SPECTRAL REFLECTANCE MODIFICATION OF NEUGEBAUER EQUATIONS

Yan Liu*

Abstract: This paper presents a spectral reflectance modification of the Neugebauer equations for predicting printing colors and dot area ratios by using spectral reflectance. These modified Neugebauer equations include all of the optical effects of color printing: absorption of ink layers, first-surface reflectance, paper reflectance, multiple internal reflections.

The equivalent spectral reflectance (ESR) is introduced to represent the optical effects of the ink layer. The reflectance of overlap ink color is equal to the product of the ESRs of its components. The application of the ESR can solve the disagreement between the theoretical prediction of spectral reflectance of overlap ink layers and measured results. The spectral reflectances of the secondary printing colors can be predicted by using ESRs of the primary printing colors. In addition, ESR can be used to predict percent trapping. Results were obtained where a perfect spectral reflectance match was made using ESRs and ink trapping parameters were determined.

Introduction

There are several methods for predicting printing color and dot area ratios: Neugebauer equations (Neugebauer, 1937), masking equations (Yule, 1967a), Kubelka-Munk theory (Kubelka and Munk, 1931) and some modified methods (Clapper, 1961, Pobboravsky and Pearson, 1972, Yule, 1951). Pollak (1955a, 1955b, 1956) assumed that ink densities are additive and gave a simplified Neugebauer equation. On the contrary, the ink densities are not additive and the spectral reflectance of overlap color is not

* Munsell Color Science Laboratory,
Rochester Institute of Technology, Rochester, New York.

equal to the product of the spectral reflectances of its ink layers, due to the existence of first-surface reflectance, multiple internal reflections, and absorption of the paper. For dealing with the disagreement of the Neugebauer equations with measured results, Yule and Colt (1951) presented an n value modification for correcting the ink densities. Viggiano (1985) presented a spectral Yule-Nielsen model and an extension model (1990) which provides an acceptable level of accuracy for many applications.

This paper presents a spectral reflectance modification of the Neugebauer equations for solving the problem of the disagreement between theoretical prediction and measured results. Based on the additivity of light intensities, we can directly get equations that are similar to the Neugebauer equations. The equations include all of the optical effects: absorption, first-surface reflectance, paper reflectance, multiple internal reflections.

The use of an equivalent spectral reflectance method for predicting printing colors and dot area ratios gives a more clear optical and colorimetric model than the use of optical densities. The spectral reflectance modified Neugebauer equations can be used to predict dot area ratios and printing colors wavelength by wavelength.

The equivalent spectral reflectance (ESR) is introduced to represent all of the optical effects of the ink layer. The equivalent spectral reflectance of an overlap ink color is equal to the product of the ESRs of its components. The ESR can theoretically solve the disagreement between predicted spectral reflectance of overlap printing ink layers and measured results.

There are several equations(Preuci, 1958, Childers, 1980, Hamilton, 1985) used for calculating ink trapping based on densities. The ESR is also be useful to predict ink trapping. By applying linear no-intercept regression to the logarithmic ESR of overlap printing ink layers, a parameter can be obtained. This parameter represents the percent trapping. In addition, this parameter can be used to predict the true spectral reflectance of the overlap printing ink layer with trapping.

Two samples are used to verify the equations introduced in this paper. One is a 3M Matchprint proof sample and the other is a press sheet on paper.

Theory

In color printing, there are three primary colors (cyan, magenta and yellow) and four secondary colors (red, green, blue and black).

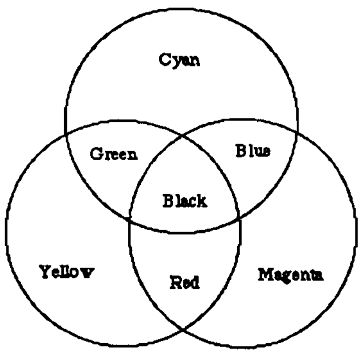

Figure 1. The primary printing colors. Cyan, magenta and yellow are the first primary printing colors. Red, green, blue and black which are produced by the corresponding overlap of the three first primary printing colors are the second primary printing colors.

Figure 1 shows the seven primary printing colors. The area covered by cyan and magenta overlap appears blue, the area covered by cyan and yellow overlap appears green, the area covered by magenta and yellow overlap appears red, and the area covered by cyan, magenta and yellow overlap appears black.

Figure 2 shows light penetrating an ink layer printed on paper and then being reflected back by paper.

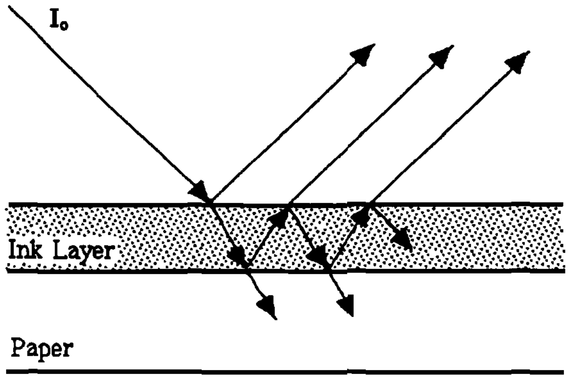

Figure 2. The Optical properties of printing ink layer. The reflected light includes first surface reflectance and multiple reflections with the paper. The light penetration length is twice of the ink layer thickness.

Based on Bouguer's law, the reflected light intensity is described by the following simplified relation:

$$I(\lambda) = I_s(\lambda)R_s(\lambda) + I_o(\lambda)[1 - R_s(\lambda)]^2 T(\lambda)R_p(\lambda) \cdot \left\{1 + R_s(\lambda)R_p(\lambda)T(\lambda) + R_s^2(\lambda)R_p^2(\lambda)T^2(\lambda) + \cdots\right\} \quad (1)$$

with,

I:	the light intensity,
I_o:	the incident light intensity,
T:	the inner absorption $\exp[-2\alpha(\lambda)d]$,
R_p:	is the spectral reflectance of paper,
R_s:	the spectral reflectance of first surface.

In equation (1), $\alpha(\lambda)$ represents the absorption coefficient of the ink layer, d represents the thickness of the ink layer, and $I_0(\lambda)R_s(\lambda)$ represents the light intensity from the first-surface reflection. Here the scattering of light is neglected due to the fact that the scattering of the ink layer is insignificant compared to the scattering of the paper.

Equation (1) can be simplified as:

$$I(\lambda) = I_0(\lambda)[1 - R_s(\lambda)]^2 T(\lambda)R_p(\lambda)\left[\frac{1}{1 - R_s(\lambda)R_p(\lambda)T(\lambda)}\right] + I_0(\lambda)R_s(\lambda). \quad (2)$$

Equation (2) can be rewritten as:

$$I(\lambda) = I_0(\lambda)[1 - R_s(\lambda)]^2 R_p(\lambda)\left\{\frac{T(\lambda)}{1 - R_s(\lambda)R_p(\lambda)T(\lambda)}\right\} + I_0(\lambda)R_s(\lambda). \quad (3)$$

For a printing ink layer, the ink thickness can be considered as constant. For convenience, $\Re(\lambda)$ can replace $\frac{\exp[-2\alpha(\lambda)d]}{1 - R_s(\lambda)R_p(\lambda)\exp[-2\alpha(\lambda)d]}$ from equation (3)

Therefore equation (3) changes to:

$$I(\lambda) = I_0(\lambda)[1 - R_s(\lambda)]^2 \Re(\lambda)R_p(\lambda) + I_0(\lambda)R_s(\lambda), \quad (4)$$

where $\Re(\lambda)$ is the light intensity after internal absorption by the printed ink layer. For the ink layer on the paper, $\Re(\lambda)$ is equal to the effect of the spectral reflectance for observer, so the $\Re(\lambda)$ is called the equivalent spectral reflectance (ESR) of the ink layer. The ESR can be calculated by:

$$\Re(\lambda) = \frac{I(\lambda) - I_0(\lambda)R_s(\lambda)}{I_0(\lambda)[1 - R_s(\lambda)]^2 R_p(\lambda)}. \tag{5}$$

Equation (5) can be simplified as:

$$\Re(\lambda) = \frac{R(\lambda) - R_s(\lambda)}{[1 - R_s(\lambda)]^2 R_p(\lambda)}, \tag{5'}$$

For two ink layers, equation (3) changes to:

$$I(\lambda) = I_0(\lambda)[1 - R_s(\lambda)]^2 R_p(\lambda) \left\{ \frac{T_1(\lambda)T_2(\lambda)}{1 - R_s(\lambda)R_p(\lambda)T_1(\lambda)T_2(\lambda)} \right\} + I_0(\lambda)R_s(\lambda), \tag{6}$$

where T_1 and T_2 are the internal absorption of layers 1 and 2.

Considering the optical effects between the two layers, equation (6) is approximately equal to:

$$I(\lambda) = I_0(\lambda)R_s(\lambda) + I_0(\lambda)[1 - R_s(\lambda)]^2 R_p(\lambda) \cdot \left\{ \frac{T_1(\lambda)}{1 - R_s(\lambda)R_p(\lambda)T_1(\lambda)} \cdot \frac{T_2(\lambda)}{1 - R_s(\lambda)R_p(\lambda)T_2(\lambda)} \right\}. \tag{7}$$

Therefore, if the ESRs of the primary colors are known, then the ESRs of the secondary colors are:

$$\begin{aligned} \Re_r(\lambda) &= \Re_m(\lambda)\Re_y(\lambda) \\ \Re_g(\lambda) &= \Re_c(\lambda)\Re_y(\lambda) \\ \Re_b(\lambda) &= \Re_c(\lambda)\Re_m(\lambda) \\ \Re_{bk}(\lambda) &= \Re_c(\lambda)\Re_m(\lambda)\Re_y(\lambda), \end{aligned} \tag{8}$$

where $\Re(\lambda)_c$, $\Re(\lambda)_m$, $\Re(\lambda)_y$, $\Re(\lambda)_r$, $\Re(\lambda)_g$, $\Re(\lambda)_b$ and $\Re(\lambda)_{bk}$ are the ESRs corresponding to cyan, magenta, yellow, red, green, blue and black ink layers respectively. For the secondary colors the ESR can also be calculated by equation (5) using measured spectral reflectance data. Since the measured data are used to calculate the ESRs, the effect of light scattering is already included in the ESRs. The ESRs can be used to predict the color of a printed sample if the dot area ratios are known, or to predict the dot area ratios if the spectral reflectance of the printed color sample is known.

Suppose that the dot area ratios of the primary printing colors are A_c, A_m, A_y, A_r, A_g, A_b, A_{bk}, A_p for cyan, magenta, yellow, red, green, blue, black and white(paper). The sum of the dot area ratios is unity:

$$A_c + A_m + A_y + A_r + A_g + A_b + A_{bk} + A_p = 1 \qquad (9)$$

Based on the additive of light intensities, the spectral reflectance $R(\lambda)$ of the printed color is:

$$\begin{aligned} R(\lambda) = {} & A_c\left[(1 - R_s(\lambda))^2 \Re_c(\lambda) R_p(\lambda) + R_s(\lambda)\right] \\ & + A_m\left[(1 - R_s(\lambda))^2 \Re_m(\lambda) R_p(\lambda) + R_s(\lambda)\right] \\ & + A_y\left[(1 - R_s(\lambda))^2 \Re_y(\lambda) R_p(\lambda) + R_s(\lambda)\right] \\ & + A_r\left[(1 - R_s(\lambda))^2 \Re_r(\lambda) R_p(\lambda) + R_s(\lambda)\right] \\ & + A_g\left[(1 - R_s(\lambda))^2 \Re_g(\lambda) R_p(\lambda) + R_s(\lambda)\right] \\ & + A_b\left[(1 - R_s(\lambda))^2 \Re_b(\lambda) R_p(\lambda) + R_s(\lambda)\right] \\ & + A_{bk}\left[(1 - R_s(\lambda))^2 \Re_{bk}(\lambda) R_p(\lambda) + R_s(\lambda)\right] \\ & + A_p R_p(\lambda). \end{aligned} \qquad (10)$$

Equation (10) is similar to the Neugebauer equations (Yule, 1967b).

In general, the first-surface reflectance of the printing primary colors are approximately the same. In equation (10), only R_s is included instead of first-surface reflectances of every colored ink layer.

Considering equation (9), equation (10) can be simplified as:

$$
\begin{aligned}
R(\lambda) = [1 - R_s(\lambda)]^2 R_p(\lambda)[&A_c\Re_c(\lambda) + A_m\Re_m(\lambda) + \\
&A_y\Re_y(\lambda) + A_r\Re_r(\lambda) + A_g\Re_g(\lambda) + \\
&A_b\Re_b(\lambda) + A_{bk}\Re_{bk}(\lambda)] + \\
&A_pR_p(\lambda) + R_s(\lambda)(1 - A_p).
\end{aligned} \tag{11}
$$

Equation (11) can be further simplified using equation (8):

$$
\begin{aligned}
R(\lambda) = [1 - R_s(\lambda)]^2 R_p(\lambda)[&A_c\Re_c(\lambda) + A_m\Re_m(\lambda) + \\
&A_y\Re_y(\lambda) + A_r\Re_m(\lambda)\Re_y(\lambda) + A_g\Re_c(\lambda)\Re_y(\lambda) \\
&+A_b\Re_c(\lambda)\Re_m(\lambda) + A_{bk}\Re_c(\lambda)\Re_m(\lambda)\Re_y(\lambda)] \\
&+A_pR_p(\lambda) + (1 - A_p)R_s(\lambda).
\end{aligned} \tag{12}
$$

In equation (12), only the ESRs of the three primary colors are included.

Equations (11) and (12) represent the spectral reflectances of a printed color of unit area. Equation (11) is more accurate but equation (12) is simpler.

For four color printing, black is added to the three primary colors. Based on the Demichel equation (Demichel, 1924), the spectral reflectance is:

$$
\begin{aligned}
R'(\lambda) &= (1 - a)R(\lambda) + aR(\lambda)\Re'_{bk}(\lambda) \\
&= R(\lambda)(1 - a + a\Re'_{bk}(\lambda)).
\end{aligned} \tag{13}
$$

Where $R'(\lambda)$ is the spectral reflectance of four color printing, $R(\lambda)$ is the spectral reflectance of three color printing, a is the ratio of the black dot area, $\Re'_{bk}(\lambda)$ is the ESR of the black ink layer.

Dot Area Ratio Prediction

For the convenience of writing matrix equations, total reflectances of primary printing colors are introduced here:

$$\begin{aligned}
R_c(\lambda) &= [1 - R_s(\lambda)]^2 \Re_c(\lambda) R_p(\lambda) + R_s(\lambda) \\
R_m(\lambda) &= [1 - R_s(\lambda)]^2 \Re_m(\lambda) R_p(\lambda) + R_s(\lambda) \\
R_y(\lambda) &= [1 - R_s(\lambda)]^2 \Re_y(\lambda) R_p(\lambda) + R_s(\lambda) \\
R_r(\lambda) &= [1 - R_s(\lambda)]^2 \Re_r(\lambda) R_p(\lambda) + R_s(\lambda) \\
R_g(\lambda) &= [1 - R_s(\lambda)]^2 \Re_g(\lambda) R_p(\lambda) + R_s(\lambda) \\
R_b(\lambda) &= [1 - R_s(\lambda)]^2 \Re_b(\lambda) R_p(\lambda) + R_s(\lambda) \\
R_{bk}(\lambda) &= [1 - R_s(\lambda)]^2 \Re_{bk}(\lambda) R_p(\lambda) + R_s(\lambda),
\end{aligned} \tag{14}$$

where R_c, R_m, R_y, R_r, R_g, R_b, and R_{bk} are the total spectral reflectances of the cyan, magenta, yellow, red, green, blue, black ink layers.

Combining equation (14) and equation (9) gives a matrix equation:

$$\begin{vmatrix} R(400) \\ R(420) \\ \vdots \\ \vdots \\ R(700) \\ 1 \end{vmatrix} = \begin{vmatrix} R_c(400) & R_m(400) & R_y(400) & R_r(400) & R_g(400) & R_b(400) & R_{bk}(400) & R_p(400) \\ R_c(420) & R_m(420) & R_y(420) & R_r(420) & R_g(420) & R_b(420) & R_{bk}(420) & R_p(420) \\ & & & & & \vdots & & \\ & & & & & \vdots & & \\ R_c(700) & R_m(700) & R_y(700) & R_r(700) & R_g(700) & R_b(700) & R_{bk}(700) & R_p(700) \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{vmatrix} \begin{vmatrix} A_c \\ A_m \\ A_y \\ A_r \\ A_g \\ A_b \\ A_{bk} \\ A_p \end{vmatrix} \tag{15}$$

where the wavelength range is from 400 to 700 nm at wavelength intervals of 20 nm. The selection of the wavelength range and interval depends on practical requirements. The matrix equation is for dot area ratio prediction, which can be solved by using regression to obtain the dot area ratios of the seven primary printing colors and paper.

This method is suitable for printing color when all eight dot area ratios of the primary printing colors (including dot area ratio of paper) can be controlled. Now it is still difficult to control the dot area ratios of the eight primary printing colors simultaneously for the technology limit of printing machines. Only the ratios of the three first primary colors can be controlled and an additional ratio of black is controlled to improve the printed color. This does not affect the use of the matrix equation (15). The eight dot area ratios of the primary printing colors can easily change to three dot area ratios of the first primary printing colors by the Demichel equations.

Weighting functions, such as color matching function, can also be included in the matrix equation to obtain improved results. For four color printing, we can simply put equation (13) in the matrix equation. The proportion, a, should be mainly dependent on the spectral reflectance. We can also obtain an optimal proportion, a, mathematically.

Ink Trapping Parameter Prediction

Ink trapping always exists in printing. Ink trapping brings some problems, such as the lightness, hue and chroma shift of the colors of overlap ink printing layers. The commonly used method for predicting ink trapping is by using ink densities. Because of the failure of additivity and proportionality of ink densities, using densities to predict ink trapping can not give high accuracy. Since the ESR includes the optical effects of printing ink layers, it can give a correct prediction of ink trapping.

For the convenience of mathematical calculation, the equations (8) can be changed to logarithmic equations:

$$\log[\Re_r(\lambda)] = \log[\Re_m(\lambda)] + \log[\Re_y(\lambda)]$$
$$\log[\Re_g(\lambda)] = \log[\Re_c(\lambda)] + \log[\Re_y(\lambda)]$$
$$\log[\Re_b(\lambda)] = \log[\Re_c(\lambda)] + \log[\Re_m(\lambda)]$$
$$\log[\Re_{bk}(\lambda)] = \log[\Re_c(\lambda)] + \log[\Re_m(\lambda)] + \log[\Re_y(\lambda)]$$

(16)

If the ink printing order is cyan, magenta and yellow and the first down ink layer has no trapping back, the equation(16) can be changed to :

$$\log[\Re_r(\lambda)] = \log[\Re_m(\lambda)] + t_y \log[\Re_y(\lambda)]$$
$$\log[\Re_g(\lambda)] = \log[\Re_c(\lambda)] + t_y \log[\Re_y(\lambda)]$$
$$\log[\Re_b(\lambda)] = \log[\Re_c(\lambda)] + t_m \log[\Re_m(\lambda)]$$
$$\log[\Re_{bk}(\lambda)] = \log[\Re_c(\lambda)] + t_m \log[\Re_m(\lambda)] + t_y \log[\Re_y(\lambda)]$$

(17)

where t_m and t_y are the parameters of ink trapping of magenta and yellow. By applying no-intercept regression, wavelength by wavelength, to equation (17), the ink trapping parameters can be obtained.

Experiment

For verifying the equations given in this paper, a 3M Matchprint proof sample and a press sheet sample were used. The ESRs of cyan, magenta, yellow, red, green and blue were calculated. The ESRs of cyan, magenta and yellow were used to predict the ESRs of red, green and blue. Finally the measured spectral reflectances of red, green and blue were compared with the calculated spectral reflectances by using the ESRs. The press sheet sample was also used to calculate ESRs of cyan, magenta, yellow, red, green and blue. Because of the existence of ink trapping, the ESRs of cyan, magenta and yellow can not directly calculate the ESRs of red, green and blue. First, no-intercept regression was used to calculate the parameters of ink trapping. Then the trapping parameters were used to correct the second ink layer ESRs to predict the ESRs of red, green and blue printed on paper. The measured spectral reflectances of red, green and blue were compared with the calculated spectral reflectances.

Figure 3 shows the spectral reflectances of primary printed colors cyan, magenta, yellow, red, green and blue on 3M Matchprint proof sample.

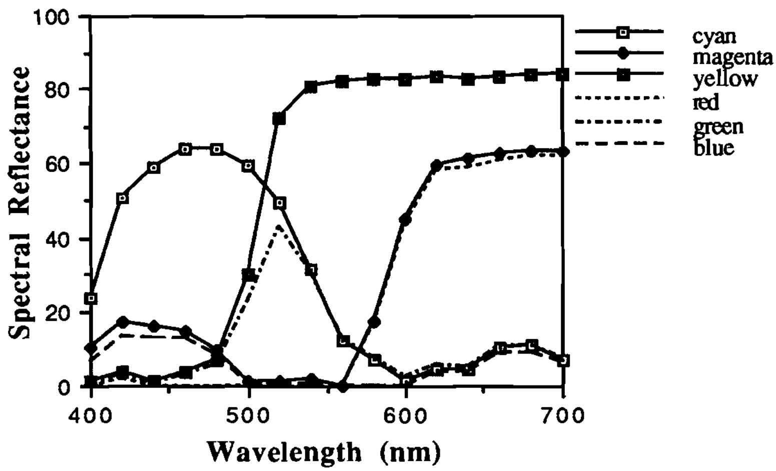

Figure 3. The spectral reflectances of primary printing colors and white of 3M Matchprint proof sample .

Figure 4 shows the ESRs of the primary printing colors:

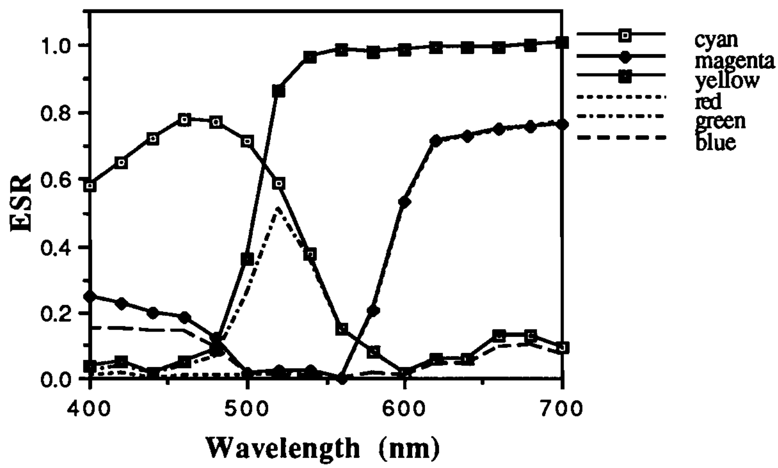

Figure 4. The ESR of primary printing colors of 3M Matchprint proof sample.

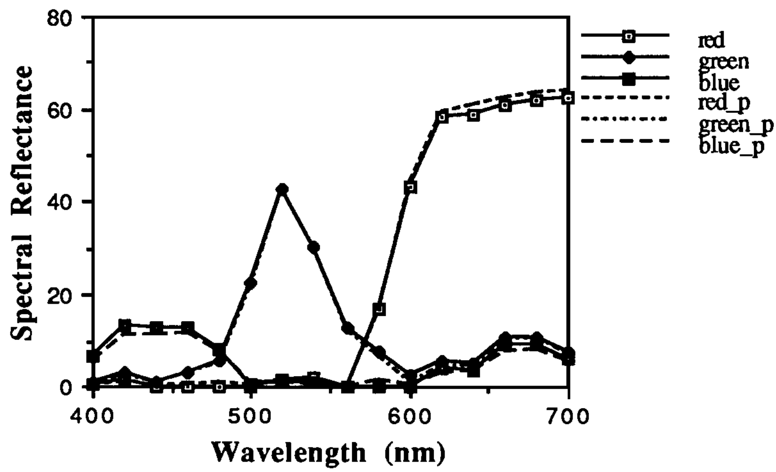

Figure 5. The measured spectral reflectances of red, green and blue and the predicted spectral reflectances of red, green and blue by ESRs of cyan, magenta and yellow. Red_p, green_p and blue_p represent the predicted spectral reflectances of red, green and blue.

The major difference between Figure 3 and Figure 4 is that the ESRs have been corrected by the spectral reflectance of paper(white), internal multiple reflections, and first surface reflectance. The ESRs are larger than the corresponding spectral reflectances of the primary printing colors.

Figure 5 shows the measured spectral reflectances and the predicted spectral reflectances of red, green and blue by using the ESRs of cyan, magenta and yellow. Figure 5 shows that the predicted spectral reflectances are very close to the measured spectral reflectances. This means the ESR is appropriate for predicting the spectral reflectances of secondary printing colors on film. On the other hand, this also means the conditions for printing on film are very close to the perfect theoretical conditions.

Figure 6 shows the spectral reflectances of primary printing colors of press sheet.

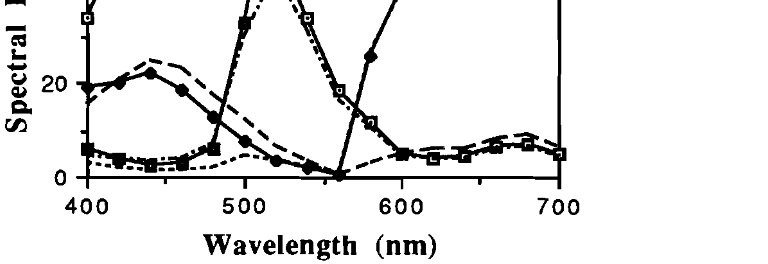

Figure 6. The spectral reflectances of primary printing colors of the press sheet.

Figure 7 shows the ESRs of the primary printing colors: cyan, magenta, yellow, red, green and blue.

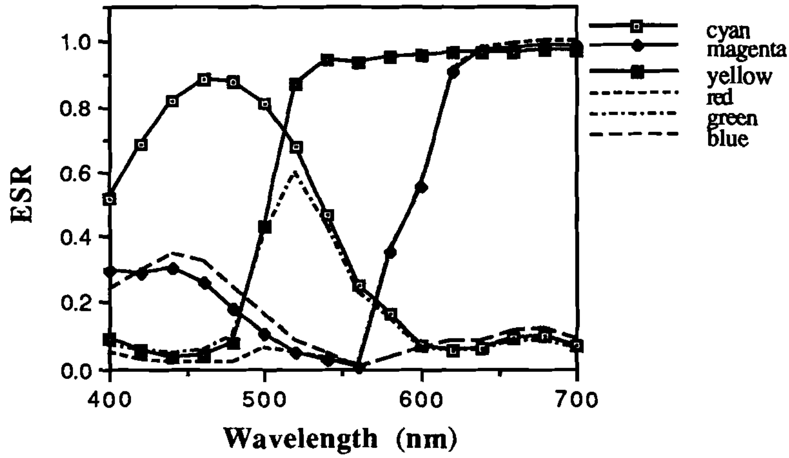

Figure 7. The ESRs of primary printing colors of press sheet.

The thicknesses of the first down color layer is assumed not to change. Because of ink trapping, the thickness of second down color layer will change. Using no-intercept regression to equation (17), we can get the results shown in Table 1:

	Cyan	Magenta	Yellow
Red	---	1.0	0.7797
Green	1.0	---	0.8550
Blue	1.0	0.6673	---

Table 1. The ink trapping parameters for red, green and blue colors of press sheet.

In Table 1, the yellow ink trapping parameter of red is 0.7797, the yellow ink trapping parameter of green is 0.8550, and the magenta ink trapping parameter of blue is 0.6673. The corresponding first down ink trapping parameters are one. The ink trapping parameter of one means that the ink layer thickness does not change.

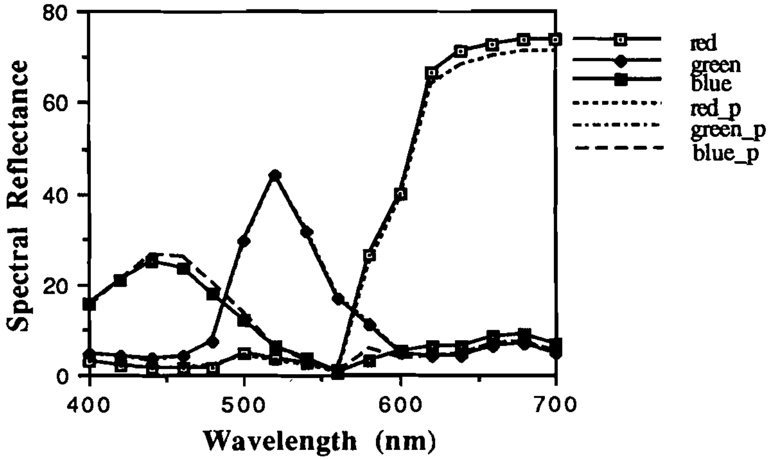

Figure 8. The measured spectral reflectances of red, green and blue and the predicted spectral reflectances of press sheet. Red_p, green_p and blue_p represent the predicted spectral reflectances of red, green and blue.

By using the ink trapping parameters in Table 1, we can calculate the spectral reflectances of secondary colors of red, green and blue. Figure 8 shows the measured and predicted spectral reflectances of red, green and blue ink layers on paper.

Discussion

The equivalent spectral reflectance (ESR) is the fundamental quantity for the spectral reflectance method. The ESRs of the primary colors can be used to calculate the ESRs of the secondary printed colors. The prediction accuracy of dot area ratios and printed colors depends on the accuracy of the ESRs.

The ESR can also be used to predict percent ink trapping. A set of trapping parameters were obtained by using linear no-intercept regression. By using trapping parameters to correct the ESRs of the second ink layer down, the predicted spectral reflectances of secondary colors match the measured spectral reflectances very well.

These spectral reflectance modified Neugebauer equations directly use the spectral reflectance of printed color to predict the color or the dot area ratios of the primary printing colors. Spectral reflectance is the most fundamental property of printed color. The spectral reflectance can transfer to any other value, such as density and k/s value, of previous color printing theories.

The thickness of the printed ink layer is fixed, the K/S value for each primary printing color is constant at each wavelength. The total K/S value of a unit printed area does not equal the sum of the product of K/S values and dot area ratios. Consequently the calculation of K/S values may be unnecessary. From this point, the spectral reflectance modified Neugebauer equations are suitable for predicting printed colors.

Because this method uses spectral reflectance to predict printed colors and dot area ratios, the result is not simply an optical density match but a spectral match.

Work is still in progress for predicting halftone colors and dot area ratios.

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