OPTICAL DENSITY AND INK FILM THICKNESS; A COMPARISON OF MODELS.

B.E. Blom[1] and T. J. Conner[2]

ABSTRACT

Many models purporting to relate the optical density of an ink film on paper to the amount of amount of applied ink have been advanced. Several of the more popular models are examined for their applicability to black ink on uncoated paper. The effects of printing pressure and speed, ink viscosity, and paper calendering are examined for their impact on the various models.

INTRODUCTION

One of the main methods of characterizing ink/paper interactions is by measurement of the optical density or reflectance of the printed ink film at specific or varying amounts of ink coverage. Terms such as "ink receptivity" and "ink mileage" are commonly used and refer to the amount of ink, usually expressed in grams of ink per square meter of print (or corresponding values of pounds per ream, ton, etc.), necessary to achieve a specified optical density or reflectance.

A more general approach has been taken by a number of authors (1-5) wherein they attempt to mathematically describe the change in optical density as a function of ink coverage. Such curves typically have the appearance shown in Figure 1. The benefit of being able to describe such curves mathematically is that the curve-fitting coefficients can potentially be related to basic ink and/or paper properties and, in some cases, the printing conditions. The data must fit the empirical model well, however, if the regression coefficients are to be useful in characterizing the interactions.

One of the more frequently encountered empirical expressions is attributed to Tollenaar and Ernst (1):

$$D = D_\infty(1-e^{-mw}) \qquad (1)$$

where: D = optical density
D_∞ = optical density of an infinitely thick ink film
w = ink coverage in grams/m^2
m = a constant

[1]James River Corporation, Neenah, WI

[2]Institute of Paper Science and Technology, Atlanta, GA

Normally D_∞ would be another constant or regression coefficient. It can also be determined independently but there may not necessarily be good agreement between the values determined by the two approaches. The authors found that "m" generally increased with printing pressure but was not a function of printing speed. D_∞ generally increased with pressure and decreased with speed, particularly at lower printing pressures.

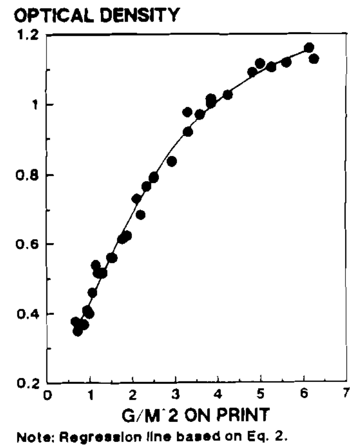

Figure 1. Typical Curve Showing Optical Density as a Function of Ink Coverage.

Another empirical expression, usually overlooked, is given by Kornerup, *et al* (2):

$$\frac{R_o - R_\infty}{R - R_\infty} = 1 + (k'w)^{n''} \quad (2)$$

where: R_o = reflectance of the paper
R_∞ = reflectance of an infinitely thick ink film
R = reflectance of the print
k', n'' = constants
w = ink coverage in grams/m^2

These authors found it necessary to restrict their data range to w= 1-4 g/m^2 to improve the fit of the data. Noteworthy is their inclusion of the residuals (the difference between predicted and experimental values). This has generally not been done by other authors. They determined R_∞ experimentally (by measurment of very thick films) as opposed to treating it as another variable (regression coefficient). In this work the authors explored the effect of pigmentation type and concentration. They found that "n" " was dependent on the colorant and that "k' " was a function of both the colorant and its concentration. These authors also present an excellent review of prior expressions.

More recently Calabro and Savagnone (3) have presented another empirical expression:

$$(1/D) = (n'/w)^h + b' \quad (3)$$

where: n', h = constants
b' = constant = $1/D_\infty$

and other variables have the same meaning as before. D_∞ could be calculated or independently determined from thick film measurements. Calabro found the expression useful over a wide range of inking/optical density levels but did not analyze the data in terms of the residuals. In a previous paper Calabro and Mercatucci (4) had reported that for newsprint, the variable "h" was equal to 1 (one). Based on regression analysis Calabro and Savagnone (3) report that "n' " is very dependent on paper, "h" is weakly dependent on certain ink properties, and that "b' " is not well correlated with either ink or paper. Their work involved various ink/paper combinations but not variations in printing conditions.

Another empirical expression, found (5) useful for newsprint, is:

$$1-(R/R_o) = a_o + a_1(1/w) \quad (4)$$

where: a_o, a_1 = constants

and the other variables have their usual meaning. This expression turns out to be a special case of Equation 2. The term "1" on the right hand side of Equation 2 was included by Kornerup (2) to insure that the reflectance approached a limiting value with increasing ink film thickness. Omission of this term in their derivation and setting of the exponent "n'"equal to 1 (one) allows the expression to be transformed into Equation 4. Blom (5), using black ink on newsprint, found Equation 4 useful for ink coverages greater than 1 g/m^2 or relative reflectances (R/R_o) less than 0.35. Based on an examination of the residuals this particular expression fit the data better than either Equation 1 or Equation 3 (with h=1).

EXPERIMENTAL

Test prints were made on the Prüfbau Printability Tester utilizing the metal printing form. Speed was varied from 2-6 meters/sec and printing pressure from 100-300 Newtons/cm. Ink temperature was maintained at 25°C and the room in which the experiments were conducted was maintained at 23°C and 50% relative humidity. Prints were prepared by initially inking the ink distribution system with an excess of ink and then making 10 successive prints. The printing form was weighed between each print to determine the amount of ink transferred to the paper. For each set of printing conditions three separate trials (in the case of some of the calendered paper only two trials were done), carried out. Each trial, 76 in total, consisted of the 10 successive prints. For each set of conditions the 20 or 30 points were combined into one data set. The order in which the 76 trials were conducted was randomly determined beforehand.

Two inks were used to prepare prints, one high viscosity and the other low viscosity. The composition and properties of the inks are shown in Table 1. Materials were supplied by the General Printing Ink division of Sun Chemical. The ink oil was a conventional 750 Saybolt Universal Second oil. Carbon was a conventional newsink furnace black. Resin was a 140°C melt point hydrocarbon resin. Viscosity was determined on a Rheometrics Mechanical Spectrometer (RMS-800) using the coni-cylinder bob and cup geometry at 25°C. Both inks exhibited some shear-thinning behavior.

Table 1. Ink Composition

	Low Viscosity	High Viscosity
Carbon	10.5%	10.5%
Ink Oil	87.5%	73.7%
Resin	-	13.8%
Dispersant	2.0%	2.0%
Visc. =	1.0Pa-S @1000sec^{-1}	2.3Pa-S @1000sec^{-1}

The paper used in this study was 50# Richmond Opaque White Vellum, a clay-filled sheet consisting of approximately 70% hardwood fibers and 30% softwoods. The paper was used as received or calendered on a laboratory steel calender (3 passes @ 1200 pounds gauge pressure). Paper properties are shown in Table 2.

Table 2. Paper Properties

	Uncalendered		Calendered
Roughness* (microns)	7.3	Side A	3.4
	6.5	Side B	3.4
Profilometer** RMS roughness	3.9		1.9
Gurley Air Resistance (sec/100 cm^3 of air)	11		29
Basis Weight (lbs/ream)	46.9		46.9
Thickness, mils	4.4		3.1

* Parker Print-Surf, S-10
**Tencor Alphastep 200 (5μ probe)

Note that there is a distinct two-sidedness to the uncalendered paper. Rather than attempt to sort the paper by side we chose to mix the paper to assure that both sides were randomly printed. This lead to a small but acceptable increase in the experimental "noise". Note also that the roughness as determined by the Parker Print-Surf and the profilometer show the same relative trend with calendering.

After printing the prints were allowed to sit for 24 hours prior to the determination of their reflectance on a Macbeth Color-Eye Spectrophotometer. The reduced (i.e. measured relative to the unprinted paper) Y-tristimulus value was used as the reflectance value and transformed, when necessary, to optical density using:

$$D = -\log(R) \qquad (5)$$

RESULTS

The data was analyzed using the appropriate SAS linear and non-linear programs. All fives cases were examined:

1. $D = D_\infty(1-e^{-mw})$

2. $H = \frac{R_o - R_\infty}{R - R_\infty} = \frac{1-R'_\infty}{R'-R'_\infty} = 1 + (k'w)^{n''}$

3a. $(1/D) = (n/w) + b$

3b. $(1/D) = (n'/w)^h + b'$

4. $1-(R/R_o) = 1-R' = a_o + a_1(1/w)$

In all cases D_∞ and R_∞ were treated as regression variables. All residuals were calculated in terms of $(D-D_{CALC})$ by first making the appropriate conversions, e.g. Equation 5.

Figures 2-6 show a comparison of the residuals for all five models on both calendered and uncalendered paper. Each graph depicts the residuals for all variations of pressure, speed, and viscosity. The reason for separating the two types of paper is that it has been observed (3) that an expression may be suitable for uncoated paper but not coated grades. While coated grades were not examined in this study the effect of the calendering could have resulted in the same phenomenon. In order that a mathematical expression be considered as giving a "good fit" to the experimental data, it is not enough that the

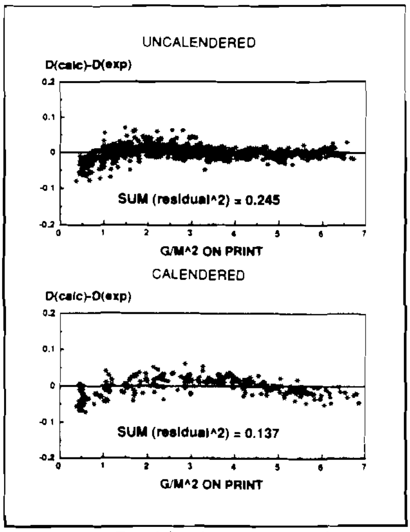

Figure 2. Residuals for Equation 1.

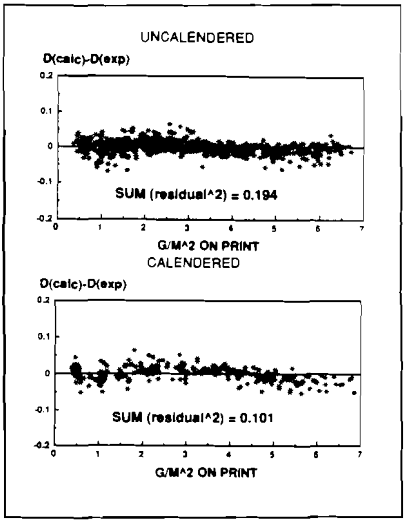

Figure 3. Residuals for Equation 2.

correlation coefficient be high; the residuals also need to be evenly distributed around the 'zero" point, over the range of observations. For Equation 1 (Figure 2) and Equation 4 (Figure 6) it can be seen that this is not the case.

Figure 4 (Equation 3a) suggests that this expression may not be valid for very rough (uncalendered) surfaces but could be useful for smoother surfaces. For Equation 3b (Figure 5) the same arguments might be made. Calabro and Savagnone (3) reported that Equation 3b was better for coated papers while Equation 3a was better for newsprint. Our results show that Equation 3b offers an improvement over Equation 3a in reducing the sum square of the residuals but does not improve their distribution about the "0" axis.

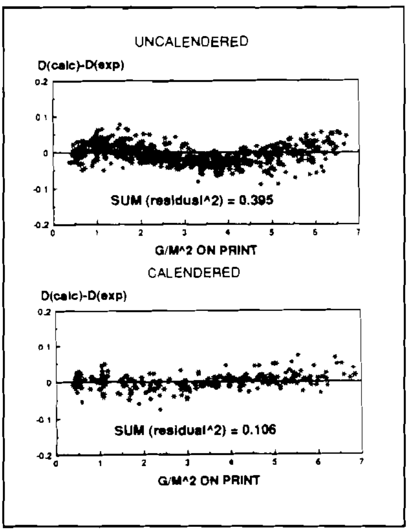

Figure 4. Residuals for Equation 3a.

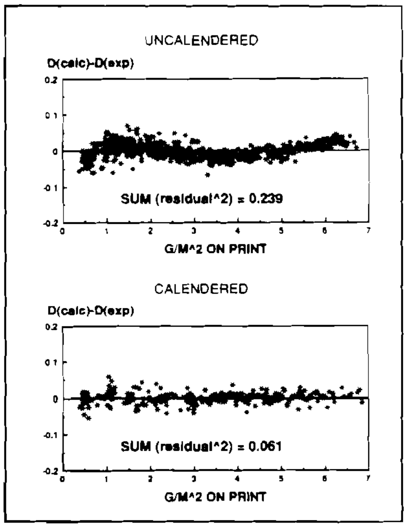

Figure 5. Residuals for Equation 3b.

Equation 2 (Figure 3) fits the uncalendered data quite well over the entire range of ink weights. With the calendered paper the residuals are larger and not as well distributed as in the previous model. In their original work Kornerup, *et al* (2) found the expression to be unsatisfactory for low ink coverages. This may be a result of their having looked only at organic colorants and not black inks or the fact that they determined R_∞ experimentally rather than treating it as another regression constant. A third possibility may be due to the fact that the left hand side of Equation 2 does not approach "0" as the ink weight approaches "0". Examination of this region of the regression line shown in Figure 1 shows a slight curvature at the lowest values.

Effect of Printing Variables on Regression Coefficients

Since Equations 2, 3a, and 3b appear to be potentially useful, their respective regression coefficients were examined to see how they varied with printing pressure and speed. Ink viscosity, over the limited range studied, was not found to be significant in its effect on any of the regression coefficients. Figures 7-16 depict some of the regression coefficients along with corresponding confidence intervals. For clarity it was not possible to show confidence intervals for all points but those shown are illustrative of the data.

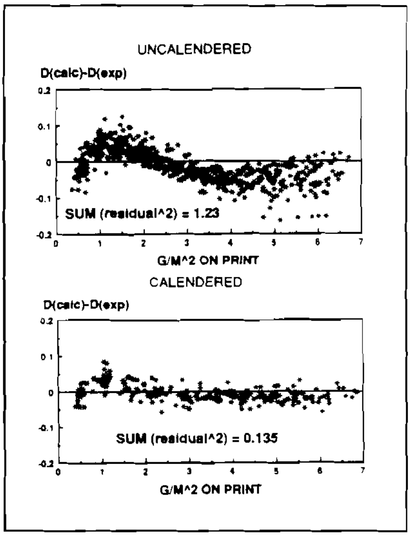

Figure 6. Residuals for Equation 4.

Equation 2

Figure 7 shows that, because of the magnitude of the error in its determination, R_∞ (labeled as R(i) in the figure), is apparently independent of pressure. Similar results were obtained at the other printing speeds. In fact, if the average value for R_∞ is used as a constant in fitting the experimental data there is little change in the distribution of the residuals although the other two regression coefficients will change to some degree.

R_∞ should represent the maximum obtainable density (minimum reflectance) that can be achieved with a particular ink. Thus, it has been common (2-4) to attempt to measure this value using very heavy ink coverages on paper or non-absorbent surfaces. In this work we have treated R_∞ (as Tollenaar and Ernst(1) did for D_∞ in their use of Equation 1) as a regression variable, since there is no reason *a priori* to think that there is a correlation between 'infinitely' thick films of ink and the higher ink coverages experimentally examined. In fact, it can be argued that such thick films, especially if not dried, would have optical properties, particularly specular gloss, that would lead to a lower than expected value of R_∞. This effect has been experimentally observed (5,7).

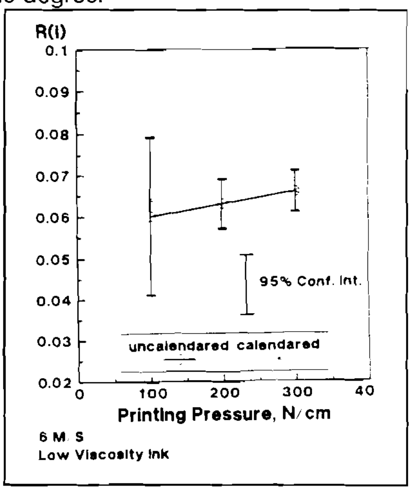

Figure 7. Variation of R_∞ (Eq. 2) with Pressure, Under Specific Conditions.

The data in Figure 7 show that low printing pressures and rough paper lead to large errors in the estimation of R_∞. Under these conditions this was true for the other regression coefficients as well. This is probably attributable to the high roughness and subsequent poor (non-uniform) ink transfer to the uncalendered sheet. Figure 8 shows a comparison of the ink transfer curves for the two papers. As is readily seen the percent transfer of ink to the uncalendered paper is well below that of the calendered paper.

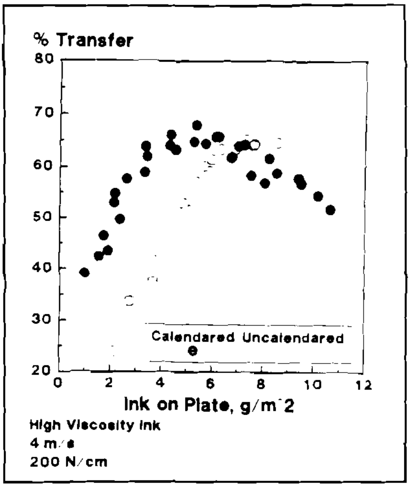

Figure 8. Effect of Calendering and Printing Pressure on Percent Transfer.

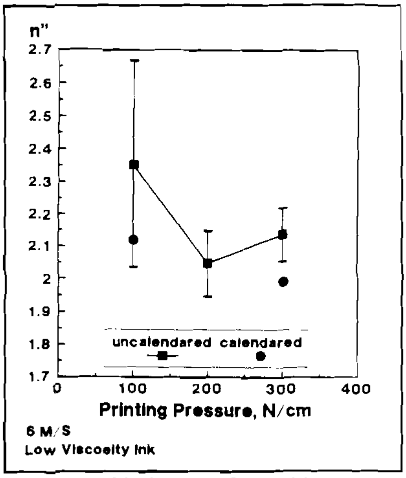

Figure 9. Variation of n" with Pressure.

Figure 9 shows typical behavior for n", the exponent in Equation 2. Here, however, the confidence interval for the calendered paper is about the same as that found with the uncalendered paper, i.e., there is again a pronounced increase in variability at lower pressures. None of the variables, except perhaps for viscosity at the highest printing pressure on the calendered paper, had a significant effect on n". Recall that Kornerup (2) found n" to be affected by colorant.

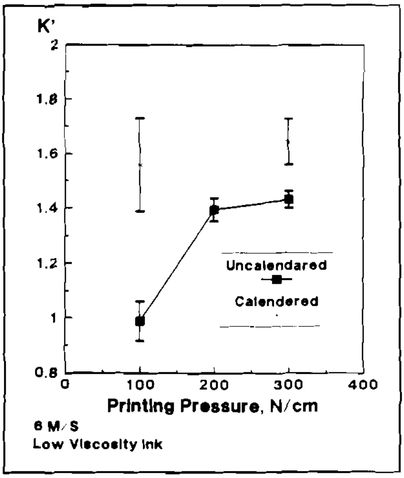

Figure 10. Variation of K' (Eq. 2) with Pressure, under Specific Conditions.

K', on the other hand, increases with pressure as shown in Figure 10. Although appearing to be reaching a plateau at 200 N/cm the data for the uncalendered paper at some other conditions continued to rise indicating that K' for the uncalendered paper would approach that of the calendered paper. This trend is consistent with the view that, under the impressions forces in the printing nip, paper roughness decreases. Thus, the values obtained on the uncalendered paper at high pressure should be similar to those obtained on the calendered paper at low pressure. Kornerup (2) had found K' dependent on colorant and concentration. In our case concentration may be replaced by the more effective utilization of pigment that is a result of greater ink hold-out. This increased hold-out is a consequence of the reduction in porosity caused by either calendering or increasing printing pressure. These points will be discussed in a later section.

Equation 3b

Figures 11 and 12 show how the regression coefficients n' and b', respectively, vary with speed and pressure for both the calendered and uncalendered paper. No effect of viscosity was seen. Calabro and Savagnone (3) had reported that "n' " was very dependent on paper and "b' " was not correlated with any variable studied. We find a strong dependence of "b' " on paper (Figure 12) but no dependence on "n' " on paper, except at the lowest printing pressure. In the case of n' and the uncalendered paper, while there appears to be a dependence on printing speed, particularly noticeable at low speed, the error estimates are too large to allow one to draw this conclusion.

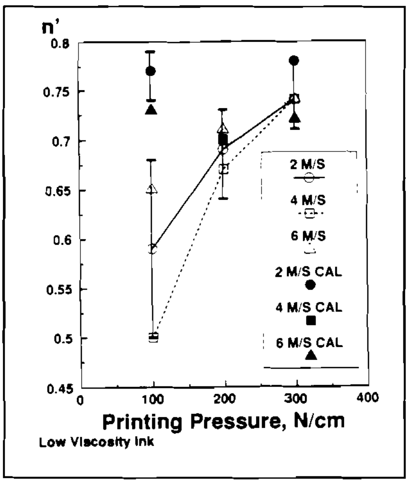

Figure 11. Variation of n' (Eq. 3b) with Pressure.

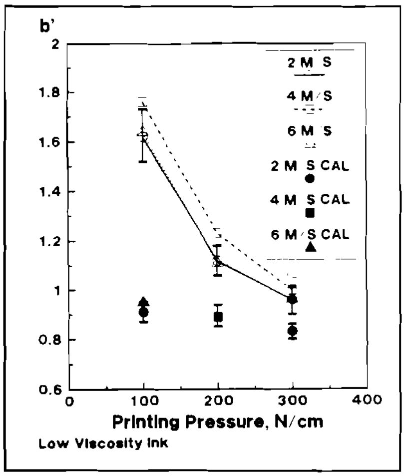

Figure 12. Variation of b' (Eq. 3b) with Pressure.

Figure 13 shows the variation in the exponent, h, of Equation 3b. Only the confidence interval for the data at 2 m/s is shown but other data points had similar sized intervals. Recall that the authors (3) found that "h" was weakly correlated with some ink properties. The lack of correlation of "h" with any properties and the large errors in its estimate suggests that "h" accounts for the majority of the variance in the data. In fact, the coefficients are closely correlated as can be seen in a typical correlation matrix shown in Table 3. As can be seen the coefficients are highly correlated for Equation 3b (lower) but less so for Equation 2.

Equation 3a

Figure 14 and Figure 15 show the effects of the variable on the coefficients "n" and "b", for Equation 3a. Most of the error is taken up by the n coefficient. The usefulness of this expression lies in its ease of use for determining the regression coefficients. The other two expressions require non-linear methods which may not be routinely available to some users.

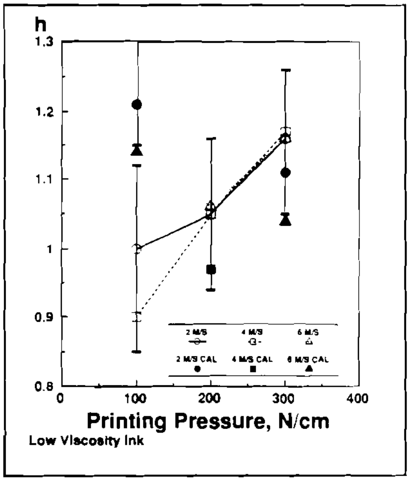

Figure 13. Variation of "h" (Eq. 3b) with Pressure.

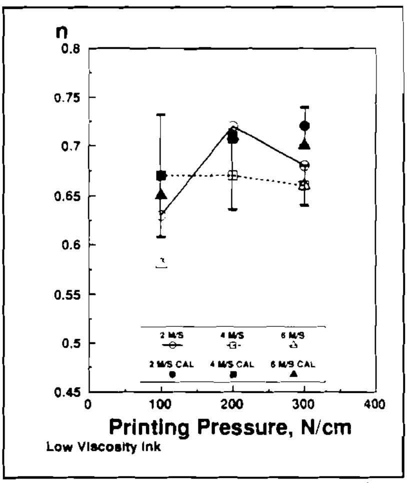

Figure 14. Variation of "n" (eq. 3a) with Pressure and Speed.

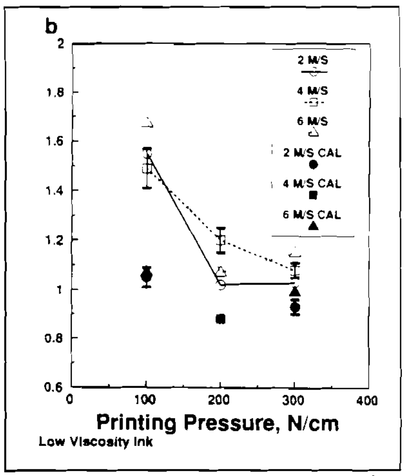

Figure 15. Variation of "b" (Eq. 3a) with Pressure and Speed.

Analysis of variance (6) showed that paper roughness and printing pressure were the most significant factors affecting the coefficients for all models. Smaller effects could sometimes be seen, e.g. only looking at calendered paper under certain operating conditions, exclusion of ink weights less than 1 gram per square meter, etc. It is possible that these effects would become more significant with smoother papers where non-uniform transfer would be less likely to affect the results.

Table 3. Asymptotic Correlation Matrix of the Parameters for Equations 2, and 3b. Data is for calendered paper, low viscosity ink, 4m/s, and 200N/cm.

Parameter	R_∞	K'	n''
R_∞	1.0	.28	.71
K'	.28	1.0	-.27
n''	.71	-.27	1.0

	n'	b'	h
n'	1.0	-.82	.94
b'	-.82	1.0	-.63
h	.94	-.63	1.0

DISCUSSION

The usefulness of these expressions can best be seen if they are used to determine what effect the various experimental conditions have on the amount of ink required to achieve a certain optical density. Figures 16 and 17 show the calculated optical densities, calculated from Equation 2, for the various experimental conditions at ink coverages of 1 and 3 grams per square meter, respectively.

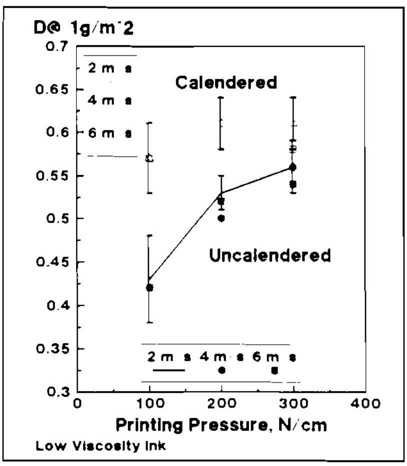

Figure 16. Effect of Printing Conditions on Optical Density at 1 g/m^2 Ink Coverage.

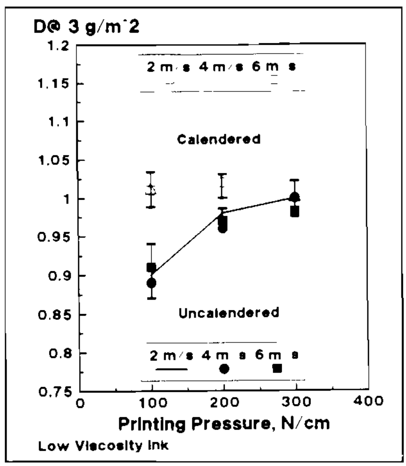

Figure 17. Effect of Printing Conditions on Optical Density at 3 g/m^2 Ink Coverage.

Certain trends can be seen. At both low (Figure 16) and high (Figure 17) ink coverages, the uncalendered paper, at the highest printing pressure, gives optical densities equivalent to the values found for the calendered paper at any printing pressure. This is consistent with the view that, during printing, the paper surface is compressed, resulting in decreased pore size (and better ink hold-out) and concurrently higher ink transfer, due to more ink-paper contact points, comparable to a smoother surface. Figure 18 shows how percent transfer increases with increasing pressure on the uncalendered paper. Figure 19 shows how, at the highest printing pressure on the uncalendered paper, the percent transfer approaches that of the lowest printing pressure on the calendered paper. Thus, roughness measurements, such as those obtained from air leak instruments (e.g. Parker Print-Surf) or profilometers, must be viewed in the context of the end-use printing parameters and the print properties being measured (7).

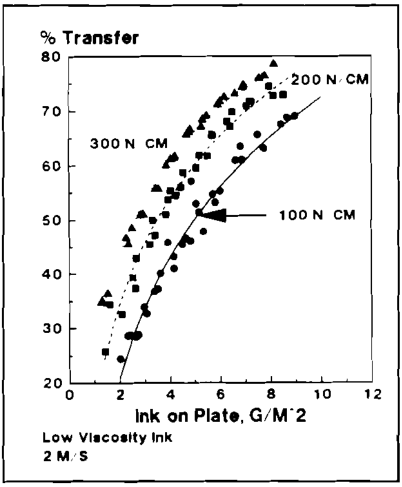

Figure 18. Percent Transfer on Uncalendered Paper.

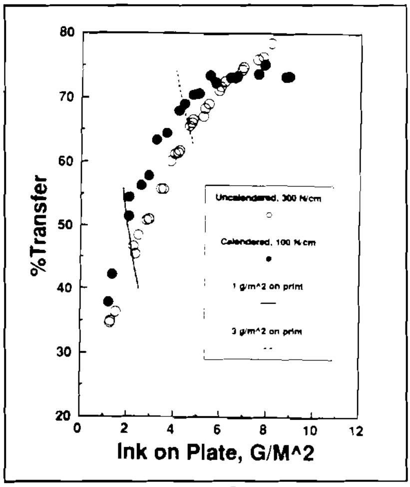

Figure 19. Effect of Calendering and Printing Pressure on Percent Transfer.

Figures 16 and 17 suggest equal conditions exist for the uncalendered paper at high printing pressure and the calendered paper at low printing pressure. Optical densities are comparable, at the same inking level, under these conditions. Regardless, print appearance, notably print smoothness, is not the same. Inspection of the prints shows:

Decreasing Smoothness-->

@ 1 g/m^2 300CAL≥ 300UNCAL> 100CAL> 100UNCAL

@ 3 g/m^2 300CAL> 100CAL≥ 300UNCAL>> 100UNCAL

Focussing on the 100CAL (Calendered, 100 N/cm printing pressure) and the 300UNCAL, we found that, at low ink coverage, calendering has created a mottle pattern due to non-uniform ink absorption. At higher ink coverages (3 g/m^2) the mottle becomes obscured.

Figure 19 shows the percent transfer for these two cases. At low inking levels the percent transfer is higher for the calendered paper because its smoothness increases the contact area with the plate. Calendering has also reduced the porosity as reflected in the maximum in the percent transfer curve. This point, corresponding to the filling of the voids, marks the transition into pure ink film splitting behavior.

De Grace and Dalphond (7) have described the printing on uncoated paper as ink transfer to paper in the compressed state followed by aspiration of some portion of the ink into the pores as they are enlarged following removal of the compression forces. Clearly, accurate ink receptivity expressions can be useful in understanding ink-paper interactions. They do not, however, give a complete picture and should be used cautiously, for the reasons shown.

ACKNOWLEDGEMENT

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