requirement is the regular tetrahedron. The vertices of such a polyhedron are all separated by the same distance, and many such tetrahedra may be fitted together so as to maintain this property throughout three-dimensional space (Figure 1).

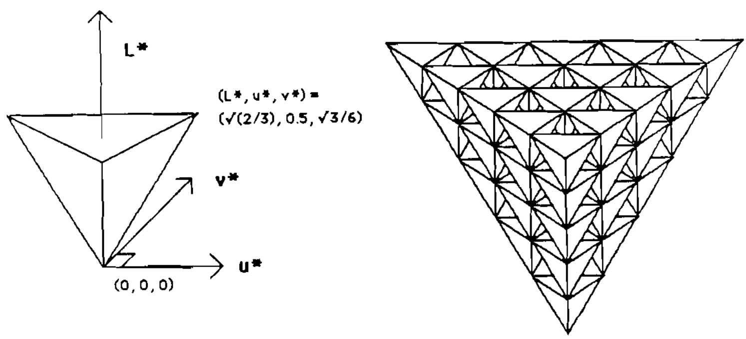

Figure 1: Tetrahedral topology for a uniform color space. At left, basic CIELuv unit; at right, construction for 6 values of L^*.

A natural way to generate CIELUV points is to begin with the origin of the space as one vertex of a tetrahedron with altitude coincident with the neutral axis. Rotating this tetrahedron to roughly correspond to the orientation of the (u',v') chromaticity diagram, and choosing a side of length 1.0 (i.e., one JND), we find that the (L^*,u^*,v^*) coordinates of the first four colors are:

$$
\begin{array}{lll}
(0.0, & 0.0, & 0.0) \\
(\sqrt{(2/3)}, & -0.5, & \sqrt{3}/6) \\
(\sqrt{(2/3)}, & 0.5, & \sqrt{3}/6) \\
(\sqrt{(2/3)}, & 0.0, & -\sqrt{3}/3).
\end{array}
$$

The rest of the space may be "grown" by constructing layers of similar tetrahedra from each of the vertices of the previous layer. Since each layer corresponds to a single value of L^*, 123 layers will be necessary to span the space. The topmost layer of colors will then have $L^* = 100.429$, and the boundaries of the space will form one large regular tetrahedron. In practice, it will be desirable to trim the boundaries of this structure at certain levels of L^*, and to extend the chromaticity range at others by means of the equilateral triangles formed by the u^* and v^* coordinates on a given layer.

Determining the Input Gamut

The medium on which the input image is produced is usually color reversal film or reflection copy from a color negative. In these cases, samples of available colors may be obtained by varying exposure of the three film layers over the allowed range. Measuring the resulting samples with a spectroradiometer allows one to determine the CIELUV coordinates of the colors. Figure 2 shows gamut data for a typical color reversal film.

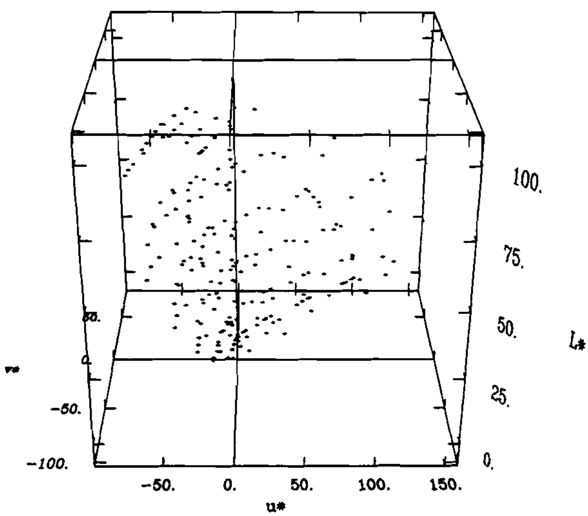

Figure 2: A typical input gamut in CIELUV coordinates.

The input color gamut is determined by the three-dimensional boundary enclosing the coordinates thus obtained. Finding this boundary is not as trivial a problem as one might think, however. It amounts to finding the convex hull of a set of points in three-dimensional space. Shamos (1978) discusses an efficient two-dimensional convex hull finding algorithm which can be extended to three dimensions. See also (Dijkstra 1976).

The gamut itself may be described in terms of the data representation discussed above. For each of the 123 levels of L^*, one may find a polygon composed of equilateral triangles in (u^*,v^*) which comes closest to fitting the boundary. A three-dimensional curve fitting or interpolation technique such as that of Bézier or B-spline surfaces (see, e.g., Newman & Sproull, 1979) must in general be applied to the convex hull in order to generate the intersections of the gamut boundaries with the L^* planes.

Determining the Output Gamut

A more elaborate method may be used to determine the shape of the gamut for the output image. In most cases, the output will be a four-ink printing process, and some transformation must be applied to the CIELUV image data in order to convert it to CMYK values, e.g., percent dot. The output gamut is always known in the CMYK space, however. For example, in the case of half-tone processes each of the inks is constrained to the region of 0% to 100% dot, so the gamut is a hypercube in ink space (Figure 3). If a maximum dot constraint on the process inks is in force, the gamut is constructed by truncating the hypercube by means of the plane satisfying the equation:

$$C + M + Y = maxdot - 100\%. \quad (3)$$

Here we have assumed the traditional practice of applying the constraint only to the process inks while allowing the black ink to span the full 0% to 100% range.

It is a fairly straightforward matter to construct a calibration target by commanding a series of C, M, Y, and K values to the printing process. Measuring this target with a spectroradiometer allows one to determine the (L^*,u^*,v^*) coordinates corresponding to the commanded inkings. Using fitting techniques or semi-physical models such as that of Neugebauer (1937), one can then determine the "forward transformation" which converts ink to CIELUV color, viz., $\mathbf{f}(C,M,Y,K) = (L^*,u^*,v^*)$.

The output gamut is obtained by applying $\mathbf{f}$ to the (possibly truncated) ink hypercube. Figure 4 shows gamut data obtained in this way for a typical proofing system. The gamut itself may be approximated by a polygon composed of equilateral triangles in u^* and v^* at each of the 123 L^* levels used in the data representation.

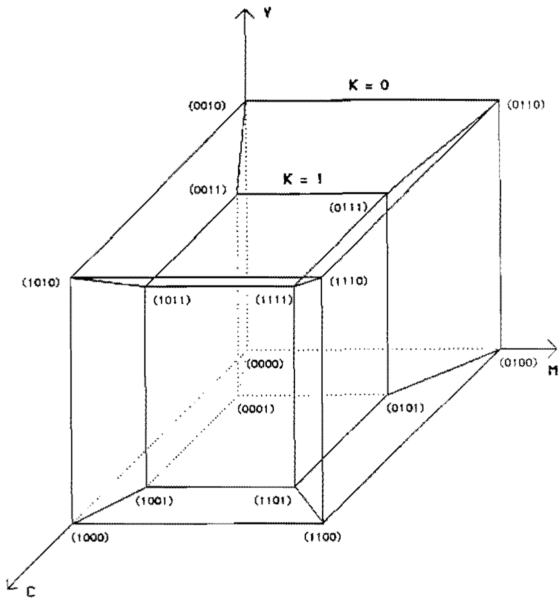

Figure 3: The ink hypercube. Vertices are labeled as (CMYK), where 0 corresponds to 0% dot and 1 to 100% dot.

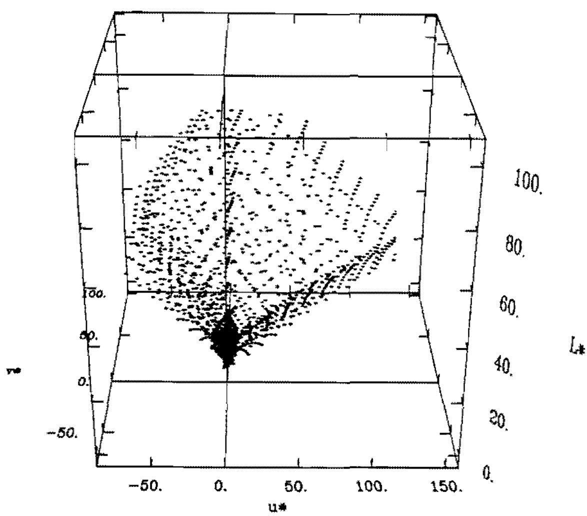

Figure 4: A typical output gamut in CIELUV coordinates.

Additionally, conversion from CIELUV data to output inkings may be accomplished by inverting **f**. This is easier said than done because the domain of **f** is a four-dimensional space and its range is three-dimensional. Thus, degeneracies will arise in the inversion process unless some constraint is applied to the ink space. When under color removal (see, e.g., Yule, 1967) or some other method (e.g., Jung, 1984) of optimizing the black printer is taken into account, these degeneracies can be resolved.

Compressing the Gamut

Usually the input gamut will contain a wider range of colors than the output gamut, which means that not all colors in the input image may be realizable on the desired output medium. The input gamut must therefore be compressed so as to entirely fit inside the output gamut. The CIELUV system takes a step toward this goal by automatically normalizing luminances in the definition of L^*. Also, CIELUV automatically maps input neutrals into output neutrals, since both are represented by $(u^*,v^*) = (0,0)$.

It is still possible, however, that for a given L^* level there will be chromaticities in the input gamut which lie outside of the output gamut. Searching through the gamuts, one can find the largest of these discrepancies and compute the factor which, when applied to the input colors, will reduce the discrepancy to zero. If (u^*_i,v^*_i) and (u^*_o,v^*_o) are respectively the input and output colors of maximum discrepancy, this factor is given by:

$$s = [(u^{*2}_o + v^{*2}_o)/(u^{*2}_i + v^{*2}_i)]^{1/2}. \tag{4}$$

If one then regenerates the input gamut with regular tetrahedra of side s rather than 1.0, while retaining the same shape, the input gamut will have been compressed to fit within the output gamut, and the compressed colors will bear perceptual relationships to each other similar to those of the original input colors. Figure 5 is a conceptual diagram of a CEPS employing the principles of this paper.

The approach we have described thus far is simple, but too naïve in practice. It allows any image produced by the input medium to be faithfully reproduced on the output device, but at the cost of quite severe compression of tonal and chromatic range. A better, but less general, method is to compute a scale factor from the gamut of a given image rather than a given input process. This requires that the CEPS maintain some form of image gamut data, such as maximum and minimum values of u^* and v^* for each of the distinguishable L^* levels. Gamut compression in this case would cause the image itself to just fit inside the printable space, so the CEPS would additionally have to keep track of color edits performed by the user which serve to expand the image gamut.

It is also possible to perform non-uniform gamut compression by replacing the scale factor s by a function which depends on the CIELUV coordinates. In this case, however, gamut compression would no longer perserve similar perceptual relationships among the colors of the input image. Hence, the compression may change the pleasingness of the image. Nevertheless, one can envision an advanced CEPS which "learns" from experience the kinds of color corrections which users are likely to make on images from certain input media when they are being adapted to certain output devices. Such a knowledge-based CEPS could modify its initial gamut compression transformation so as to anticipate user preferences.

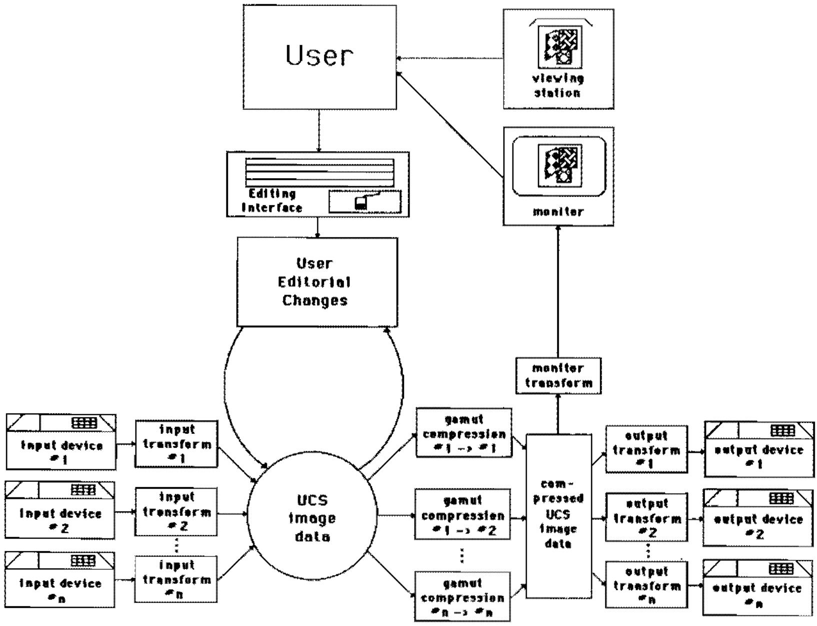

Figure 5: Block diagram of a CEPS performing explicit gamut compression in a uniform color space.

Conclusion

We have described some of the principles involved in the adaptation of images to printing media, especially as regards substitutions for image colors not available on a given output device. In particular, we have shown that a colorimetric approach based on a uniform color space provides an objective measure of the faithfulness of a reproduction. Clearly, much of the power of this approach lies in the possiblity of automatic color correction. Such corrections are made possible by transcending the detailed properties of colorant (e.g., ink) and working in terms of the perceptual colors available to the input and output media.

Eikonix Corporation, a Kodak company, manufactures and sells the

Designmaster® 8000, a digital CEPS which employs colorimetric principles not unlike those described in this paper for the representation and reproduction of images.

Acknowledgments

The authors wish to express our gratitude to Wayne Kearsley for his invaluable technical assistance in the Eikonix color laboratory. We would also like to thank Christopher Pearson, John Taylor, and Leonard Gibbons for their help in collecting and analyzing gamut data.

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