BLACK PRINTER, UCR AND UCA
— GRAY COMPONENT REPLACEMENT —

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Abstract: Theoretical considerations on black printer by using linear color reproduction model which consists of Demichel relations and Neugebauer equations are discussed. Gray component replacement is defined clearly by a combination of black printer, under color removal and under color addition as three elements. Analytical expression of necessary operation is given as a form: $(a_I - a_K)/(1 - a_K)$ by good approximation, where a_I is a dot area of any color component and a_K is a dot area of black ink. As seen in this relation, the numerator shows UCR and the denominator indicates UCA after introduced black printer a_K.

History of GCR

The principle of four color printing was suggested theoretically by Yule (1940) related with pure subtractive color mixture and the theory was correctly applied for three layer color photography and photographic color separation in graphic arts technology as four separations before screening. The theory which suggests full UCR could be applied to subtractive color mixture because it assumes density additivities, and therefore there are discrepancies pointed out by Pollak (1955) in applications for halftone images.

Practical solutions for the above discrepancies were summarized in Yule's book (1967) one as skeleton black without UCR and another as moderate UCR both to avoid or recover pale colors when we applied full UCR in shadow parts of prints. These practices were described using color densities of photographic separations under the condition of photographic

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characteristic curves.

When people started developments of color scanners, there were two approarches to treat color informations. One is color density model which could be stated as simulations of photographic color separation processes and served for duplications in certain accuracy. Another is several efforts to practice Neugebauer model (1937) which uses colorimetric values in new electronic information processes. In practice, the capabilities of electronic computers at that time were not enough to realize applicabilities of this theory in speed and also several unsolved problems, such as four color printings, solutions for dot gain and others, disturbed further studies on possible applications.

Handling of four color printings by almost of color scanners was therefore based upon use of color densities and simulation of skeleton black and moderate UCR in low black rate. Recent years, use of higher rate black was discussed and practiced by getting flexible operations of electronic processings in scanners. We are facing now rather confused statements between several venders who manufacture scanners. Few ones among them claim linear treatments.

Four color printing is generally aiming substitution of certain amount of color inks by black one. If we assume the same color reproduction starting from one set of three color dot area combination, solution could be a unique one in theoretical sense. However, there are many "unique" solutions named by different manners. Almost of them were published by the form of instrument instructions and principles were not fully exposed in technical society. Now people are treating four color printings or gray component replacement by using explanations which are expressed by commercial terms instead of technical ones.

The author found a way to handle this problem and obtained useful information to use in practical processes, starting from linear treatment. The four color printings or GCR is a combination of black printer: a_K, UCR: $(a_I - a_K)$ and UCA: $1/(1-a_K)$ in mathematical expression. This result explains skeleton black and moderate UCR as extreme approximations of real solution.

From Three Colors to Four Colors

Suppose we have already a set of halftone dot areas a_I: (a_Y, a_M, a_C). Tristimulus values of a color corresponding to this set are given by equations (1) which represent averaged additive color mixture given by

Neugebauer (1937) knowing tristimulus values (X_i, Y_i, Z_i) of primary and secondary colors by inks.

$$\left.\begin{aligned}X&=a_wX_w+a_yX_y+a_mX_m+a_cX_c+a_rX_r+a_gX_g+a_bX_b+a_kX_k\\Y&=a_wY_w+a_yY_y+a_mY_m+a_cY_c+a_rY_r+a_gY_g+a_bY_b+a_kY_k\\Z&=a_wZ_w+a_yZ_y+a_mZ_m+a_cZ_c+a_rZ_r+a_gZ_g+a_bZ_b+a_kZ_k,\end{aligned}\right\}\quad(1)$$

where a_i are effective areas of eight color areas which are given by equations (2).

$$\left.\begin{aligned}a_w&=(1-a_Y)(1-a_M)(1-a_C)\\a_y&=a_Y(1-a_M)(1-a_C)\\a_m&=(1-a_Y)\,a_M\,(1-a_C)\\a_c&=(1-a_Y)(1-a_M)\,a_C\\a_r&=a_Y\,a_M\,(1-a_C)\\a_g&=a_Y(1-a_M)\,a_C\\a_b&=(1-a_Y)\,a_M\,a_C\\a_k&=a_Y\,a_M\,a_C\,.\end{aligned}\right\}\quad(2)$$

Equations (2) are named as Demichel relations since these were described by Demichel (1924) long time ago. On the contrary, the set of dot areas could be given as solution of Neugebauer equations or by different means based upon color density information. In this paper, we are considering mainly the same color reproduction between three color printings and four color ones.

Starting from a three color printing (a_Y, a_M, a_C), if one adds black printer a_K, we get a set of dot area for corresponding four color printing (a'_Y, a'_M, a'_C, a_K), where a_K is between zero and the minimum area among the set (a_Y, a_M, a_C). Relations between (a_Y, a_M, a_C) and (a'_Y, a'_M, a'_C, a_K) are given by,

$$\left.\begin{aligned}a'_Y&=\frac{a_Y-a_K}{1-a_K}\\a'_M&=\frac{a_M-a_K}{1-a_K}\\a'_C&=\frac{a_C-a_K}{1-a_K}\end{aligned}\right\}\quad(3)$$

These relations were given by comparison of Neugebauer relations for three color and four color reproductions, and detail is given in Appendix of this paper.

Use of block dyes was assumed to obtain equations (3) for simple presentation of the principle, therefore, these are approximations for real cases. Departure from this theory depends on difference of real dyes from block dyes and also on value of a_K. Numerical examples are illustrated in Table 1.

Table 1. Errors in Non-Block Magenta Dye (x=0.45, y=0.40)

Y-value	P	Theoretical UCA	Simulated UCA					
			Block-Dye Set			M(non-Block) & Block Y,C-Dye		
			a_Y	a_M	a_C	a_Y	a_M	a_C
0.12	0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.2	1.1962	1.1962	1.1962	1.1962	1.1982	1.1962	1.1962
	0.4	1.4881	1.4881	1.4881	1.4881	1.4957	1.4881	1.4881
	0.6	1.9686	1.9686	1.9686	1.9686	1.9950	1.9686	1.9686
	0.8	2.9072	2.9072	2.9072	2.9073	3.0222	2.9071	2.9073
	1.0	5.5568	5.5568	5.5556	------	6.6758	5.5558	------
0.60	0.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	0.2	1.0205	1.0205	1.0205	1.0205	1.0231	1.0205	1.0205
	0.4	1.0418	1.0418	1.0418	1.0418	1.0474	1.0418	1.0418
	0.6	1.0640	1.0640	1.0640	1.0640	1.0730	1.0640	1.0640
	0.8	1.0872	1.0872	1.0872	1.0873	1.1000	1.0872	1.0873
	1.0	1.1114	1.1114	1.1114	------	1.1285	1.1114	------

Each relation consists of two parts in practical sense. One is UCR expressed by the numerator as,

$$\text{UCR}: (a_I - a_K), \tag{4}$$

which has already been well defined and been familiar with people. Besides the above, now we have second factor which is multiplicative to the above factor and corresponding to UCA as,

$$\text{UCA}: 1/(1-a_K), \tag{5}$$

and this number increases continuously with respect to a_K, from unity with $a_K=0$ to large number with a_K unity. The author hesitates to use the word "UCA" for this factor because operation in equation (3) is not additional but multiplicative. However, "under color addition" is used recently among people to claim the same action to improve faded color in higher rate black printer applications, and therefore, this reciprocal number of the denominator in equations (3) could be called UCA.

As results of analysis, now we can define a structure of gray component replacement (GCR) clearly as follows.

$$\text{GCR}\begin{cases}\text{Black Printer} & : \text{any rate, } a_K \\ \text{UCR} & : \text{full, } (a_I - a_K) \\ \text{UCA} & : 1/(1-a_K)\end{cases}$$

These three components are necessary always as a set of operations. Application of full UCR without UCA gives us pale color because colorants are not enough to keep same color between three color and four color reproductions. It is very important to notice that the values of UCA are continuous and increasing coefficient according to change of the black printer.

Considerations

Needs of UCA in practice have been pointed out through increasing interests on high rate black printers as experienced results of color scanner utilization mainly in Europe. Theoretical necessity of UCA proved in the Appendix is caused by existing factor $(1-a_K)$ in Demichel relations for four color printings. This factor shows decrease of primary and secondary color areas in screen angled color printing because of covering effect of black ink. For high rate of black, only parts of a three color ink areas appear between black ink areas and rest parts are covered by black ones. Dot after full UCR is just enough area in such way in that ink of one area is exposed without covering effect of black ink. From the above point of view, Demichel relations are very basic and also very important to have good understandings on GCR in graphic arts. These relations are ordinarily read as a first part of Neugebauer equations by people. The author stresses these relations as an independent principle by giving scholar's name by whom these relations are stated at the first.

One of equations (3) is illustrated in Figure 1. In this figure, a'_I is shown as a function of a_K for different a_I. At the first, experimental results by Schwartz et alii (1985) are illustrated in figure 2. There is some degree of agreement between the theory and the experiments. Less quantity of a'_I needed in experiments compared with theory is caused by, in certain degree, increasing density effect of laminated ink layers. In another way, the theory is an approximation and we need further studies on more detailed and accurate theory for practical applications.

In Figure 1., straight lines as a chord between two axis points for each curve show results of UCR without UCA. The theoretical curves draw arcs because of UCA. Several lines starting from vertical axis having number by percentage show cases of moderate UCR. In this case, line crosses only once with the theoretical curve. Furthermore, it is easy to see in the figure

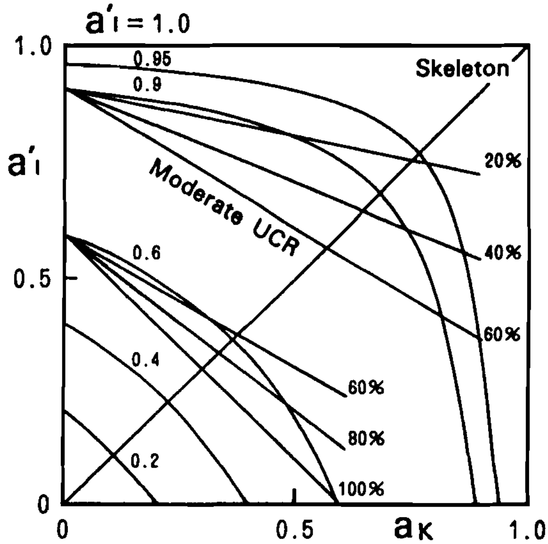

Figure 1. Change of a'_I vs a_K

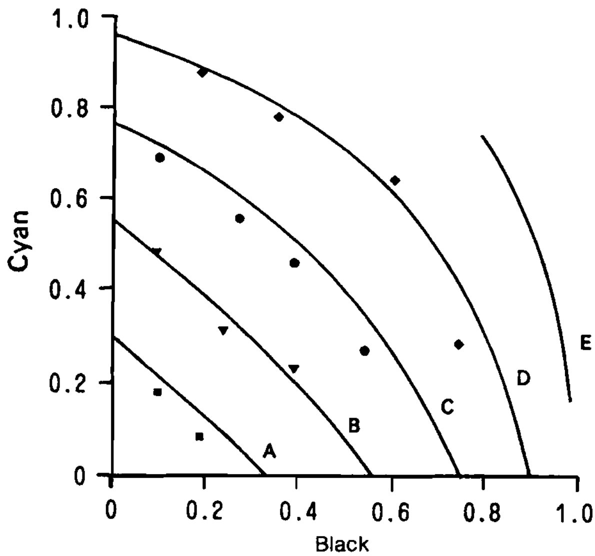

Figure 2. Experiments after Schwartz et alii

that lower UCR rate is needed for higher black printer rate. As the result of these observations, moderate UCR is cosidered as a kind of one point approximation of real needs. Curves for large a_t and high black rate approach to the upper right corner of illustration, and this means that result of GCR is nearly the same with no-UCR. Skeleton black printer is again approximation for the correct way as the extreme end.

If we apply this principle to practical printings, ink saving by GCR is less than expected for shadow part. Calculation of total ink amount and cost can be obtained easily by using equations.

As was pointed out in previous section, need of UCA is based upon Demichel relations. His relations are necessary for graphic arts because of screen angles and psydo-random overlappings of dots. There are several different dot designs in new non-impact printing techniques. Comparison of three component combination for different dot designs are shown in Table 2. Classic explanation for black printer based upon laminated layers is not existed in practice, UCA is only needed for graphic arts (Demichel), corner black need only UCR and concentric dot design need only replacement of the smallest dot by black ink without UCR and UCA.

Table 2. Combination of Three Components of GCR for Different Dot Design

Dot Design	Black Printer	UCR	UCA
Lamination	hypothetical		
Concentric	0—100% replacement	non	non
Corner Black	0—100%	100% of B.P.	non
Demichel	0—100%	100% of B.P.	$1/(1-a_k)$

Conclusions

Structure of GCR was analytically clarified in this paper. In graphic arts, one uses screen angles between four colors to avoid coarse moire effects of different kinds and Demichel relations are very fundamental to consider and analyse many problems related with color reproduction. Neugebauer equations have two faces in applications. One side is calculation of reproduced color from known dot areas of three or four color printings as calculation of averaged additive color mixture by using colorimetric quantities. Another important job is to give three dot areas to realize given tristimulus values as solutions of three-dimensional simultaneous equations. In his theory, again, Demichel relations had important contributions. The result in this paper is one of interesting examples of Demichel and Neugebauer theory.

Structure of GCR as a result in this paper is explained by straightforward way with Demichel relations which suggest overlapping effects between color dots. This simple result is very much depend upon the use of block dyes as approximation. Departures of color reproduction from approximated model in the case of real dyes are not quite large in practice. Needs of three component in GCR, they are a_K, $(a_I - a_K)$ and $1/(1-a_K)$ and structure given by equations (3) and also reasons of needs could be considered from many different sides based on Demichel and Neugebauer theory. Very intrinsic function of this colorimetric model compared with color density model is linear relations between color stimuli and dot areas, but, in color density treatments, additivity and linearity failures (Yule, 1967) are fatal to obtain understandable or clear views to consider GCR projects.

In this presentation, improvement of black by four color printings is ignored to run simple way to find inside mechanisms of GCR in color reproduction processes. Further investigations will follow on such practical and detailed applications, and those efforts are very necessary to use the theory in practical prepress works.

Acknowledgement

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Appendix : Analytical Study on GCR

For given three color printings, one has a set of dot areas (a_Y, a_M, a_C), each of them represents yellow, magenta and cyan. According to Demichel (1924), statistically averaged eight color areas a_i (w : white, y : yellow, m : magenta, c : cyan, r : red, g : green, b : blue, and k : black respectively) are given by,

$$a_w = (1-a_Y)(1-a_M)(1-a_C) \qquad \text{(A1.1)}$$
$$a_y = a_Y\ (1-a_M)(1-a_C) \qquad \text{(A1.2)}$$
$$a_m = (1-a_Y)\ a_M\ (1-a_C) \qquad \text{(A1.3)}$$
$$a_c = (1-a_Y)(1-a_M)\ a_C \qquad \text{(A1.4)}$$
$$a_r = a_Y\ a_M\ (1-a_C) \qquad \text{(A1.5)}$$
$$a_g = a_Y\ (1-a_M)\ a_C \qquad \text{(A1.6)}$$
$$a_b = (1-a_Y)\ a_M\ a_C \qquad \text{(A1.7)}$$
$$a_k = a_Y\ a_M\ a_C \qquad \text{(A1.8)}$$

Using these quantities, tristimulus values of reproduced color are given as mentioned by Neugebauer (1937) as,

$$X = a_w X_w + a_y X_y + a_m X_m + a_c X_c + a_r X_r + a_g X_g + a_g X_b + a_k X_k \qquad \text{(A2.1)}$$
$$Y = a_w Y_w + a_y Y_y + a_m Y_m + a_c Y_c + a_r Y_r + a_g Y_g + a_b Y_b + a_k Y_k \qquad \text{(A2.2)}$$
$$Z = a_w Z_w + a_y Z_y + a_m Z_m + a_c Z_c + a_r Z_r + a_g Z_g + a_b Z_b + a_k Z_k, \qquad \text{(A2.3)}$$

where (X_i, Y_i, Z_i) is a set of tristimulus values corresponding to eight color patches.

In the case of four color printing, extension of the above relations was made by Hardy and Wurzburg (1948) as follows.

$$a'_w = (1-a'_Y)(1-a'_M)(1-a'_C)(1-a_K) \qquad \text{(A3.1)}$$
$$a'_y = a'_Y\ (1-a'_M)(1-a'_C)(1-a_K) \qquad \text{(A3.2)}$$
$$a'_m = (1-a'_Y)\ a'_M\ (1-a'_C)(1-a_K) \qquad \text{(A3.3)}$$
$$a'_c = (1-a'_Y)(1-a'_M)\ a'_C\ (1-a_K) \qquad \text{(A3.4)}$$
$$a'_r = a'_Y\ a'_M\ (1-a'_C)(1-a_K) \qquad \text{(A3.5)}$$
$$a'_g = a'_Y\ (1-a'_M)\ a'_C\ (1-a_K) \qquad \text{(A3.6)}$$
$$a'_b = (1-a'_Y)\ a'_M\ a'_M\ (1-a_K) \qquad \text{(A3.7)}$$
$$a'_k = a'_Y\ a'_M\ a'_C\ (1-a_K) \qquad \text{(A3.8)}$$
$$a'_{wk} = (1-a'_Y)(1-a'_M)(1-a'_C)\ a_K \qquad \text{(A3.9)}$$

$$a'_{yk} = a'_Y \; (1-a'_M)\,(1-a'_C) \; a_K \qquad \text{(A3.10)}$$
$$a'_{mk} = (1-a'_Y) \; a'_M \; (1-a'_C) \; a_K \qquad \text{(A3.11)}$$
$$a'_{ck} = (1-a'_Y)\,(1-a'_M) \; a'_C \; a_K \qquad \text{(A3.12)}$$
$$a'_{rk} = a'_Y \; a'_M \; (1-a'_C) \; a_K \qquad \text{(A3.13)}$$
$$a'_{gk} = a'_Y \; (1-a'_M) \; a'_C \; a_K \qquad \text{(A3.14)}$$
$$a'_{bk} = (1-a'_Y) \; a'_M \; a'_C \; a_K \qquad \text{(A3.15)}$$
$$a'_{kk} = a'_Y \; a'_M \; a'_C \; a_K \qquad \text{(A3.16)}$$

Calculation of reproduced color (X', Y', Z') by four color printings is written by the same form as equations (A2). We have sixteen equations for a'_i, but (A3.8) - (A3.16) could be considered as one black area as an approximation.

Task of this work is to find relations between a'_I and a_I for to achieve same color reproduction,

$$X' = X \qquad \text{(A4.1)}$$
$$Y' = Y \qquad \text{(A4.2)}$$
$$Z' = Z. \qquad \text{(A4.3)}$$

We have three equations in (A4) but, as one see from equations (A1) and (A2), contributions of a_I for X, Y and Z have exactly the same form. Therefore, it is only necessary to use one equation from (A4).

In the case of X if one substitutes (A1) into (A2), still we have too many factors X_i and it is difficult to obtain clear view between X' and X. X is expanded as,

$$\begin{aligned} X = &(1-a_Y)(1-a_M)(1-a_C)X_w + a_Y(1-a_M)(a-a_C)X_y \\ &+(1-a_Y)a_M(1-a_C)X_m + (1-a_Y)(1-a_M)a_cX_c \\ &+a_Ya_M(1-a_C)X_r + a_Y(1-a_M)a_CX_g \\ &+(1-a_Y)a_Ma_CX_b + a_Ya_Ma_CX_k \end{aligned} \qquad \text{(A5)}$$

To continue further considerations, it is necessary to introduce certain assumptions to modify the above equation to be simpler. Block dyes as primary colors are useful to apply, because of the simple relations as,

$$X_w = X_r + X_g + X_b \qquad \text{(A6.1)}$$
$$X_y = X_r + X_g \qquad \text{(A6.2)}$$
$$X_m = X_r + X_b \qquad \text{(A6.3)}$$
$$X_c = X_g + X_b \qquad \text{(A6.4)}$$
$$X_k = 0 \qquad \text{(A6.5)}$$

then one can reduce number of factors to three (X_r, X_g, X_b) instead of eight.

To keep same X value for any combinations of a_I, it is necessary to keep also contributions by X_r, X_g and X_b as same values respectively. Thus summarized contribution by four X_r as $X_{(r)}$ is calculated and reduced to simple result as,

$$X_{(r)}=\Big\{(1-a_Y)(1-a_M)(1-a_C)+a_Y(1-a_M)(1-a_C)$$
$$+(1-a_Y)a_M(1-a_C)+a_Y a_M(1-a_C)\Big\}X_r$$

$$=(1-a_C)X_r, \qquad \text{(A7)}$$

and similar calculation is given for four color reproduction as,

$$X'_{(r)}=(1-a'_C)(1-a_K)X_r. \qquad \text{(A8)}$$

Then equation (A4.1) is expressed by using (A7) and (A8),

$$(1-a_C)=(1-a'_C)(1-a_K), \qquad \text{(A9)}$$

and solution to get a'_C from (A9) has a form

$$a'_C=\frac{a_C-a_K}{1-a_K}. \qquad \text{(A10.1)}$$

In the same manner, two other relations :

$$a'_M=\frac{a_M-a_K}{1-a_K} \qquad \text{(A10.2)}$$

$$a'_y=\frac{a_y-a_K}{1-a_K}, \qquad \text{(A10.3)}$$

are obtained from condition $X'_{(g)}=X_{(g)}$ and $X'_{(b)}=X_{(b)}$ respectively.

The above solutions (A10) are approximation for hypothetical spectral characteristics of colorants so called block dyes. Without this simplification it is impossible to obtain analytical solution. Errors occured by real dyes because of the above approximation in printing results are simulated and tabulated in Table A1 using dyes characteristics shown in Table A2. In these cases, cyan dot areas are minimum and p is expressing rate of black printer compared with cyan dot area. Theoretical UCA is then given by,

$$UCA=\frac{1}{1-p\cdot a_C}. \qquad \text{(A11)}$$

and simulated UCA is calculated from the solutions of Neugebauer equations for given rate of black printer, as comparison of the solutions with full UCR cases. Errors are very small for smaller p and noticeable for larger p. In practical applications, it is necessary to check deviation of reproduced color caused by dot size error.

Results shown in Table A1 indicate that our approximation (A10) is applicable for low black rate exactly but with certain errors for high black rate. Solutions of Neugebauer equations are recommended to obtain accurate answers for high black rate. Use of approximation is valuable to give concrete understandings on the structure of GCR as three components

Table A1. Comparison of Appoximation and Neagebauer Solution

Chromaticity coordinates	Y-value	P	Theoretical UCA	Simulated UCA		
				a_Y	a_M	a_C
x=0.46 y=0.34	0.12	0.0	1.0000	1.0000	1.0000	1.0000
		0.2	1.1667	1.1657	1.1685	1.1672
		0.4	1.4002	1.3969	1.4060	1.4020
		0.6	1.7504	1.7414	1.7667	1.7564
		0.8	2.3344	2.3079	2.3824	2.3594
		1.0	3.5029	3.4034	3.6860	------
	0.36	0.0	1.0000	1.0000	1.0000	1.0000
		0.2	1.0133	1.0129	1.0149	1.0180
		0.4	1.0269	1.0262	1.0298	1.0399
		0.6	1.0409	1.0398	1.0455	1.0709
		0.8	1.0553	1.0538	1.0616	1.1375
		1.0	1.0701	1.0681	1.0783	------
x=0.37 y=0.35	0.12	0.0	1.0000	1.0000	1.0000	1.0000
		0.2	1.2076	1.2089	1.2129	1.2079
		0.4	1.5239	1.5289	1.5460	1.5257
		0.6	2.0648	2.0825	2.1516	2.0731
		0.8	3.2008	3.2833	3.7052	3.2642
		1.0	7.1157	8.4359	569.6521	------
	0.72	0.0	1.0000	1.0000	1.0000	1.0000
		0.2	1.0044	1.0055	1.0095	1.0418
		0.4	1.0088	1.0111	1.0193	1.1094
		0.6	1.0132	1.0168	1.0293	1.2415
		0.8	1.0177	1.0226	1.0396	1.6316
		1.0	1.0222	1.0284	1.0502	------

because of clear expressinos on relations between dot areas as equations (A10). And also, errors are smaller than expected for real dye characteristics.

It is interesting to point out that we are using only one equation from equations (A4) and also that three cases corresponding to Y, M and C are the result of splitting out of this one equation by characteristics of block dyes. Concept of block dyes has been used in graphic art technology for long time to help people's ideas to simplify and to make color reproduction processes visible in people's mind. The above case is one of new examples of usefulness of block dyes.

Table A2. Tristimulus Values of Model Color Patches

	X	Y	Z
Yellow	0.800	0.900	0.118
Magenta	0.750	0.384	0.695
Cyan	0.229	0.292	0.750
Red	0.650	0.411	0.080
Green	0.210	0.380	0.202
Blue	0.210	0.105	0.560
Black	0.050	0.050	0.050

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