CAVEATS IN THE USE OF ΔH^* AND ΔC^* IN COLOR DIFFERENCE ANALYSIS AND CORRECTION †

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Abstract

The CIE uses ΔE^* as an estimate of an overall color difference between two colors. However, it is usually desirable to segregate an overall difference into differences due to hue, chroma, and/or lightness. The CIE suggests the quantity $(\Delta E^*)^2$ to be considered as the sum of $(\Delta L^*)^2$, $(\Delta C^*)^2$, and $(\Delta H^*)^2$, where the correlates correspond to either the 1976 CIELAB or CIELUV color spaces. The quantity ΔH^* is supposedly a psychometric of hue difference as defined by $\Delta H^* = [(\Delta E^*)^2 - (\Delta L^*)^2 - (\Delta C^*)^2]^{1/2}$. Since ΔH^* is defined as a remainder, this definition is valid only to the extent that ΔE^* comprises exclusively ΔL^*, ΔC^*, and ΔH^*, and that ΔL^*, ΔC^*, and ΔH^* are mutually independent compositionally, both psychophysically and psychometrically. It will be shown that the present definition of ΔH^* lacks psychometric independence of chroma, and therefore ΔH^* does not represent a difference due to *only* hue difference. Similarly, because C^* is dependent on L^*, a significant ΔL^* can affect ΔC^* so that ΔC^* might not represent a difference due to *only* chroma difference. Such deficiencies can suggest an incorrect interpretation and subsequent adjustment to colorants.

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Introduction

In paints, textiles, and plastics, a variety of pigments or dyes is used to achieve a large gamut of colors. However, in the halftone printing industry, color reproduction is usually achieved with only three chromatic and one achromatic inks, but additional inks can be used for special color rendition. Furthermore, the phenomenon of "dot gain"[1] complicates the direct applicability of the color science applied to the relative color homogeneity of paints, textiles, and plastics.

In printing, off-press proofs are used as a reference guide of acceptability. A color difference between the proof and the press sheet can be due to many factors. A common one is that the color separation scanner might be based on reproduction primaries colorimetrically different from either the proof colors or press inks, or both. Secondly, the primaries of the proof and press can be different, while one agrees well with the scanner primaries. Thirdly, there can be differences in the optical characteristics between the proof and ink on paper. Therefore, it is usually necessary to overcome these factors so that the press sheet will be acceptable. Colorimetric analysis in conjunction with knowledge of the separation, printing and proofing processes can help accomplish this. Although colorimetry seems so sophisticated that its use will guarantee correct results, it is not without its pitfalls, and one must always be cognizant of these pitfalls to avoid misdirection.

There are many color difference formulas, the CIE 1976 CIELAB and CIELUV spaces[2] being two of the common ones. In both systems, an overall color difference ΔE^* is deemed comprising visual perceptual differences in lightness, chroma, and hue. Thus, by knowing the principal contributors to ΔE^*, the appropriate correction can be tried. However, a large ΔH^* might not truly be due to *only* a *large hue* difference, and similarly ΔC^* might not be due to *only* a chroma difference. Then again, they might be. This paper elucidates circumstances where ΔH^* and ΔC^* can be misleading and therefore require careful interpretation to avoid erroneous conclusions about the true contributions to a color difference. A previous analysis[3] for ΔH^* is included with that for ΔC^* to show the subtle interdependence rather than independence among psychometric lightness, chroma, and hue differences.

ΔH^* Definitions

The CIE 1976 a,b hue difference ΔH^*_{ab} is defined[2] via (1), and the ΔE^*_{ab} therein is defined via (2). Although the CIE 1976 CIELAB color space is used herein, the arguments which follow apply also to the CIE 1976 CIELUV color space. Therefore, "ab" subscripts will be omitted except where it is necessary to distinguish between CIELAB and CIELUV.

$$(\Delta H^*)^2 = (\Delta E^*)^2 - (\Delta L^*)^2 - (\Delta C^*)^2 \qquad (1)$$

$$(\Delta E^*)^2 = (\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2 \qquad (2)$$

"For small colour differences away from the achromatic axis" [2], ΔH^* can be approximated by equation (3), where Δh is the difference between the CIE hue angles in degrees.

$$\Delta H^* = C^* \Delta h(\pi/180°) \qquad (3)$$

Three difficulties immediately arise concerning (3). First, what does "small" mean quantitatively so that (3) is still valid? Similarly, how far away is "away from"? Secondly, if the two colors have different C^*s, which one is used in (3)? As will be shown, the C^* given in (3) implies that both C^*s are equal. A third difficulty in (3) is that ΔH^* is not psychometrically independent of C^*, but instead made dependent on C^*. This is due to a fundamental error in (1) and not merely to (3) being an approximation of (1). Therefore, as will be shown, the non-validity of (3) as an approximation lies not principally in the magnitude of Δh or the C^*s involved, but rather in the definition of ΔH^* in (1). The implied premise in (1), that the subtraction of $(\Delta L^*)^2$ and $(\Delta C^*)^2$ from $(\Delta E^*)^2$ leaves a quantity comprising *only* hue difference, is false. While ΔH^* does contain the contribution of hue difference, it cannot be considered the contribution of *only* hue difference to ΔE^*. It will contain also a contribution due to the chromas.

Substituting the terms on the right side of (2) into (1) for $(\Delta E^*)^2$, gives (4). According to (4), ΔH^* is independent of ΔL^*.

$$(\Delta H^*)^2 = (\Delta a^*)^2 + (\Delta b^*)^2 - (\Delta C^*)^2 \qquad (4)$$

It will be helpful to represent a general color difference case in Figure 1 and to distinguish between two more color difference metrics. In Figure 1, let color 1 have coordinates a_1^*, b_1^* and chroma C_1^*, and color 2 have coordinates a_2^*, b_2^* and chroma C_2^*. The angle θ between the two chroma lines is the difference in their hue angles (*i.e.*, $h_2 - h_1$). It may be assumed that L^* is the same for both colors as in a Value plane of Munsell colors, although it isn't necessary.

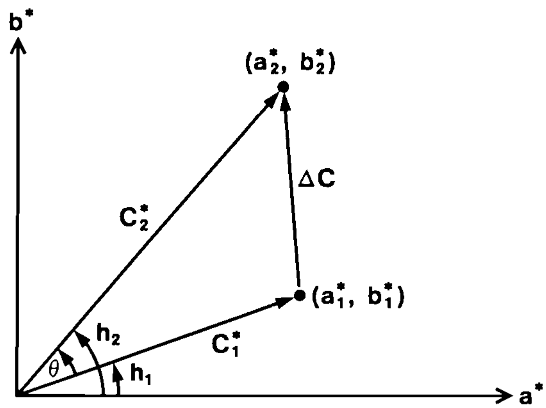

Figure 1. Hue, chroma, and chromaticity difference between two colors in a CIELAB a*b* diagram.[3]

It is important to distinguish between two color difference metrics, ΔC^* and ΔC, which are not the same. Psychometric chroma C^* is defined[2] by (5). It follows from (5) that *chroma* difference ΔC^* is defined by (6). The author follows the notation of MacAdam[4] and defines *chromaticity* difference ΔC via (7). These reiterations of well-known definitions might seem pointless to many readers of this journal, but it has been the author's experience that the right side of (7) is sometimes erroneously equated with ΔC^* as a result of simply placing Δ in front of all the variables in (5). Substituting (7) into (4) gives (8).

$$C^* = [a^{*2} + b^{*2}]^{1/2} \tag{5}$$

$$\Delta C^* = C_2^* - C_1^* \tag{6}$$

$$\Delta C = [(\Delta a^*)^2 + (\Delta b^*)^2]^{1/2} \tag{7}$$

$$\text{where } \Delta a^* = a_2^* - a_1^*, \text{ and } \Delta b^* = b_2^* - b_1^* .$$

$$(\Delta H^*)^2 = (\Delta C)^2 - (\Delta C^*)^2 \tag{8}$$

From Figure 1, ΔC is simply the distance between the points (a_1^*, b_1^*) and (a_2^*, b_2^*), and hence, from plane geometry, the relation in (7). Since the a^*b^* plane represents chroma and hue (although a^* and b^* *per se* do not), the difference (ΔC) between two points in the plane not having the same hue angle will be due to both hue difference and chroma, even if the chromas are the same. A tempting geometric interpretation of (8), from Figure 1, is that ΔC seems a hypotenuse defined by ΔH^* and ΔC^*, where ΔH^* is the length from the smaller C^* to the larger C^*, intersecting the larger C^* at a point equal to the smaller C^*. ΔC^* is then the difference as an extension along the larger C^*, and ΔC becomes the length joining ΔH^* and ΔC^*. ΔC is not a Pythagorean hypotenuse because ΔH^* and ΔC^* (as just geometrically defined) cannot be orthogonal if $\theta > 0$. However, when $\Delta C^* = 0$, $\Delta H^* = \Delta C$, the chromaticity difference, or geometrically, from Fig. 8, the base of an isosceles triangle opposite angle θ. For increasing C^*s at constant θ and $\Delta C^* = 0$, ΔH^* increases.

In order to separate chroma contribution from hue difference contribution, it is necessary to define ΔC another way, in terms of C_1^*, C_2^*, and hue difference only. Since the position of a color in an a^*b^* plane can be represented as a vector of magnitude C^* and direction angle h, ΔC can be thought of as the vector difference in terms of the colors' polar coordinates; whereas, (6) is the scalar difference in only chroma. Thus, from basic vector principles (the Law of Cosines), ΔC can also be defined by (9), where $\theta = h_2 - h_1$.

$$(\Delta C)^2 = C_1^{*2} + C_2^{*2} - 2C_1^* C_2^* \cos\theta \tag{9}$$

Substituting (9) into the right side of (8) for $(\Delta C)^2$ yields (10).

$$(\Delta H^*)^2 = 2C_1^* C_2^* (1 - \cos\theta) \tag{10}$$

Equation (10) is analytically accurate for any chromas and hue angles and thus shows the inherent inclusion of chroma in ΔH^* as a consequence of the definition in (1). An example of the consequence of (10) is that two pairs of colors from lines of constant hue and thus having essentially the same difference in hue but different chromas would have a considerable difference between their ΔH^*, but it would be incorrect to conclude that the pair with the much larger ΔH^* has a much larger difference in perceived hue. The observed, larger *total* color difference for the higher chroma pair would be due to *both* their hue difference *and* magnitudes of their chromas.

Hue as used herein refers to association with dominant wavelength. Since it is known that chroma scales of constant perceived hue are not perfect straight lines in any psychometric color space, it is not implied herein that equal differences in psychometric hue angles always constitute the same perceived hue difference or the same difference in dominant wavelengths. Such nonlinearity further complicates correlation of ΔH^* to perceived hue-only difference, especially among colors having the same perceived hue but greatly different chromas. However, for chromatic colors, if ΔH^* is zero, then their hue angles will be the same. The principal point herein is that a meaningful interpretation of ΔH^* as a correlate of hue-only difference requires consideration of its inherent dependence on chroma.

An approximation for ΔH^* according to (10) for small hue angle differences can be obtained by substituting the Taylor series expansion of $\cos\theta$, truncated to two terms, which results in (11), where the hue angle difference Δh in degrees has been substituted for θ.

$$(\Delta H^*)^2 = C_1^* C_2^* (\Delta h\, \pi / 180°)^2 \tag{11}$$

Equation (11) agrees with (3) if $C_1^* = C_2^*$. The C* in (3) may be considered the geometric mean of C_1^* and C_2^*. In any case, however, even from both (10) and (11), ΔH* is still dependent on the chroma of the colors, and thus, the subtraction of $(\Delta L^*)^2$ and $(\Delta C^*)^2$ from $(\Delta E^*)^2$ as suggested in (1) does not leave a quantity dependent on *only* hue. From a mathematical point of view, the error arises in trying to treat psychometric variables represented by the *polar* coordinates C* and h of a *cylindrical* coordinate system as *rectangular* coordinates also mutually orthogonal with L*. Equation (1) implies a Cartesian coordinate system; otherwise, there is no *justification* for using exponents = 2, when a linear relation would be even more "desirable".

To eliminate a dependence on chroma, ΔH* can be redefined according to (12). Equation (12) is merely (8) divided by $C_1^* C_2^*$, and (13) is an approximation of (12) from dividing (11) by $C_1^* C_2^*$. However, (12) does *not* represent the remainder from the right side of (1). Equation (13) indicates that a mathematically proper psychometric hue difference quantity independent of chroma is nothing more than the difference in CIE hue angles expressed in radians.

$$(\Delta H_{12}^*)^2 = \frac{(\Delta C)^2 - (\Delta C^*)^2}{C_1^* C_2^*} \tag{12}$$

$$\Delta H_{13}^* = \Delta h \pi / 180° \tag{13}$$

To show the possible errors in the use of ΔH* as defined by (1), four Munsell color samples of Munsell Value = 6 were spectrophotometrically measured $0°/d$ from 380 nm to 700 nm in 10 nm intervals, specular component excluded, and their 1976 CIELAB parameters calculated assuming CIE Illuminant C and the 2° CIE Standard Observer. The colors were two pairs, each pair's samples having the same Munsell Hue but differing in Munsell Chroma (C_M) by two. The color samples are qualitatively represented (not to scale) in Figure 2. Their CIELAB metrics are given in Table I. In Table II are the various metrics related to determining psychometric hue difference as discussed already between the four possible pair combinations having nearly identical hue angle difference.

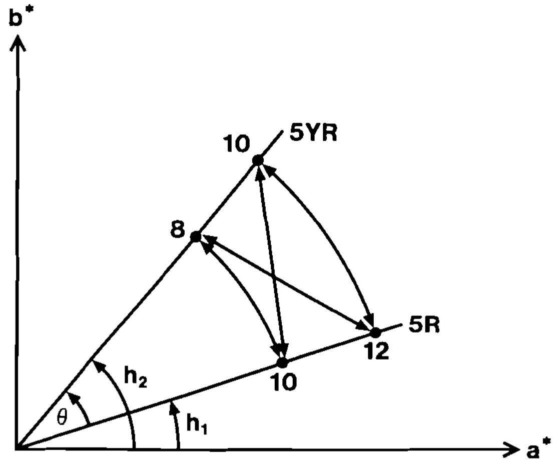

Figure 2. Qualitative representation of four Munsell colors in a CIELAB a*b* diagram.[3]

Table I[3]

CIELAB colorimetrics for the Munsell samples represented in Figure 2

Munsell	L^*	a^*	b^*	C_{ab}^*	h_{ab}
5R 6/10	61.51	38.87	18.52	43.06	25.48°
5R 6/12	60.72	47.01	22.58	52.15	25.66°
5YR 6/8	60.46	23.34	37.91	44.52	58.38°
5YR 6/10	60.63	28.45	48.53	56.25	59.62°

Table II[3]

Colorimetric differences from Table I

Difference Pairs	$(\Delta C)^2$	$(\Delta C^*)^2$	ΔH_8^*	ΔH_{12}^*	ΔH_{13}^*	$\cos\theta$	$\cos(\Delta h)$
5YR 6/8 –5R 6/10	617.15	2.13	24.8	0.5664	0.5742	0.8395	0.8396
5YR 6/10–5R 6/12	1,017.88	16.81	31.6	0.5842	0.5927	0.8295	0.8294
5YR 6/10–5R 6/10	1,009.18	173.98	28.9	0.5872	0.5959	0.8276	0.8276
5YR 6/8 –5R 6/12	795.28	58.22	27.2	0.5634	0.5711	0.8413	0.8413

ΔH_8^* is from (8); ΔH_{12}^* is from (12); ΔH_{13}^* is from (13)

In Table II, $\cos\theta$ is calculated from (10), and $\cos(\Delta h)$ from the h's in Table I. The agreement between $\cos\theta$ and $\cos(\Delta h)$ proves the validity of the derivation and the explicit dependence of ΔH_8^* on the chromas of the samples. More importantly, the ΔH_8^* values for the four pairs are significantly different, even though all pairs represent perceived equal hue differences and have essentially equal differences in CIELAB hue angle. It is obvious that ΔH_8^* [derived from (1)] is an improper metric for only hue difference, while ΔH^* from (12) or (13) suggests much less difference between the pairs due to *only* hue, since the latter has no dependence on the C^* of either color in the difference pair. One possibly undesirable aspect of defining ΔH^* according to (12) regards scaling. The values of

ΔH^*_{12} in Table II are small even for such large differences in h's. If larger numbers for ΔH^*_{12} are desirable, the results can be scaled higher by, for example, using % (*i.e.*, x 100), or simply by using Δh in degrees, which is usually done already, if the only use would be for comparison and not computation.

From the foregoing, it be be seen that ΔH^* will correlate best with a difference in perceived hue when C^* of all the colors is the same (assuming for now that L^* is also constant). When the C^*s are different, there can be sizable differences in ΔH^* among pairs of colors for which there is no perceived difference in hue. It can, therefore, be risky to infer that differences in ΔH^* correlate with differences in perceived hue difference.

ΔC^* and ΔL^*

While the foregoing ignored the effect of ΔL^* being not zero, a nonzero ΔL^* might affect the validity of attributing the magnitude of a ΔC^* to a difference in only chroma. The reason comes from the explicit dependence of both C^*_{uv} and C^*_{ab} on L^*, which is sometimes overlooked since colorimetric parameters and differences are usually automatically computed and tabulated by computer controlled instruments. C^*_{uv} equals $13L^*$ times the distance from the reference white to the color in the 1976 u'v' chromaticity diagram. C^*_{ab} also depends on L^*, but less simply, because of using $(Y/Y_n)^{1/3}$ for both lightness and a*b* coordinates.

The effect of a nonzero ΔL^* on ΔC^* when the perceived difference in chroma is small can be evaluated using Munsell colors having a small difference in Munsell Chroma (*e.g.*, $\Delta C_M = 2$) and a difference in L^* (by a change in Munsell Value). Colorimetric values of Munsell samples of the colors 5R 5/10, 5R 5/12, 5YR 7/8, and 5YR 7/10 were determined as for the other Munsell colors and are given in Table III in both CIELAB and CIELUV. From Table IV, the effect on the magnitude of a ΔC^* due to a large ΔL^* is shown in Figures 3 and 4, where the difference in Munsell Chroma (ΔC_M) is kept the same (*e.g.*, 2). For the difference pairs (5R 6/10 – 5R

5/12}, {5R 6/12 – 5R 5/10}, {5YR 6/8 – 5YR 7/10}, and {5YR 6/10 – 5YR 7/8}, the magnitude of ΔC^*_{ab} and ΔC^*_{uv} is substantially different for both 5R and 5YR hues, where there is also a significant ΔL^*. However, where ΔC_M is also 2, but ΔL^* is nearly zero as for the two pairs {5R 6/12 – 5R 6/10} and {5R 5/12 – 5R 5/10}, and the two pairs {5YR 6/10 – 5YR 6/8} and {5YR 7/10 – 5YR 7/8}, ΔC^*_{ab} and ΔC^*_{uv} are nearly the same for each pair of hues, as might be desired for a given hue. Thus, the magnitudes of ΔC^* values are not necessarily attributable to a difference in *only* chroma if there is also a significant difference in L^*. Since the CIE ΔH^* has been shown earlier to have a dependence on C^*, and C^*, in turn is dependent on L^*, it follows that ΔH^* can also be affected by significant ΔL^*. Therefore, to help optimize color difference analysis and correction, it would be good to make a color in question have the same L^* as the reference color before attempting corrections in chroma and hue.

It is not intended to correlate ΔC_M with ΔC^*, or C_M with C^*, since there is generally not a uniform correlation. A good example is in Table I, where the C_{ab}^* for 5YR 6/8 is larger than the C_{ab}^* for 5R 6/10, even though the latter color has a larger Munsell Chroma. A small or constant ΔC_M is used only as a control of the difference in the perceived chroma variable of the color order system used in the comparisons.

The principal point herein is to show that the interdependence of L^* and C^* can lead to a metric ΔC^* whose magnitude can be determined more by an achromatic difference in L^* than a true difference in chromaticity due to only a chroma difference. A relevant example in printing is that the addition of an ideal achromatic "black" to a color will cause a decrease in both C^* and L^*, although saturation (s^*) will remain constant in the CIELUV system. One might otherwise wonder why chroma should decrease due to a purely achromatic change.

Table III

CIELAB and CIELUV colorimetrics for Munsell samples

Munsell	L^*	C_{ab}^*	h_{ab}	C^*_{uv}	h_{uv}
5R 6/10	61.51	43.06	25.48°	75.10	13.44°
5R 6/12	60.72	52.15	25.66°	92.59	12.61°
5YR 6/8	60.46	44.52	58.38°	71.41	35.75°
5YR 6/10	60.63	56.25	59.62°	87.23	34.63°
5R 5/10	51.68	46.20	27.07°	79.86	13.58°
5R 5/12	51.48	55.68	26.07°	98.21	11.88°
5YR 7/8	71.50	45.68	64.48°	74.62	41.52°
5YR 7/10	71.62	57.12	64.23°	90.72	39.63°

Table IV

Colorimetric Differences from Table III

Munsell Difference	ΔC_M	ΔE^*_{ab}	ΔC^*_{ab}	ΔL^*	ΔE^*_{uv}	ΔC^*_{uv}
5R 6/10 – 5R 5/10	0	10.49	-3.14	9.83	10.93	-4.76
5R 6/10 – 5R 5/12	2	16.13	-12.62	10.03	25.30	-23.11
5R 6/12 – 5R 5/10	2	10.97	5.95	9.04	15.67	12.73
5R 6/12 – 5R 5/12	0	9.90	-3.53	9.24	10.88	-5.62
5R 6/12 – 5R 6/10	2	9.13	9.09	-0.79	17.55	17.49
5R 5/12 – 5R 5/10	2	9.52	9.48	-0.20	18.53	18.35
5YR 6/8 – 5YR 7/8	0	12.09	-1.16	-11.04	13.64	-3.21
5YR 6/8 – 5YR 7/10	2	17.60	-12.60	-11.16	22.95	-19.31
5YR 6/10 – 5YR 7/8	2	15.76	10.57	-10.87	19.27	12.61
5YR 6/10 – 5YR 7/10	0	11.93	-0.87	-10.99	13.91	-3.49
5YR 6/10 – 5YR 6/8	2	11.79	11.73	0.14	15.88	15.82
5YR 7/10 – 5YR 7/8	2	11.45	11.44	0.12	16.33	16.10

ΔC_M= difference in Munsell Chroma

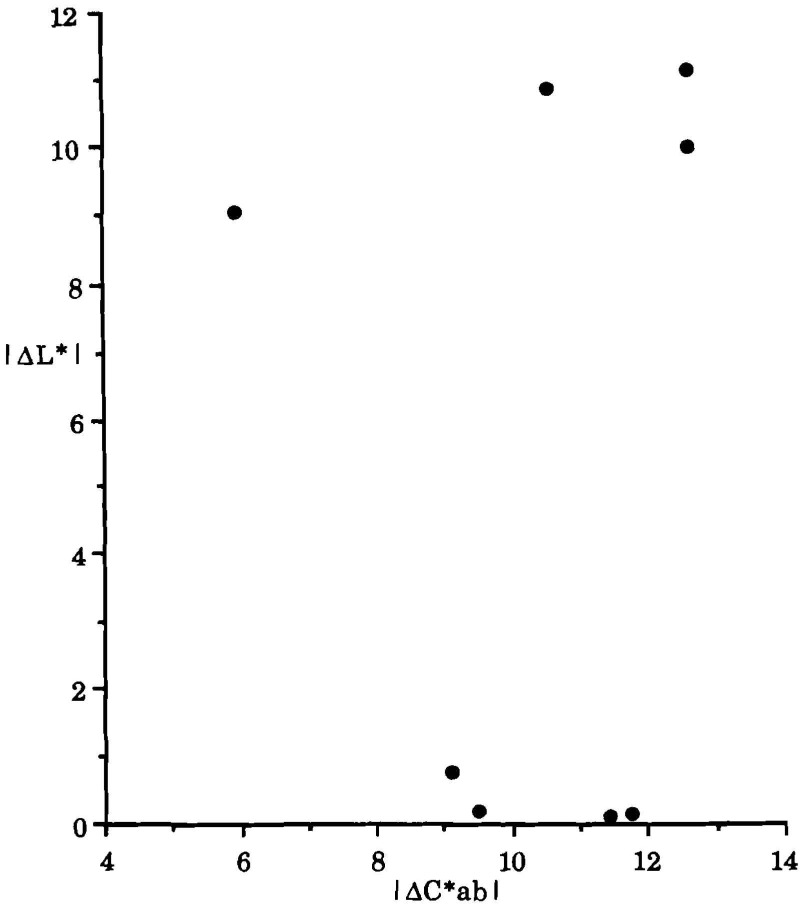

Figure 3. $|\Delta L^*|$ vs. $|C^*_{ab}|$ for the Munsell difference pairs in Table IV where $\Delta C_M = 2$.

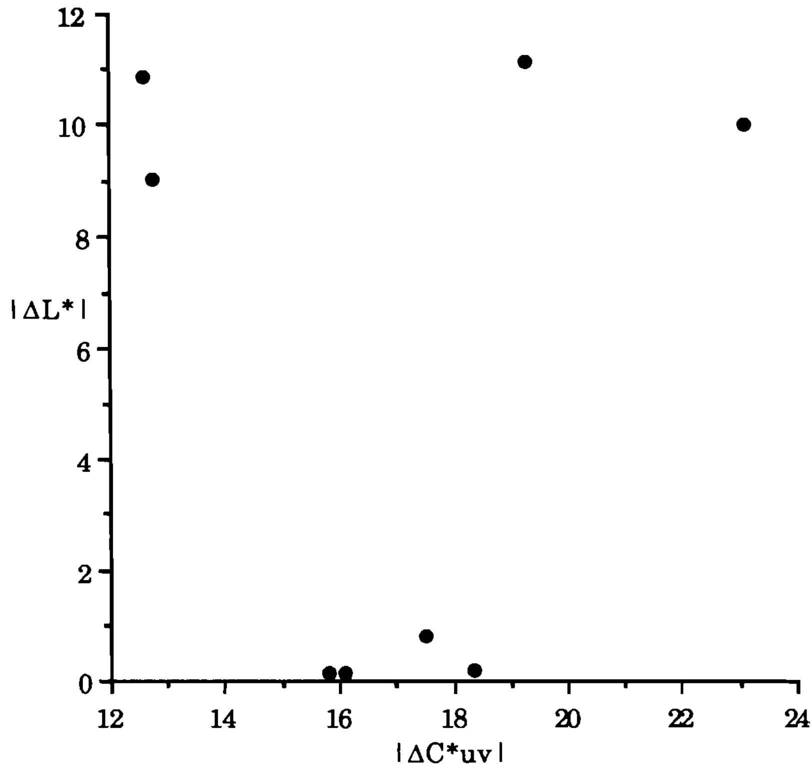

Figure 4. $|\Delta L^*|$ vs. $|C^*_{uv}|$ for the Munsell difference pairs in Table IV where $\Delta C_M = 2$.

Table IV also suggests a distributive property for ΔL^* and ΔC^*, as for other spatial coordinates in general. For example, ΔL^* and ΔC^* for the difference pair (5R 5/12 – 5R 5/10) can be determined by subtracting the corresponding ΔL^* and ΔC^* values of the difference pairs (5R 6/12 – 5R 5/12) and (5R 6/12 – 5R 5/10) since the Munsell color 5R 6/12 is common to both. In general, ΔL^* and ΔC^* for the unknown difference pair (B–C) will be the difference in ΔL^* and ΔC^* for the difference pairs (A–B) and (A–C) as if corresponding to the difference (A–C) – (A–B). This property can be useful where only ΔL^* and ΔC^* values are given for many but not all possible difference pairs, and some absent difference pair values related to specific combinations of colorants and/or papers are desired. However, considering the interdependences just discussed, the results of such combinations could be better or worse than the input values as to representing true differences.

Summary

The present CIE hue difference metric ΔH^* includes not only hue difference, but also chroma, and therefore it can be erroneous to infer that the magnitude of ΔH^* is due to *only* a hue difference. Thus, differences in ΔH^*s will seldom correlate well with perceived differences due to *only* hue difference when the metric variables L^* and C^* are not the same for all the colors. Another definition of ΔH^* which eliminates any psychometric dependence of hue difference on chroma was given (Eq. 12), which further suggests that the difference in CIELAB or CIELUV hue angles expressed in radians is a better quantitative perceptual correlate of *only* hue difference. Analogously, ΔC^* might not accurately represent a difference due to *only* chroma when ΔL^* is significant. A significant ΔL^* will complicate segregating an overall color difference into true hue-only and chroma-only differences.

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