A SIMPLE ALTERNATIVE TO THE NEUGEBAUER EQUATIONS FOR COMPUTING DOT AREA AND DOT GAIN COLORIMETRICALLY

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Abstract

Dot area (and thus dot gain) is usually determined by application of the Murray-Davies equation to density values. While generally useful for common process inks and colorants, density-based dot area has the disadvantages of (1) not being applicable in general to other colors unless there is a filter response matched to the specific color; (2) being limited to CIE illuminant A due to current standard filter responses; and (3) not being based on visually perceived gain. These disadvantages can be overcome somewhat by using colorimetric densities based on tristimulus values, but it is not always obvious which tristimulus values to use or how. Using the Neugebauer Equations for a single color on paper is better, but the Neugebauer approach is not computationally simple and is not easily incorporated into instrument software. Also, the Neugebauer approach, being a linear combination of CIE tristimulus values, maintains the same hue of the primary solid color and might not be able to accurately match tints whose hue has shifted from the primary's hue (e.g, magentas). This paper will present a simple method and formula for computing dot (or colorant) area colorimetrically from reference-white normalized CIE tristimulus values, which is applicable to any color, allows correlation to visual perception under any light source, and also has a significant computational and colorimetric advantage over the Neugebauer approach. The method is based on the amount of white present instead of the amount of colorant.

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Introduction

In the reproduction of color by halftone and halftone-like printing processes, pixel elements comprise discrete areas, or "dots", of various colorants (e.g., Y, M, C, K). Ideally, the dots of these colorants are uniform and behave independently in some relation to their physical area with no interaction among them. However, in reality, because the dots are not uniform and because of scattering by the colorant's substrate (e.g., a reflective paper base) and/or scattering within the colorant material itself (opacity), the reflectance behavior is no longer dependent on only the actual physical colorant area[1,2]. Scattering interactions among adjacent colorant areas can also occur, affecting the color rendering. The deviation from ideal reflectance behavior is generally called "dot gain", a phenomenon well known in the graphic arts literature. For example, see Huntsman[3] and references therein.

Densitometric Dot Area

The phenomenon of dot gain in halftone printing can affect color reproduction both tonally and colorimetrically, and if accurate color reproduction is to be accomplished, the amount of dot gain as a function of dot area must be well characterized. Dot area is usually determined by densitometry, using a filter whose characteristic color is complementary to the process color evaluated because of the subtractive nature of the colorants used. Black is measured with respect to tone (lightness), not hue or chroma. Density values have a positive correlation with the amount of colorant present. Densitometric area, however, by focussing on only the wavelength region where the color has very little reflectance, ignores the color's reflectance which actually stimulates the human visual system. By knowing the reflectance (or transmittance) of the solid colorant used and the reflectance (or transmittance) of the reflective (transparent film) substrate, the area of colorant in a halftone pattern relative to a solid area (solid = 100% area) can be estimated. Densitometric reflectance can be defined generally as in (1), where $\Pi(\lambda)$ represents the spectral product for the densitometer filter bandpass, light source, and photodetector, and $R(\lambda)$ is the reflectance at wavelength λ.

$$R = \int_{\lambda} \Pi(\lambda) R(\lambda) d\lambda \qquad (1)$$

Murray[4] published a relationship to determine colorant tint density in a binary mixture assuming ideal behavior; that is, the total radiance from a halftone area is the weighted sum of the radiances from the areas of colorant and paper, the weights being the relative areas of the colorant and substrate in a pixel of unit area. A basic linear relation for dot area in terms of reflectance (R) is given in Eq. (2). This relation can also be given in terms of density (D) (Eq. (3)), by using D = –logR. Equation (3) is commonly known as the Murray-Davies ("M-D") formula, although it is not the original form of Murray's relation[4]. Equations (1), (2), and (3) are strictly valid for the total reflectance as measured by integrating sphere, specular-component-included geometry. In practice, the reflectance within a limited solid angle is used, usually 0°/45° or 45°/0° measurement geometry by standard (ISO 5/4). The reflectance of glossy, matte, or textured samples will differ considerably between integrating sphere and ISO 5/4 geometries. In (2), (3), and throughout this paper, %A is the effective, relative percent area of the solid colorant in a halftone or dithered "tint" (combination of colorant of %A effective area and substrate) area, and the subscripts t, s, and o will refer to the "tint" area, the colorant's solid area, and the substrate area, respectively. Since the principles herein are analogously applicable to transparent substrates, only reflective substrates (e.g., paper, coated and metallic surfaces) will be detailed.

$$\%A = \frac{R_o - R_t}{R_o - R_s} \bullet 100 \qquad (2)$$

$$\%A = \frac{1 - 10^{-(D_t - D_o)}}{1 - 10^{-(D_s - D_o)}} \bullet 100 \qquad (3)$$

The difference between %A from (2) or (3) and some reference percent dot area for the color is called the dot gain. If the reference area is the actual physical dot area in the printed image, the dot gain is usually considered optical dot gain. If the reference area is the dot area in the separation films, the dot gain is usually considered the total (physical + optical) dot gain of the reproduction process.

Although fairly simple to implement, there are some deficiencies in using the densitometric approach. One is that, since only the major filter density is used, the color's reflectance outside the densitometer filter's bandpass is ignored, and some colors can have nearly the same density but noticeably different perceived color [See Huntsman[5], figures 1 and 2.]. A second deficiency is that, by standard, virtually all densities are based on an incandescent source, which might not correlate with visual perception under another source. A third deficiency is that densitometer filters are usually matched to the specific colorants and do not correspond to the human colorimetric responses. Thus, the use of densitometry to determine dot gain and tone reproduction of colors can be erroneous, or at least misleading, with respect to correlation with visually perceived dot gain and its effects. However, densitometry is excellent for determining ink film thickness when properly matched to the appropriate absorption of the color. This paper describes a method for overcoming these deficiencies for dot area by densitometry by quantitatively estimating the effective dot area of a color in a halftone pattern, and thus its dot gain, colorimetrically. The method utilizes CIE tristimulus values[6] X, Y, and Z, normalized to the substrate as the reference white and also the principles of adaptation and scaling by the human visual system to an image's reference white.

Colorimetric Dot Area

Although densitometry and the M-D formula have some deficiencies with regard to accurate color reproduction, they do involve some principles also utilized in colorimetry. The first principle is that a visual response can be characterized as the combined interaction of an illuminant, an object's reflectance spectrum, and a perceiver's visual response function. For a person as the perceiver, a person's response can be characterized by CIE color matching functions[6] ("cmf"). The combined response of the cmf, illuminant, and object's reflectance spectrum can be represented by the CIE tristimulus values X, Y, and Z, the mathematical relation for the X value being given in (4). Equation (4) is usually approximated by a summation based on discrete spectrophotometric data. The value of the summation will depend on the wavelength range, wavelength interval, instrument bandpass, and measurement geometry for the $R(\lambda)$ data. Analogous relations hold for Y and Z, differing by the cmf used. From studies of visual adaptation, human perception adapts to spectrally near-whites and even non-spectrally-white "memory whites" as perceived white, which can be considered the colorimetric visual reference white. Other colors become scaled visually to the reference white. Colorimetric variables for the reference white are usually denoted with a subscript n, and the tristimulus values for the reference white are also derived from (4), the difference being that values of the reference white reflectance are used. For the perfect white diffuser, $R(\lambda) = 1$ at all wavelengths. For color science in the graphic arts, the reference white is usually the perfect white diffuser because most commercial software packages seldom allow the user to specify the reference white. However, in the graphic arts, the paper will usually be the appropriate *visual* reference white for images, because, due to adaptation, other colors in the image are visually scaled relative to the paper white [See Hunt[7] , pp.114–116.].

$$X = \int_{\lambda} \bar{x}(\lambda)S(\lambda)R(\lambda)d\lambda \tag{4}$$

The tristimulus values are directly proportional to the object's reflectance. Neugebauer[8] relied on this principle in developing his equations, which say that the tristimulus values of a pixel area are equal to the weighted sum of the tristimulus values of the color areas, the weights being the relative fractional area of each "primary" color. The Neugebauer equations relating an arbitrary color having tristimulus values X^*, Y^*, Z^* to n "primary" colors, each having tristimulus values X_i, Y_i , Z_i, are given in (5). The a_i coefficients are the relative fractional areas for the primary color i. A Neugebauer "primary" color is not only yellow, magenta, and cyan, but rather all the different areas of color, including secondary colors (red, green, and blue) as well as combinations of all these with black, and the substrate. In color reproduction, the usual approach is to solve (5) simultaneously for the a_i coefficients, knowing X_i, Y_i , and Z_i, usually from empirical measurements. The a_i coefficients used by Neugebauer were initially based on areas determined by Demichel according to random distribution. Today, the a_i coefficients are often based on the M-D formula, or are modified with the Yule-Nielsen equation, or some other modification, to allow for variables such as dot gain, the periodic distribution from defined screen angles, and line frequency. For examples, see [9–22].

$$X^* = \sum_{i}^{n} a_i X_i \qquad Y^* = \sum_{i}^{n} a_i Y_i \qquad Z^* = \sum_{i}^{n} a_i Z_i \tag{5}$$

If it is desired to determine dot area colorimetrically for a single color, the relations in (5) become a two component system (the paper and the colorant on the paper), which can be re-written as in (6), where W ("white") represents the substrate (e.g., paper), i.e., no colorant; X_s, Y_s, and Z_s are the tristimulus values of the solid (or maximum) colorant; and X_t, Y_t, and Z_t are the tristimulus values of the tint of the solid color, whose dot area is to be determined.

$$X_t = a_w X_w + a_s X_s \qquad Y_t = a_w Y_w + a_s Y_s \qquad Z_t = a_w Z_w + a_s Z_s \tag{6}$$

Solving (6) for a_s gives the effective dot area of the colorant tint, and its dot gain can be determined by subtracting the reference dot area. However, determining a_s requires the simultaneous solution of three equations, even though it is much easier for only one color and paper. A second deficiency in solving (6) is that because the relations are linear, if the hue of the tint is not the same as the solid, (6) might not be accurately solvable. Such is the case for tints of magenta, whose hue shifts significantly with tone. Matching a midtone magenta by the Neugebauer approach will usually result in the inclusion of some cyan because the magenta tint will be slightly bluer than the solid magenta. Thus, a method of determining dot area colorimetrically that does not require solving three equations or is unaffected by hue shift with tone would be very useful.

One basis of the new method for colorimetric dot area is the similarity of the tristimulus values as from (4) and reflectance as from (1). Thus, one can rewrite (2) with each tristimulus value as the metric basis as in (7) because the tristimulus values can be considered weighted integrated reflectance, equivalent to the Rs in (2). The subscript n in (7) is analogous to the o subscript in (2) and (3), because both subscripts usually refer to the substrate ("white"). Equation (7) is a linear equivalent to using colorimetric densities (–log of the resultant tristimulus value) in (3) to determine dot area without making the logarithmic transformation. It is simpler to keep variables in a linear relation, especially if interpolation might become involved.

$$\%A = \frac{X_n - X_t}{X_n - X_s} \bullet 100 \tag{7a}$$

$$\%A = \frac{Y_n - Y_t}{Y_n - Y_s} \bullet 100 \tag{7b}$$

$$\%A = \frac{Z_n - Z_t}{Z_n - Z_s} \bullet 100 \tag{7c}$$

However, there is a problem in (7) in that it is not likely that the relations for X, Y, and Z will all yield the same value of %A because the rate of change of each tristimulus value with the amount of colorant is not necessarily the same. While the choice is easier for most normal process colors, the choice is not necessarily obvious for special "spot" colors, even if colorimetric densities are used. Thus, two of the three relations in (7) could give the wrong area, and so it becomes necessary to know which relation in (7) is the right one to use. In color science, it is sometimes considered that an arbitrary color is a mixture of a "pure" chromatic color and a pure achromatic ("white") color, e.g., in a CIE *xy* chromaticity diagram. It is reasonable that the amount of white component in a binary color mixture is also a valid correlate for the mixing of a colorimetric white with a primary chromatic color, especially for halftone-like colorant areas, which are usually on an essentially "white" substrate (e.g., white paper, clear film). From (7), one makes use of the fact that for a single color on a substrate, $a_W + a_S = 100\%$. While much effort is usually focussed on determining a_S, often requiring careful matching of a filter response to the specific color, it is not necessary to do so. If one can determine a_W, one has also determined a_S, because $a_S = 100\% - a_W$, and, therefore, one does not have to directly measure only the amount of the colorant. Letting W represent a suitable white component metric, then (7) can be rewritten as (8), with the subscripts having the same implication. It is important that W_n be for the actual substrate used (e.g., paper white) because the limits for the white component of the tint (W_t) are bounded by the white component of the substrate (W_n) and the white component of the solid color (W_s). If W_n for the perfect white diffuser is used where reflective substrates are involved, then additional mathematics are necessary to determine the same proportion of the perfect white diffuser in the tint, paper, and solid color, which is unnecessary. Equation (8) is also intuitive. In a plot of the suitable W against dot area, %A is simply the ratio of the line segment (W_n – W_t) to the total line (W_n – W_s), because increasing the amount of colorant moves W_t from W_n (zero colorant area or no colorant) to W_s (100% colorant area or maximum colorant).

$$\%A = \frac{W_n - W_t}{W_n - W_s} \bullet 100 \tag{8}$$

Determining the White Components

To utilize (8), it is necessary to know how to determine the white component metrics therein. Presently used color *difference* spaces (e.g., CIELAB, CIELUV, U*V*W*, CMCII, FMCII, ANLAB, Hunter Lab, etc.) do not provide separate, quantitative achromatic ("white") and chromatic metrics from a set of tristimulus values simply because these spaces are concerned with color difference and not with quantifying separate achromatic and chromatic metrics, which can be useful for color reproduction. However, a recently developed color space for graphic arts, MOTR[23], does quantify separate achromatic and chromatic components from a set of tristimulus values and considers a color to be a linear mixture of these components. Here, as in the MOTR model[23], the achromatic (white) component can be the smallest tristimulus value, but there are also other psychophysical considerations in deciding whether usual tristimulus values are the most appropriate for use in a relation like (8).

Although it is sometimes believed that a color with equal tristimulus values is perceived as a neutral (a tone of white), such is not true, except in special cases. For example, in a 3M Matchprint™ II Commercial colors proof, the XYZ tristimulus values of a ca. 15% dot area of "blue" from cyan and magenta for the 10° CIE Standard Observer and Illuminant D50 were spectrophotometrically determined to be 50.71, 51.58, and 51.54, respectively. When viewed under essentially D50 illumination, the perceived color is definitely bluish, not neutral. Because of adaptation, the tristimulus values of the reference white represent perceived white, and these values are usually not equal. Thus, in XYZ space, perceived white would usually not be at (100,100,100). A perceived neutral at any tone level should, therefore, have tristimulus values in the same proportion as the reference white's tristimulus values.

While this circumstance is not a problem mathematically because a transformation can be made to allow for a displaced origin, it is easier to work in a coordinate system with an origin (black) at (0,0,0), and white at (100,100,100). This can be easily accomplished by normalizing all tristimulus values to the reference white's tristimulus values. Psychometrically, this is essentially a von Kries transformation[24] to allow for some adaptation. This normalization can be done by dividing all tristimulus values by the reference white's corresponding tristimulus values, which is different from a translational displacement to new coordinates. These ratios are the same as in CIELAB. This normalization is equivalent to producing a unit normal basis set of coordinate vectors. If desired to keep comparable magnitudes, one can multiply the ratios by 100 to have normalization to 100 rather than 1. Normalization to 100 will be assumed hereafter unless otherwise specified. The reference white will then always have normalized tristimulus values of (100,100, 100), and, very conveniently, a neutral will have equal white-normalized tristimulus values. These reference-white-normalized tristimulus values have the advantage that the amount of white will be the smallest tristimulus value because a color can be thought of as the result of the additive mixing of the normalized reference white and an ideal, pure (no white) chromatic color.

For example, let the reference white (e.g., paper) have tristimulus values (88.3, 87.2, 82.5) and a tint area have tristimulus values (45.1, 37.3, 15.4) for a given illuminant and Standard Observer. If both sets of tristimulus values are normalized to the paper as reference white, then the resulting white-normalized tristimulus values (X', Y', Z') are (100,100,100) for the paper and (51.08, 42.78, 18.67) for the color. The white-normalized tristimulus values of the white component become obvious upon inspection since they are all equal to the smallest white-normalized tristimulus value. For the object, the white-normalized tristimulus values of its white component would therefore be (18.67, 18.67, 18.67), and the white-normalized ideal chromatic component would be (32.41, 24.11, 0) [=(51.08, 42.78, 18.67) – (18.67, 18.67, 18.67)]. Moreover, since the normalized white scale now goes from (0,0,0) to (100,100,100), there is 18.67% white in the mixture, and 81.33% (100 – 18.67) of an ideal, "pure" (no white component) color whose tristimulus values are (39.85, 29.64, 0) [= (32.41, 24.11 ,0) /.8133]. In condensed notation, (51.08, 42.78, 18.67) = .1867(100,100, 100) + .8133(39.85, 29.64, 0). If the object color was a 60% dot area of this ideal color, then the colorimetric dot gain would be 21.33% (= 81.33% – 60%). Thus, for the case of reference-white normalized colorimetric variables, (8) can be expressed as in (9), where $W_{t/n}$ and $W_{s/n}$ are the reference-white normalized (to 100 here) colorimetric white component metric (e.g., tristimulus value) for the tint and solid (or maximum) colorant areas, respectively. Allowance for non-zero white components in real (non-ideal) solid colors is made through the $W_{s/n}$ term in (9).

$$\%A = \frac{100 - W_{t/n}}{100 - W_{s/n}} \bullet 100 \tag{9}$$

Since a normalized white component is arrived at by equal scaling for all terms in (8), the %A area in (9) will be the same value as calculated by (8) if the correct tristimulus value for the white component value is chosen. The use of (8) simply has the risk that the wrong tristimulus value set might be chosen for a given reference white, illuminant, and colorant. Thus, it is better to make an appropriate transformation first and then choose the appropriate basis variables. A computer algorithm or electronic circuitry for determining dot area is also simplified because one can routinely first normalize a color's tristimulus values from the tristimulus values of the reference white and then choose the appropriate white component (e.g., with the use of the MINIMUM function) as the smallest of the white-normalized tristimulus values of the color.

Determining Colorimetric Dot Area and Dot Gain

To examine the considerations of which tristimulus value to use and the effect of reference-white normalization, dot area scales of yellow and magenta from a 3M Matchprint™ II Commercial colors proof were used. In Tables 1 and 2 are tristimulus values, based on CIE Illuminant D50 and the 2° Standard Observer, before (X, Y, Z) and after normalization to the paper (X', Y', Z'), for yellow and magenta halftone tone scales, respectively. In Tables 3 and 4 are dot areas calculated from (9), based on the white-normalized tristimulus values (X', Y', Z') for the yellow and magenta halftone

tone scales in Tables 1 and 2, respectively. "% Film Area" is the nominal dot area in the film and is also the corresponding dot area measured on the paper. Also included for comparison are the M-D dot areas for Status T ("Area(T)") and DIN 16536 narrow band ("Area(NB)") from (3). From Table 3, it is obvious that the dot areas based on the X' and Y' white-normalized tristimulus values are not well behaved because they exceed 100% for the 95% area. Area based on Z', the smallest white-normalized tristimulus value, is well behaved, decreases monotonically, and agrees with the M-D areas. For yellows it seems that Z' values are better than X' and Y' for determining dot area according to (9). However, in Table 2, the smallest white-normalized tristimulus values for magenta are Y', and colorimetric areas in this case should be based on Y' values. The appropriate white-normalized tristimulus value for calculating colorimetric dot area can be different for different colorants but for typical process colors will usually be the tristimulus value corresponding to the complementary hue of the colorant. Thus, one can *usually* (but not always!) use Z' ("blue") for yellows, Y' ("green") for magentas, and X' ("red") for cyans. A significant advantage of this method is that it chooses the correct white component even if one is measuring a green, orange, or purple spot color.

Table 1

Tristimulus Values (2°, D50) and Tristimulus Values Normalized to Paper (X', Y', Z') for a 3M Matchprint™ II Commercial Yellow

% Film Area	X	Y	Z	X'	Y'	Z'
100	73.12	78.92	7.05	84.44	87.76	9.81
95	73.04	78.80	7.71	84.35	87.63	10.74
90	73.47	79.26	9.23	84.85	88.14	12.86
80	74.30	80.12	11.96	85.80	89.10	16.65
70	75.10	80.98	15.28	86.72	90.06	21.28
60	76.04	81.90	19.30	87.82	91.07	26.87
50	76.74	82.55	23.21	88.62	91.80	32.33
40	78.40	84.16	28.93	90.54	93.59	40.30
30	79.05	84.53	35.51	91.29	94.00	49.46
20	81.39	86.53	44.46	93.99	96.23	61.92
10	83.74	88.43	54.70	96.70	98.34	76.18
5	83.69	87.92	60.10	96.64	97.77	83.70
0	86.59	89.92	71.80	100.00	100.00	100.00

Table 2

Tristimulus Values (2°, D50) and Tristimulus Values Normalized to Paper (X', Y', Z') for a 3M Matchprint™ II Commercial Magenta

% Film Area	X	Y	Z	X'	Y'	Z'
100	34.95	18.16	15.39	40.36	20.19	21.43
95	35.48	18.81	15.99	40.97	20.92	22.27
90	36.81	20.49	17.57	42.51	22.79	24.47
80	39.70	24.21	20.81	45.85	26.92	28.98
70	42.83	28.23	24.28	49.46	31.40	33.81
60	45.26	31.40	27.00	52.26	34.92	37.61
50	48.75	35.88	30.83	56.29	39.89	42.94
40	54.85	43.86	37.54	63.35	48.78	52.29
30	60.27	51.20	43.36	69.60	56.94	60.39
20	67.02	61.08	50.90	77.39	67.93	70.89
10	72.75	69.70	57.52	84.01	77.51	80.12
5	79.90	79.41	65.01	92.27	88.30	90.54
0	86.59	89.92	71.80	100.00	100.00	100.00

Table 3

Colorimetric (2°, D50) and Densitometric (Status T, DIN 16536 NB) % Dot Areas for a 3M Matchprint™ II Commercial Yellow from Table 1

% Film Area	Area(X')	Area(Y')	Area(Z')	Area(T)	Area(NB)
100	100.0	100.0	100.0	100.0	100.0
95	100.6	101.1	99.0	99.0	99.0
90	97.4	96.9	96.6	96.6	96.7
80	91.2	89.1	92.4	92.3	92.6
70	85.3	81.3	87.3	87.1	87.6
60	78.3	73.0	81.1	80.9	81.6
50	73.1	67.0	75.0	74.8	75.6
40	60.8	52.4	66.2	65.9	67.0
30	56.0	49.0	56.0	55.7	56.8
20	38.6	30.8	42.2	41.8	43.0
10	21.2	13.6	26.4	26.2	27.2
5	21.6	18.2	18.1	17.9	18.7
0	0.0	0.0	0.0	0.0	0.0

Table 4

Colorimetric (2°, D50) and Densitometric (Status T, DIN 16536 NB) % Dot Areas for a 3M Matchprint™ II Commercial Magenta from Table 2

% Film Area	Area(X')	Area(Y')	Area(Z')	Area(T)	Area(NB)
100	100.0	100.0	100.0	100.0	100.0
95	99.0	99.1	98.9	99.2	99.2
90	96.4	96.7	96.1	96.9	97.0
80	90.8	91.6	90.4	92.0	92.1
70	84.7	86.0	84.2	86.6	86.7
60	80.0	81.5	79.4	82.3	82.4
50	73.3	75.3	72.6	76.4	76.6
40	61.5	64.2	60.7	65.5	65.9
30	51.0	54.0	50.4	55.5	55.8
20	37.9	40.2	37.0	41.4	41.7
10	26.8	28.2	25.3	28.8	29.2
5	13.0	14.7	12.0	15.5	15.9
0	0.0	0.0	0.0	0.0	0.0

The yellows studied here did not show much difference in the dot gains, but other yellows might show differences in the gains. That there is close agreement between colorimetric and densitometric dot areas is not surprising when one realizes that in determining the "density of Y, M, and C", one is actually determining the density of the color complementary to Y, M, and C. The density of these complementary colors (B, G, R), in effect, correlates to the amount of the white component in Y, M, and C. The problem, however, with densitometry, as already discussed, is that filtration responses cannot be matched exactly to only the complementary color for all Y, M, and C colorants used, and throughout the total gradation of these colorants, or to other colorants in general.

As an example of the impact of the illuminant on selecting the appropriate normalized tristimulus values, let's consider the same magenta in Tables 2 and 4 for estimating whether a change from D50 to incandescent illumination might affect the perceived visual dot area and gain. One might expect some noticeable effect in such a change in illuminants because of the red-blue components comprising magenta colors and the relatively very small amount of blue region radiance in incandescent light. Tristimulus data analogous to that in Table 4 were re-computed from the reflectance spectra of the magenta tint areas, based on the 2° Standard Observer and Illuminant A, and are given in Table 5.

In Table 5, the smallest white-normalized tristimulus values for this magenta under incandescent light are the Z' values, not the Y' values in Table 2. This change suggests that the lack of blue wavelengths in incandescent light tends to diminish the perception of the blue component of the magenta, causing a change in perceived hue (an increase of ca. 10° in h_{ab}), with a consequential change not only in the magnitude of the white component, but also which white-normalized tristimulus value represents the white component. A second characteristic in Table 5 is that for the 5% and 10% dot areas, the smallest white-normalized tristimulus value changes from Z' to Y', likely due to the dominance of the paper. Colorimetric dot area for these 5% and 10% dot areas should, therefore, be based on $W_{s/n}$ for Y'. The densitometric dot areas from Table 4 are included in Table 6 for ease of comparison. While the colorimetric dot areas from the 20% to 60% tint areas in Table 4 were ca. 1% less than the densitometric based dot areas, the colorimetric dot areas in Table 6 for these same tint areas have become about 3% to 4% less than the densitometric dot areas. A comparison of colorimetric dot gains based on illuminants D50, D65, and A with Status T and DIN 16536 narrow band densitometric based dot gains is shown in Figure 1. In Figure 1, the gains for D50 and D65 are so close they overlap, but for illuminant A, the gain is considerably less than the other gains.

The reflectance spectra for this magenta at selected dot areas are shown in Figure 2, where the Status T response is superimposed. From Figure 2, one can see that as area of the magenta increases, the blue and red regions change disproportionately, the red region becoming more dominant as the amount of magenta increases. This increasing ratio of the red/blue components is responsible for the significant hue shift toward red as magenta area increases. Although these spectra are for halftones, this same principle of a red hue shift with increasing density applies also for intaglio gravure printing because ink film thickness increases with density.

Table 5

Tristimulus Values (2°, A) and Tristimulus Values Normalized to Paper (X', Y', Z') for a 3M Matchprint™ II Commercial Magenta

% Film Area	X	Y	Z	X'	Y'	Z'
100	46.95	23.77	6.30	47.37	26.39	20.29
95	47.49	24.39	6.57	47.92	27.07	21.13
90	48.86	25.97	7.25	49.30	28.83	23.34
80	51.83	29.45	8.66	52.30	32.69	27.87
70	55.05	33.22	10.17	55.55	36.88	32.74
60	57.51	36.18	11.36	58.03	40.17	36.56
50	61.13	40.38	13.03	61.68	44.82	41.93
40	67.38	47.85	15.95	67.99	53.12	51.36
30	72.92	54.67	18.50	73.58	60.69	59.56
20	79.57	63.70	21.82	80.29	70.71	70.25
10	85.16	71.52	24.74	85.93	79.40	79.65
5	92.53	80.54	28.03	93.36	89.42	90.23
0	99.10	90.08	31.06	100.00	100.00	100.00

Table 6

Colorimetric (2°, A) and Densitometric (Status T, DIN 16536 NB) % Dot Areas for a 3M Matchprint™ II Commercial Magenta from Table 5

% Film Area	Area(X')	Area(Y')	Area(Z')	Area(T)	Area(NB)
100	100.0	100.0	100.0	100.0	100.0
95	99.0	99.1	98.9	99.2	99.2
90	96.3	96.7	96.2	96.9	97.0
80	90.6	91.4	90.5	92.0	92.1
70	84.5	85.7	84.4	86.6	86.7
60	79.7	81.3	79.6	82.3	82.4
50	72.8	75.0	72.9	76.4	76.6
40	60.8	63.7	61.0	65.5	65.9
30	50.2	53.4	50.7	55.5	55.8
20	37.5	39.8	37.3	41.4	41.7
10	26.7	28.0	25.5	28.8	29.2
5	12.6	14.4	12.3	15.5	15.9
0	0.0	0.0	0.0	0.0	0.0

One significant advantage of colorimetric area from (8) or (9) is that, unlike densitometric area, it is not limited to normal single primary colors (Y, M, C, K) but can estimate area for single colorant tone scales for any arbitrary color, such as the normal secondary hues (red, green, blue), or PANTONE®, COLORCURVE®, or Munsell® colors in general, for which appropriate densitometer filters seldom exist, because the method is based on the white component and is thus independent of the hue of the color. Also, if part of the colorant's significant reflectance region happens to cross the slopes of the spectral response of the filter (which often occurs with yellows and Status T response), a hue shift with tone gradation could cause one to misconstrue a densitometer reading for this hue shift as affecting the amount of colorant present, thereby possibly causing the calculated effective area to be in error. Similarly, this method can have utility over the Neugebauer method, because, being based on linear additivity of tristimulus values of the solid color, Neugebauer-calculated tones will have the same hue as the primary color. The Neugebauer method can have reduced accuracy where there is a significant hue shift with gradation (e.g., magentas).

Another advantage of this method over densitometry is its interpretation of bluish and cyan-like inks. The ISO/ANSI Status T and DIN 16536 cyan responses are compared to the 2° $\bar{x}$ response for D50 and A illuminants for a cyan in Figure 3. The DIN 16536 cyan response was multiplied by the relative power distribution of illuminant A in order to compare it with Status T, whose spectral product response has illuminant A incorporated in it. The CIE $\bar{x}$ cmf has a reasonable response in the blue region of the spectrum. Normally, densitometric area is based on only the red portion of the cyan spectrum and ignores the blue region. Also, the long wavelength portion of $\bar{x}$ is broader than the minimum reflectance region of the cyan. Thus, normal densities for cyan will be greater than for a $\bar{x}$-cmf-based response. Figure 3 shows also that a colorimetric response includes the effect of different illuminants. Colorimetric dot area will, therefore, correlate better with what a person visually perceives than will densitometric dot area. Also, colorimetric dot area can better achieve gray balance because the colorimetric effect of the blue region of the CIE $\bar{x}$ cmf and illuminant on perception is included.

A comparison of colorimetric and densitometric dot gains for a 3M Matchprint™ II Commercial proofing cyan is shown in Figure 4. As in Figure 1, the densitometric gains are the highest. However, for the colorimetric gains, the largest gain is for illuminant A, the smallest for D65, the reverse order of the magenta in Figure 1. This reversal of gain magnitude for different illuminants could cause the visual color balance to change when viewed under these different illuminants. Thus, colorimetric gain can predict possible impact of illuminant on gain and color balance; whereas, densiometric gain might not. For some colorants, the appropriate W can change with area for certain illuminants (e.g., magenta & A). Plots of the appropriate W vs area were usually found to be essentially linear where only optical gain dominates but to be "kinked" where physical gain is significant (e.g., ink), which can be used to infer about physical gain.

Summary

The method herein for determining dot area colorimetrically differs substantially from modified Neugebauer-based methods, and from the Neugebauer approach in general in that they involve all three CIE XYZ values in solving multiple equations simultaneously. This new method demonstrates the use of tristimulus values normalized to the substrate as the reference white to determine the white component present and to use the white component to indirectly determine the effective area of the colorant. This method is not concerned with the hue of the colorant but rather with determining how much relative *area* of the pixel element's white substrate is occupied by a colorant, or, from another perspective, simply is not white. Thus, any binary halftone-like or dithered tint of a colorant can be considered to comprise only two areas: (1) the substrate ("white") and (2) the colorant ("not-white"). Since the sum of their relative percent areas at any gradation equals 100, the effective, relative area of the colorant can be determined from a metric of the relative amount of the substrate ("white"), regardless of the hue of the colorant.

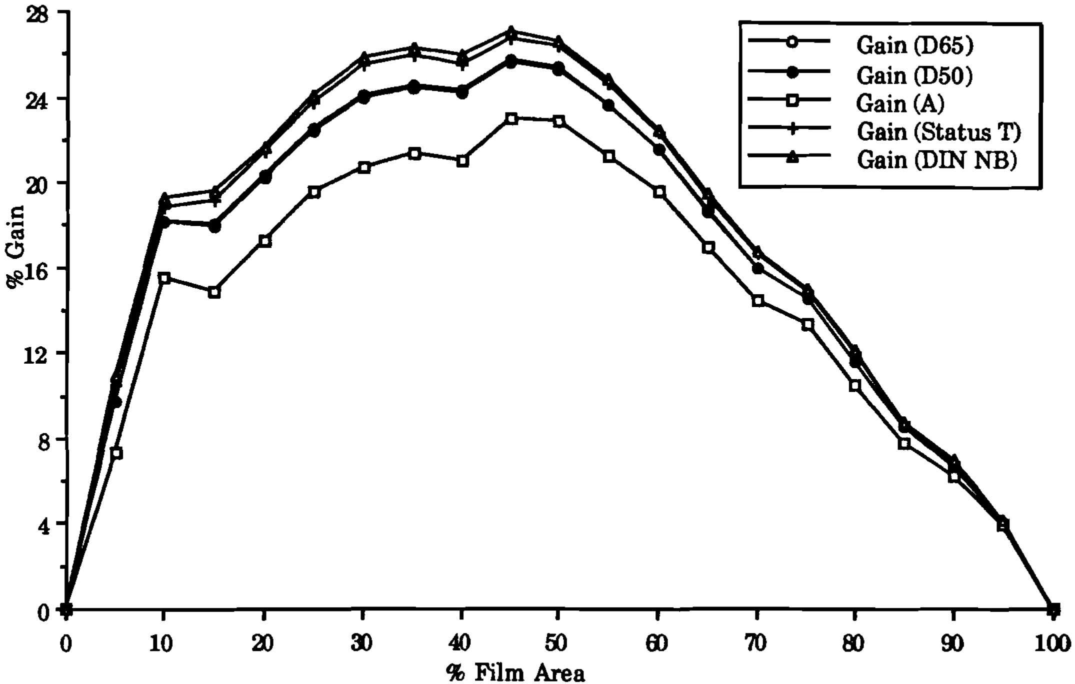

Figure 1. Colorimetric and densitometric % dot gains for a 3M Matchprint™ II magenta color.

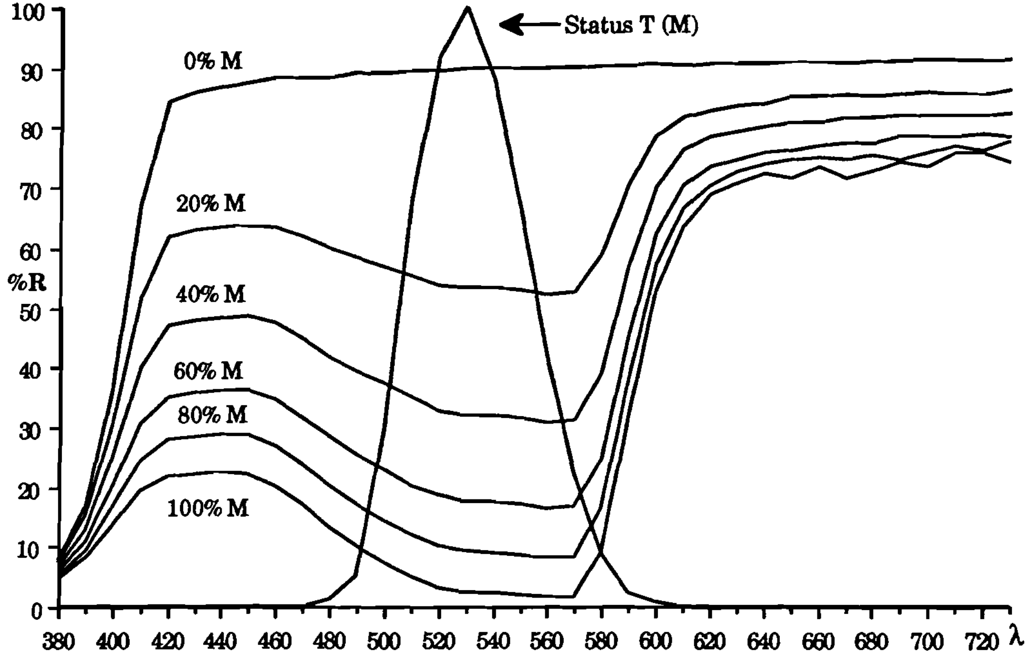

Figure 2. Reflectance spectra of % dot areas of a 3M Matchprint™ II magenta and Status T response.

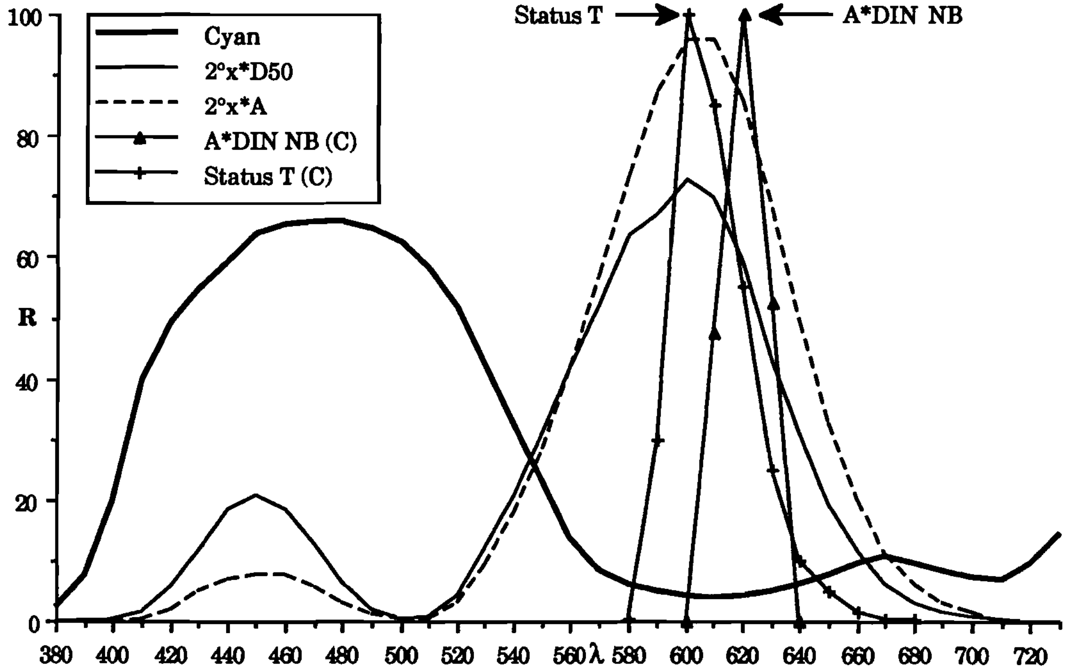

Figure 3. Colorimetric and densitometric responses for a cyan color.

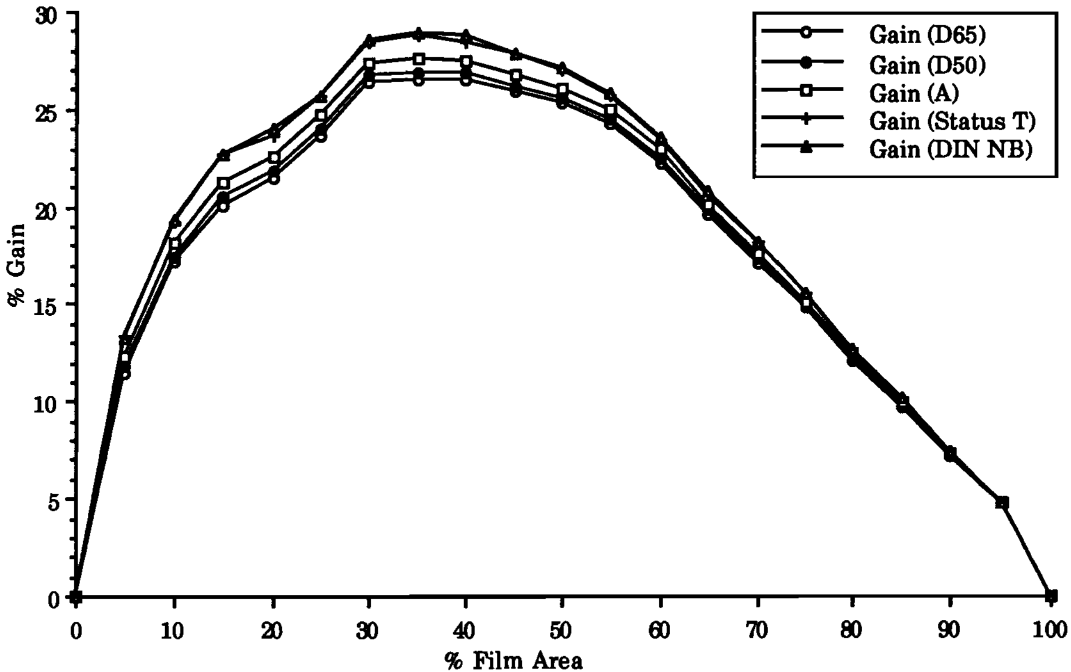

Figure 4. Colorimetric and densitometric % dot gains for a 3M Matchprint™ II cyan color.

Literature Cited

1. J. A. C. Yule, W. J. Nielsen, "The Penetration of Light into Paper and Its Effect on Halftone Reproduction", TAGA, pp.65–76 (1951).

2. F. R. Clapper, J. A. C. Yule, "The Effect of Multiple Internal Reflections on the Densities of Halftone Prints on Paper", *J. Opt. Soc. Am.*, **43**, pp.600–603 (1953).

3. J. R. Huntsman, "A New Model of Dot Gain and Its Application to a Multilayer Color Proof", TAGA, pp.83–108 (1987). Also in *J. Imag. Tech.*, **13**, 5, pp.136–145 (Oct. 1987).

4. A. Murray, "Monochrome Reproduction in Photoengraving", *J. Franklin Inst.*, **221**, pp.721–744 (1936).

5. J. R. Huntsman, "Colorimetric Analysis Methodology for Graphic Arts", SPSE 6th Int. Cong. Adv. Non-Impact Printing Technol., (Oct. 1990).

6. *Colorimetry*, 2nd Ed., Publ. No. 15.2, 1986, CIE, Vienna, Austria.

7. R. W. G. Hunt, *Measuring Colour*, Halsted Press, 1987.

8. H. E. J. Neugebauer, "Die Theoretischen Grundlagen des Mehr Farbendruckes", *Z. Wiss. Photo.*, **36**, pp.73–89 (1937).

9. A. C. Hardy, F. L. Wurtzburg, Jr., "The Theory of Three-Color Reproduction", *J. Opt. Soc. Am.*, **27**, 7, pp.227–240 (July 1937).

10. A. C. Hardy, F. L. Wurtzburg, Jr., "Color Correction in Color Printing", *ibid.*, **38**, 4, pp.300–307 (April 1948).

11. J. A. C. Yule, R. Colt, "Colorimetric Investigations in Multicolor Printing", TAGA, pp.77–82 (1951).

12. F. R. Clapper, J. A. C. Yule, "Reproduction of Color With Halftone Images", *ibid.*, pp.1–14 (1955).

13. V. L. Marquart, J. S. Rydz, "The Application of the Neugebauer Equations to Electronic Color Correction", *ibid.*, pp.15–27.

14. M. L. Pearson, "The Use of the Color Gamut of an Ink Set in the Selection of Process Inks", *ibid.*, pp.59–75 (1964).

15. I. Pobboravsky, M. Pearson, "Computation of Dot Areas Required to Match a Colorimetrically Specified Color Using the Modified Neugebauer Equations", *ibid.*, pp.65–77 (1972).

16. P. H. Chappuis, "A Modified Factorization of the Neugebauer Equations to Compute Accurate Mask Characteristics for Neutral Balance", *ibid.*, pp.256–281 (1977).

17. J. A. S. Viggiano, "The Color of Halftone Tints", *ibid.*, pp.647–661 (1985).

18. J. A. S. Viggiano, "Modeling the Color of Multi-Colored Halftones", *ibid.*, pp.44–62 (1990).

19. K. Sayanagi, M. Tamune, "Neugebauer Equations in Color Reproduction System", SPIE Neugebauer Memorial Seminar on Color Reproduction, SPIE Vol. 1184, pp.20–23 (Dec. 1989).

20. R. A. Holub, W. Kearsley, "Color to Colorant Conversions in a Colorimetric Separation System", *ibid.*, pp.24–35.

21. K. Kanamori, H. Kotera, "Analysis of Color Gamut for Hardcopy Based on Neugebauer Theory", *ibid.*, pp.36–42.

22. S. Honjo, S. Taguchi, T. Suzuki, "Dot Gain in Halftone Images", *ibid.*, pp.43–49.

23. J. R. Huntsman, "A Planar-Vector Based Color Space for Graphic Arts Color Analysis and Reproduction", *Color Res. Appl.*, **14**, 5, pp.240–259 (1989).

24. J. von Kries, in *Handbuch d. Physiol. Optik*, Vol. II, pp.366–369, Voss, Hamburg, 1911.