MODELING OF WEB TENSION RESPONSE AND ITS APPLICATION TO NEWSPAPER PRINTING

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Abstract: This paper discusses web tension response to changes in various press conditions from an analytic approach. The approach uses mathematical derivations to obtain a better understanding of some ambiguous observations made in the press room. The approach also provides a practical means to relate and therefore predict web tension response to changes in several press parameters. While the application was of a special concern for newspaper production, one may find it applicable to other press products.

The derivations start with a general, but ideal case in which web slip was excluded. The first result is the web continuity equation. Further derivations show how to predict web tension by relating the continuity equation to press conditions such as speed gain and speed change. The derivations continue with a discussion of how the web-roller interactions, including web slip, cause deviation of the tension from the level predicted in the ideal case. Also discussed is the effect of tension level change on cutoff registration, which provides a foundation of the low cost open-loop cutoff control system.

A set of field test data was included to illustrate some of the conclusions drawn from the approach.

INTRODUCTION

Web leads in newspaper presses are generally more flexible and longer than in other presses. A combination of higher press speed and thinner newsprint makes the newspaper printing operation more sensitive to web break than commercial printing. A successful press operation needs careful web tension control from the reel stand to the folder. The infeed may control web tension effectively between the paper roll and offset unit. But it has limited control capability after the offset unit. Normally the tension after the offset unit is manipulated by the outfeed tension control device and the folder. This mathematical model provides a means of predicting how web tension in various sections of the press respond to the manipulations often done by the press operators.

The basic mathematical model to describe web tension response in a web transport system is the web continuity equation. It may contain such variables as paper properties,

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press geometry, and press operation parameters depending on the conditions being investigated. While similar models have been discussed frequently before, they seldom were developed into a practical form useful in the field investigation. It is this paper's intention to emphasize the practical aspect of the model. This goal required finding the closed form solution of the continuity equation based on the practical conditions.

The general web tension model involves nonlinear equations. One may confront some difficulties in finding a solution in closed form from these general equations. However, by using some simplifying assumptions the solutions in closed form for some conditions are obtainable. Among them are the solutions for web tension response to outfeed tension control devices, velocity change, and the accompanied cutoff shifts in newspaper printing. Like most other theoretical approaches, this mathematical model needs to be verified with testing. APPENDIX B lists some of the Goss Colorliner's field test data to be compared with the computed data. Readers are encouraged to make their own comparisons between predicted and measured data relative to various parameters.

BASIC ASSUMPTIONS AND SYMBOLS

1. Basic assumptions:

   a. Paper is linear elastic material.
   b. Moisture content on the paper remains unchanged during its traveling from the last offset unit through the press.
   c. The infeed (or reel stand) is perfect so that no tension fluctuation due to continuous change of paper roll inertia is considered.

2. Symbols and units:

F	force; lb.
g	velocity gain
I	inertia; $lb.in^2$
L	linear span distance; in.
L_0	free web length in to a span L; in.
N	normal force; lb.
q	web flow amount equivalent to zero tension; in.
r	radius; in.
S	slip; in./sec.
T	tension; lb.
V	roll surface speed; in./sec.
δ	differential amount
ϵ	elongation rate; in./per in. long per lb. tension
θ	angle; rad.
μ	frictional coefficient
τ	time; sec.
ϕ	angle; rad.
ω	angular velocity; rad./sec.

FUNDAMENTAL MATH MODEL

In a web transport system a driven roller pulls the web with the frictional force. At steady state the web speed falls behind the driven roller's surface speed at a steady rate as the web slips over the roller. The slip may be minimized or prevented by maximizing friction through a nipping mechanism (as in the offset unit). Wrap angle also influences the amount of pull. Also, in newspaper presses slip is designed to exist in the folder (necessary for multi-web operation).

The web continuity equation describes the relation of web tension variation and web flow rate in a segment (span) of a press. To understand how slip affects tension, one should start with a study of the continuity equation and the application to a web transported by the driven rollers with tight nipping mechanisms (no slip) as the ideal case. Then the conclusion from the ideal case can be modified with the effects of slip and web/roller interaction.

1. Continuity Equation

Consider a traveling web in a span as shown in Fig. 1. In a simplest case the span is fixed (no dancer or compensator) and the slip at each nip is zero. The web in the span L_2 stretches an amount δ under the tension T_2.

$$\delta = T_2 \epsilon \quad ; \quad L_{02} = \frac{L_2}{1+\delta} = \frac{L_2}{1+T_2\epsilon} \tag{1}$$

The net flow rate in the span L_2 is

$$\frac{dL_{02}}{d\tau} = \frac{dq_1}{d\tau} - \frac{dq_2}{d\tau} = \dot{q}_1 - \dot{q}_2 \tag{2}$$

Since slip=0, web speed equals roller's surface speed,

$$\dot{q}_1 = \frac{V_1}{1+T_1\epsilon} \quad ; \quad \dot{q}_2 = \frac{V_2}{1+T_2\epsilon} \tag{3}$$

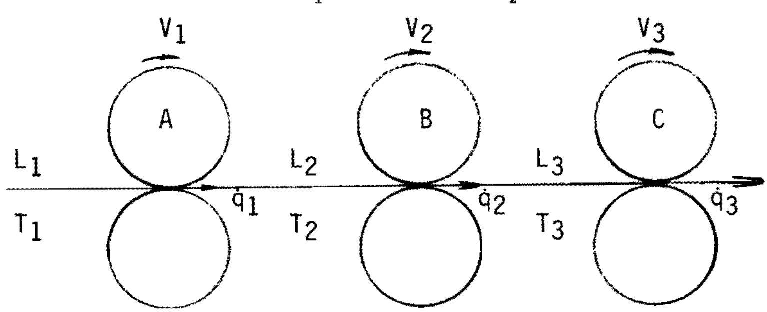

Figure 1. A web transported by driven rollers with high nip pressures (no slip).

From Eq. (1), (2) and (3) a simplest continuity equation is derived as:

$$\frac{dT_2}{d\tau}=(\dot{q}_2-\dot{q}_1)\ \frac{(1+T_2\epsilon)^2}{L_2\epsilon}=(\frac{V_2}{1+T_2\epsilon}-\frac{V_1}{1+T_1\epsilon})\ \frac{(1+T_2\epsilon)^2}{L_2\epsilon} \tag{4}$$

In the situation where the dancer mechanism or automatic compensator is built into the span, the span L_2 is no longer fixed. From Eq. (1),

$$\frac{dL_{o2}}{d\tau}=\frac{1}{(1+T_2\epsilon)^2}\left[(1+T_2\epsilon)\ \frac{dL_2}{d\tau}-L_2\epsilon\frac{dT_2}{d\tau}\right]=\dot{q}_1-\dot{q}_2 \tag{5}$$

This leads to the continuity equation for non-fixed span,

$$\frac{dT_2}{d\tau}=\left(\frac{V_2}{1+T_2\epsilon}-\frac{V_1}{1+T_1\epsilon}\right)\frac{(1+T_2\epsilon)^2}{L_2\epsilon}-\frac{1+T_2\epsilon}{L_2\epsilon}\ \frac{dL_2}{d\tau} \tag{6}$$

2. Solutions And Discussions

The continuity equations derived above are nonlinear differential equations. The followings are the examples showing how to predict web responses (such as tension and cutoff registration shift) to some conditions by solving the equations.

The complete solution of the equation for a span of interest requires information of upstream velocity and tension. Theoretically to know exact tension level and realtime tension variation of a mid-span, a series of equations from infeed performance through each span following the infeed needed to be solved. However the solution for a mid-span alone for a particular condition from the general continuity equation is obtainable with reasonable assumptions as stated previous.

a. Response To Speed Gain

Refer to Fig. 1. With an ideal infeed and constant press speed, V_1 and T_1 remain unchanged after a step change of speed gain g is imposed on the roller B (relative to the roller A). The continuity equation becomes

$$\frac{dT_2}{d\tau}=\left(\frac{V_2}{1+T_2\epsilon}-\dot{q}_1\right)\frac{(1+T_2\epsilon)^2}{L_2\epsilon}=\frac{V_2}{L_2\epsilon}\ (1+T_2\epsilon)-\dot{q}_1\frac{(1+T_2\epsilon)^2}{L_2\epsilon} \tag{7}$$

where $dq_1/d\tau = V_1/(1+T_1\mu) =$ constant.

The general solution is expressed as Eq. (8) (see APPENDIX A for detailed derivation), where C is the integral constant to be determined by initial conditions.

$$T_2=\frac{1}{\epsilon}\left[\frac{1}{\left(C-\frac{\dot{q}_1}{V_2}\right)e^{-\frac{V_2}{L_2}\tau}+\frac{\dot{q}_1}{V_2}}-1\right] \tag{8}$$

A uniform initial condition (g=0 for τ <= 0) leads to

$$T_{2,init.}=T_1 \quad ; \quad V_2=V_1 \quad for\ \tau\leq 0 \quad ; \quad V_2=V_1(1+g) \quad for\ \tau>0 \tag{9}$$

Therefore,

$$C-\frac{\dot{q}_1}{V_2}=\frac{1}{1+T_1\epsilon}-\frac{\dot{q}_1}{V_2} \quad ; \quad C=\frac{1}{1+T_1\epsilon} \tag{10}$$

$$\begin{aligned} T_2&=\frac{1}{\epsilon}\left[\frac{1}{\left(\frac{1}{1+T_1\epsilon}-\frac{\dot{q}_1}{V_2}\right)e^{-\frac{V_2}{L_2}\tau}+\frac{\dot{q}_1}{V_2}}-1\right]\\ &=\frac{1}{\epsilon}\left[\frac{1}{\frac{g}{1+T_1\epsilon}\frac{V_1}{V_2}e^{-\frac{V_2}{L_2}\tau}+\frac{\dot{q}_1}{V_2}}-1\right] \end{aligned} \tag{11}$$

As time passes, $\tau \gg L_2/V_2$, the tension will settle to a steady value.

$$\begin{aligned} T_2&\approx\frac{1}{\epsilon}\left(\frac{V_2}{\dot{q}_1}-1\right)=\frac{1}{\epsilon}\left[\frac{V_1(1+g)-\dot{q}_1}{\dot{q}_1}\right]\\ &=\frac{1}{\epsilon}\left[\frac{(1+g)(1+T_1\epsilon)\dot{q}_1-\dot{q}_1}{\dot{q}_1}\right]\\ &=\frac{1}{\epsilon}(T_1\epsilon+g+gT_1\epsilon)=T_1(1+g)+\frac{g}{\epsilon}\\ &\approx T_1+\frac{g}{\epsilon} \end{aligned} \tag{12}$$

The initial condition affects only the exponential term which becomes insignificant as $\tau \gg L_2/V_2$. This indicates that the steady state tension of a span on which a speed gain is imposed is equal to approximately the upstream tension plus a ratio of the relative speed gain and paper elongation rate. This result holds regardless of the initial tension level, speed, or how the gain is imposed (step or continuous).

Next is a discussion of the web tension response in the span following the one with the imposed step gain. Refer to Fig. 1 for the continuity equation for the span between roller B and C (L_3).

$$\frac{dT_3}{d\tau}=(\dot{q}_3-\dot{q}_2)\,\frac{(1+T_3\epsilon)^2}{L_3\epsilon}$$
$$=\left(\frac{V_3}{1+T_3\epsilon}-\frac{V_2}{1+T_2\epsilon}\right)\frac{(1+T_3\epsilon)^2}{L_3\epsilon} \quad (13)$$

By substituting T_2 with Eq. (11), Eq. 13 becomes

$$\frac{dT_3}{d\tau}=\frac{V_3(1+T_3\epsilon)}{L_3\epsilon}-\frac{V_2(1+T_3\epsilon)^2}{L_3\epsilon(1+T_2\epsilon)}$$
$$=\frac{V_3(1+T_3\epsilon)}{L_3\epsilon}-\frac{(1+T_3\epsilon)^2}{L_3\epsilon}\left[\dot{q}_1+\frac{gV_1}{1+T_1\epsilon}e^{-\frac{V_2}{L_2}\tau}\right] \quad (14)$$
$$=\frac{V_3(1+T_3\epsilon)}{L_3\epsilon}-\frac{1}{L_3\epsilon}(1+T_3\epsilon)^2\dot{q}_1\left(ge^{-\frac{V_2}{L_2}\tau}+1\right)$$

A general solution to this equation is (see APPENDIX A for detailed derivation):

$$T_3=\frac{1}{\epsilon}\left[\frac{1}{\dfrac{\dot{q}_1 g}{L_3\left(\dfrac{V_3}{L_3}-\dfrac{V_2}{L_2}\right)}e^{-\frac{V_2}{L_2}\tau}+\left(C-\dfrac{\dot{q}_1}{V_3}-\dfrac{g\dot{q}_1}{L_3\left(\dfrac{V_3}{L_3}-\dfrac{V_2}{L_2}\right)}\right)e^{-\frac{V_3}{L_3}\tau}+\dfrac{\dot{q}_1}{V_3}}-1\right] \quad (15)$$

where C is the constant of integration to be determined by the initial conditions. Since we are concerning the tension response of the span L_3 to a gain imposed on the span L_2, the previous initial conditions are still adoptable. We also set the initial tension of the span L_3 as $T_3=T_{3,init}$ for τ = 0, which leads to

$$C=\frac{1}{1+T_{3,init}\epsilon} \quad (16)$$

$$T_3=\frac{1}{\epsilon}\left[\frac{1}{\dfrac{\frac{\dot{q}_1}{L_3}g}{\frac{V_3}{L_3}-\frac{V_2}{L_2}}e^{-\frac{V_2}{L_2}\tau}+\left(\dfrac{1}{1+T_{3,init}\epsilon}-\dfrac{\frac{g\dot{q}_1}{L_3}}{\frac{V_3}{L_3}-\frac{V_2}{L_2}}-\dfrac{\dot{q}_1}{V_3}\right)e^{-\frac{V_3}{L_3}\tau}+\dfrac{\dot{q}_1}{V_3}}-1\right] \quad (17)$$

Eq. (17) represents the complete tension response of span L_3. The two exponential terms dominate the transient response which is determined by press geometry (L_2, L_3), paper elongation rate as well as running conditions (V_2, V_3, $dq_1/d\tau$, T_{3i} and g). The steady state arrives as $\tau \gg (L_2/V_2)$ and $\tau \gg (L_3/V_3)$, and the values of the two exponential terms diminish.

$$T_3 \approx \frac{1}{\epsilon}\left[\frac{1}{\frac{\dot{q}_1}{V_3}}-1\right]=\frac{1}{\epsilon}\left(\frac{V_3}{\dot{q}_1}-1\right)=T_{3,init} \quad (18)$$

Eq. (18) represents the steady state tension. Since it does not contain such parameters as L_2, L_3, speed gain, or initial tension, it is inferred that the steady state tension is not influenced by the gain imposed on the upstream spans but by the ratio of web speed of the span and the web flow rate released from the infeed. (Here a perfect infeed is assume; $dq_1/d\tau$ = constant.)

A set of field test data shown in Table 1 is used to illustrate conclusions from the above derivations. The tension measurement test was conducted on a Goss Colorliner. The listed gains are for a drag roller driven by a variable speed drive. Since the slip between the web and the roller exists and varies with speed, the gains in the Table are the roller's speed gain, not the real web speed gain. Notice that the tension before the drag roller vary with the gains while the tensions after the roller remain basically the same.

Speed	Gain	Tension: Before Drag Roller T_2	Tension: After Drag Roller T_3
30 kiph	3.5%	162 lb.	67 lb.
	3	164	69
	2	158	68
	1	151	67
	0.5	145	68
	0.25	129	62
	0.	123	64
	-0.25	123	64
	-0.5	101	64
50 k	3.5	194	62
	3	188	62
	2	178	62
	1	161	62
	0.5	153	62
	0.25	146	61
	0.	139	62
	-0.25	136	62
	-0.5	-	62
60 k	3.5	200	58
	3	192	60
	2	181	58
	1	165	58
	0.5	152	58
	0.25	147	59
	0.	135	59
	-0.25	138	60
	-0.5	99	59

Table 1. Field tension test data.

b. Tension Response To Velocity Change

As the press speeds up (or slows down), the driven rollers accelerate (or decelerate) and the web flow rate in each span varies accordingly. During accelerating,

$$V_n = V_{n,0} + a_n \tau \tag{19}$$

where $V_{n,0}$ is the driven roller's initial surface speed, a_n is the acceleration which is normally a constant. Referring Fig. 1 and Eq. (7), the tension response in L_2 is obtained by replacing V_1 and V_2 with EQ. (19).

$$\begin{aligned} \frac{dT_2}{d\tau} &= \left[\frac{V_2}{1+T_2\epsilon} - \frac{V_1}{1+T_1\epsilon}\right]\frac{(1+T_2\epsilon)^2}{L_2\epsilon} \\ &= \left[\frac{V_{2,0}+a_2\tau}{1+T_2\epsilon} - \frac{V_{1,0}+a_1\tau}{1+T_1\epsilon}\right]\frac{(1+T_2\epsilon)^2}{L_2\epsilon} \end{aligned} \tag{20}$$

The solution is (Ref. 1)

$$T_2 = \frac{1}{\epsilon}\left[\frac{1}{e^{-\frac{V_{2,0}\tau + a_2\tau^2/2}{L_2}}\left[\int e^{\frac{V_{2,0}\tau + a_2\tau^2/2}{L_2}}\left[\frac{V_{1,0}+a_1\tau}{L_2(1+T_1\epsilon)}\right]d\tau + C\right]} - 1\right] \tag{21}$$

With an ideal infeed able to hold the upstream tension very steadily, $T_1(\tau) \approx$ constant. Given a uniform initial velocity (no gain at point B) and uniform velocity change rate, $V_{2,0}=V_{1,0}$; $a_2=a_1$; $T_{2,init}=T_1$. Adding these conditions into Eq. (21) leads to

$$T_2(\tau) = T_{2,init} = T_1 = constant \tag{22}$$

This becomes a non-uniform condition for the initial state in which there is a speed gain g at point B. To maintain the same gain at B during acceleration the speed at B is

$$\begin{aligned} V_2(\tau) &= V_{2,0} + a_2\tau = (1+g)\,V_1(\tau) = (1+g)\,[V_{1,0} + a_1\tau] \\ &= (1+g)\,V_{1,0} + (1+g)\,a_1\tau \end{aligned} \tag{23}$$

With Eq. (23), Eq. (21) is further simplified as

$$T_2(\tau)=\frac{1}{\epsilon}\left[\frac{1}{\frac{1}{(1+T_1\epsilon)(1+g)}(1-e^{-\frac{V_{2,0}\tau+a_2\tau^2/2}{L_2}})+\frac{1}{1+T_{2,init}\epsilon}e^{-\frac{V_{2,0}\tau+a_2\tau^2/2}{L_2}}}-1\right] \quad (24)$$

For steady state (during speed change) as the exponential terms diminish,

$$\begin{aligned}T_2(\tau)&=T_1+\frac{g}{\epsilon}+gT_1\\&\approx T_{2,init}+gT_1\end{aligned} \quad (25)$$

Eq. (22) and (25) suggest that the web tension response to speed change should be insignificant unless under conditions in which either a large number of idle rollers are involved, driven roller with web slip is included, or infeed sensitive to speed changes.

c. Cutoff Registration

In most cases a change of tension level within a span of traveling web is accompanied by cutoff registration shift. Depending on the causes of the tension change, various parameters are involved in determining the shift amount. The following shows an example of cutoff shift due to a step gain.

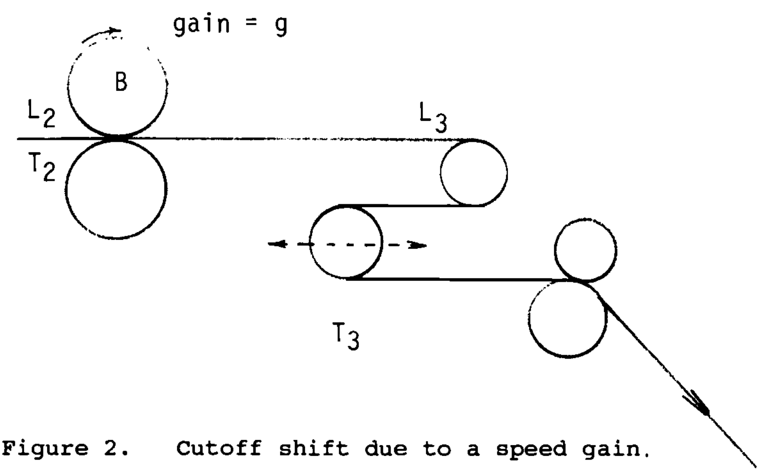

Figure 2. Cutoff shift due to a speed gain.

In Fig. 2, a step gain imposed on roller B stretches the web and increases the tension level in span L_2. The result is lower tension in span L_3 and higher web flow rate passing through the folder during the transient period. As steady state approaches the printed image shifts towards lead edge. The compensator can be used to adjust the image back to original position (relative to the lead edge). To determine the shift amount, imagine an automated compensator built in the span L_3. This compensator automatically makes adjustments so that the tension in the span remain unchanged during the transient period. Then the adjusted amount equals the shift amount.

Recall Eq. (7) for the continuity equation for span L_3 with compensator,

$$\frac{dT_3}{d\tau}=(\frac{V_3}{1+T_3\epsilon}-\frac{V_2}{1+T_2\epsilon})\frac{(1+T_3\epsilon)^2}{L_3\epsilon}-\frac{1+T_3\epsilon}{L_3\epsilon}\frac{dL_3}{d\tau} \tag{26}$$

Since tension in this span would remain unchanged,

$$\frac{1+T_3\epsilon}{L_3\epsilon}\frac{dL_3}{d\tau}=\left(\frac{V_3}{1+T_3\epsilon}-\frac{V_2}{1+T_2\epsilon}\right)\frac{(1+T_3\epsilon)^2}{L_3\epsilon} \tag{27}$$

$$\therefore \quad \frac{dL_3}{d\tau}=\left(\frac{V_3}{1+T_3\epsilon}-\frac{V_2}{1+T_2\epsilon}\right)(1+T_3\epsilon) \tag{28}$$

Substitute T_2 with Eq. (17) into Eq. (28),

$$\begin{aligned}\frac{dL_3}{d\tau}&=V_3-V_2\left(\frac{\dot{q}_1}{V_2}+\frac{V_1}{V_2}\frac{g}{1+T_1\epsilon}e^{-\frac{V_2}{L_2}\tau}\right)(1+T_3\epsilon)\\&=V_3-\left(\dot{q}_1+\dot{q}_1 g e^{-\frac{V_2}{L_2}\tau}\right)(1+T_3\epsilon)\end{aligned} \tag{29}$$

The solution is

$$\Delta L=L_3(\tau)-L_3(0)=\frac{g}{1+g}\frac{L_2}{1+T_1\epsilon}\left(e^{-\frac{V_2}{L_2}\tau}-1\right) \tag{30}$$

The cutoff shift amount is expressed as

$$\Delta L=-\frac{g}{1+g}\frac{L_2}{1+T_1\epsilon}\approx -gL_2 \tag{31}$$

The negative sign means the image shifts away from trailing edge. A negative gain at L_2 would shift the image towards trailing edge.

d. Transient Response

The web transient responses to changes of various conditions are expressed as the equivalent complete solutions of continuity equations. For example, Eq. (11) and (21) are the transient response to step gain and speed change respectively. Unlike steady state, transient responses are dominated by speed and press dimensions.

It is commonly understood that a disturbance (such as gain) imposed onto a higher speed level would cause a faster (shorter) transient response. However, the speed alone can't be a clear indication of transient duration. It is the ratio of speed and span (V_3/L_3 or V_2/L_2, or both) which determines the transient period. See APPENDIX B for the examples of transient responses of tension (T_2) with speed gain imposed and the downstream tension (T_3).

WEB/ROLLER INTERACTION

The above derivations do not consider the interaction between web and roller. To be closer to reality the considerations in this section should be added into previous discussions.

1. Idle Roller

Very often the web is led through the idle roller between the nips or driven rollers as shown in Fig. 3. The idler is actually driven by the difference between T_1 and T_2. When the web speed reaches a steady state (so does the idler) the required (T_1-T_2) is only an amount to overcome idler's frictional torque.

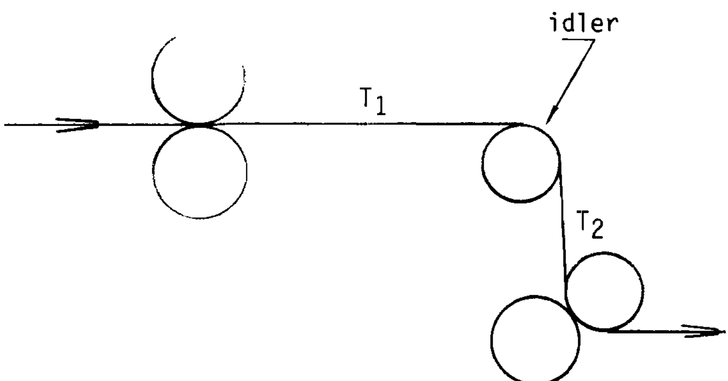

Figure 3. Web led through an idler.

Fig. 4 shows the free body diagrams for the roller and web. For this diagram the Capstan equation states that

$$\frac{T_2}{T_1} = e^{\mu\theta} \tag{32}$$

here μ is a constant for a given steady condition. For the roller,

Figure 4. Free body diagram for the roller and web.

$$\delta(drive\ torque) = r(dF) = r(dT) = r(\mu dN) = r(Td\theta)\mu$$
$$\therefore\ (drive\ torque) = \int_0^\theta r dF = \int_0^\theta r(Td\phi)\mu = r\mu\int_0^\theta T_1 e^{\mu\phi} d\phi \tag{33}$$
$$= r\mu\frac{T_1}{\mu}e^{\mu\phi}\Big|_0^\theta = r(T_2 - T_1)$$

where ϕ is introduced as the integrand. From the force balance for the roller,

$$(drive\ torque - frictional\ torque) = r(T_2 - T_1) - (frictional\ torqu$$
$$= I\frac{d\omega}{d\tau} \tag{34}$$

The main sources of the idler's frictional torque are the supporting components' (bearings) rotational friction, which normally varies with speed, among other parameters. To simplified the expression, it is assumed the frictional torque is a function of speed f(ω) (Ref. 2).

$$T_2 = T_1 + \frac{I\dot{\omega} + f(\omega)}{r} \tag{35}$$

$$\mu = \frac{1}{\theta}\ln\left[1 + \frac{I\dot{\omega} + f(\omega)}{T_1 r}\right] \tag{36}$$

When the web and idler are running at a steady speed and the slip between them is negligible Eq. (35) and (36) are further simplified as

$$T_2 = T_1 + \frac{f(\omega)}{r} \quad ; \quad \mu = \frac{1}{\theta}\ln\left[1 + \frac{f(\omega)}{T_1 r}\right] \tag{37}$$

It is understood that T_1 is still governed by the continuity equation. T_2 is higher than T_1 to overcome the frictional effect and is speed related. During a speed change part of the difference between T_1 and T_2 will compensate for the idler's inertia, which increases the difference in accelerating (or decreases in decelerating). The slip between web and idler may become obvious.

2. Web Slip At Steady State

The slip occurs when the nip pressure is low. In the sense that a slip changes web tension, it can be regarded as a gain subject to the balance of tensions T_2, T_3, and friction as shown in Fig. 5.

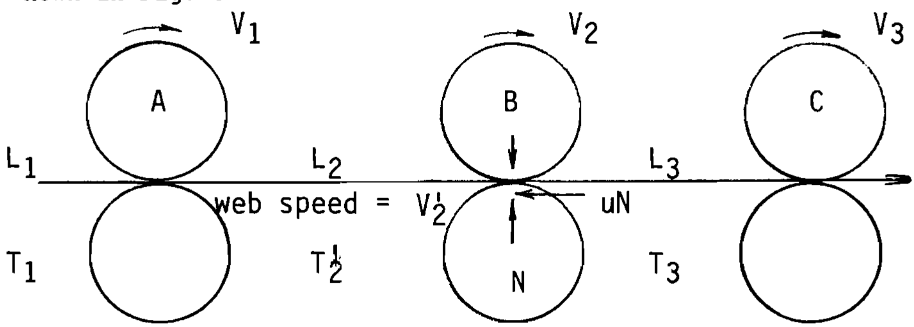

Figure 5. Web transported by driven rollers with low nip pressure.

When $T_3 < T_2 - \mu N$, slip occurs until it reaches a new balance.

$$T_3 = T_2' + \mu N \tag{38}$$

where T_2' is the new balanced tension. In the steady state the web slips continuously to maintain the balance and the gain at point B varies accordingly.

$$\begin{gathered} \textit{high nip pressure (no slip)} \qquad T_2 = T_1 + \frac{g}{\epsilon} \\ \textit{low nip pressure (slip)} \qquad T_2' = T_1 + \frac{g'}{\epsilon} \\ S = V_2 - V_2' = V_1(1+g) - V_1(1+g') = V_1(g-g') \\ T_2 - T_2' = \frac{(g-g')}{\epsilon} = \frac{1}{\epsilon}\frac{S}{V_1} \end{gathered} \tag{39}$$

where g' is the real gain for web, V_2' the web speed.

The difference in web speed V_2 without slip and V_2' with slip, and equivalently in real gain g and g', causes a difference in predicted tension T_2 (Eq. 12) and real tension T_2'. However, based on Eq. (12), when tension measurement data are available the real gain and slip can be estimated.

3. Frictional Coefficient

The Capstan equation is valid only for the steady state in which μ is a constant. P. Kornmann reported that μ is not a constant in the sense of Coulomb's law of friction as the results of several experimental investigations (Ref. 3). For the web transport system μ may be formulated in terms of tension level and web speed for each steady state. Insert Eq. (12) and (18) into Capstan equation,

$$e^{\mu\theta}=\frac{T_2'}{T_3}=\frac{T_1+\frac{g'}{\epsilon}}{\frac{1}{\epsilon}\left(\frac{V_3}{q_1}-1\right)}=\frac{(T_1\epsilon+g')\,V_1}{V_3\,(1+T_1\epsilon)-V_1} \tag{40}$$

The steady state dynamic frictional coefficient is

$$\mu=\frac{1}{\theta}\ln\left[\frac{(T_1\epsilon+g')\,V_1}{V_3\,(1+T_1\epsilon)-V_1}\right]=\frac{1}{\theta}\ln\left[\frac{T_2'\epsilon V_1}{V_3\,(1+T_1\epsilon)-V_1}\right] \tag{41}$$

The above two equations indicated that the frictional force between the web and the roller varies with the web speed even if the nipping force between both remain the same. It is inferred that the slip (when allowed by low nip pressure) and therefore the real gain are speed dependent. The evidence can be found from Table 1 in which the same drag roller's gain at various speeds causes various tension levels (due to various real gains).

CONCLUSION

The presented math model provides clear explanations to what have been observed regarding web tension and web/roller friction in the press room. Supported by field test data, the continuity equation is used to relate the web tension variations to various press parameters. The results show that:

* A change of gain causes a change of tension level in the span where the gain is imposed, the steady state tension downstream remain unchanged.
* A change of tension level in a span causes a shift in cutoff registration. The length of that span has a dominant effect on the amount of the shift.
* The web/roller dynamic frictional coefficient varies with speed, gain, and tension level.
* The slip between a roller and the web can be estimated by measuring the web tension.

The web was treated as a linear elastic material. In reality the elongation rate varies with the tension level and moisture content (Ref. 4). The elongation rate is to be treated as a variable in additional studies to investigate the attenuation caused by offset units. Further studies are also intended that will include the interaction between webs in dynamic multi-web systems.

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APPENDIX

A. Solution of the Continuity Equation

1. The General Solution of Eq. 7 - For Step Speed Gain Imposed Span L_2

Repeat the Eq. 7 here.

$$\begin{aligned}\frac{dT_2}{d\tau} &= (\dot{q}_2-\dot{q}_1)\,\frac{(1+T_2\epsilon)^2}{L_2\epsilon}\\ &= \left(\frac{V_2}{1+T_2\epsilon}-\frac{V_1}{1+T_1\epsilon}\right)\frac{(1+T_2\epsilon)^2}{L_2\epsilon}\\ &= \frac{V_2}{L_2\epsilon}(1+T_2\epsilon)-\frac{V_1}{1+T_1\epsilon}\,\frac{1}{L_2\epsilon}(1+T_2\epsilon)^2\end{aligned} \tag{A.1}$$

Let $U^{-1}=1+T_2\epsilon$,

$$\frac{dU}{d\tau}=\frac{d}{d\tau}\left(\frac{1}{1+T_2\epsilon}\right)=\frac{-\epsilon\frac{dT_2}{d\tau}}{(1+T_2\epsilon)^2} \tag{A.2}$$

$$\therefore\quad \frac{dT_2}{d\tau}=-\frac{1}{\epsilon}(1+T_2\epsilon)^2\frac{dU}{d\tau}=-\frac{1}{\epsilon}U^{-2}\frac{dU}{d\tau} \tag{A.3}$$

By combining Eq. (A.1) and (A.3),

$$-\frac{1}{\epsilon}U^{-2}\frac{dU}{d\tau}=\frac{V_2}{L_2\epsilon}U^{-1}-\left(\frac{V_1}{1+T_1\epsilon}\right)\frac{1}{L_2\epsilon}U^{-2} \tag{A.4}$$

By eliminating the common factor U^{-2}/ϵ, Eq. (A.4) is simplified as

$$\Rightarrow \quad -\frac{1}{\epsilon}\frac{dU}{d\tau}=\frac{V_2}{L_2\epsilon}U-\frac{\dot{q}_1}{L_2\epsilon}$$
$$\Rightarrow \quad \frac{dU}{d\tau}+\frac{V_2}{L_2}U=\frac{\dot{q}_1}{L_2} \tag{A.5}$$

Eq. (A.5) becomes a first order linear differential equation of U. The solution for U is

$$\begin{aligned} U &= e^{-\int\frac{V_2}{L_2}d\tau}\left[\int e^{\int\frac{V_2}{L_2}d\tau}\left(\frac{\dot{q}_1}{L_2}\right)d\tau+C\right] \\ &= e^{-\frac{V_2}{L_2}\tau}\left[\int e^{\frac{V_2}{L_2}\tau}\left(\frac{\dot{q}_1}{L_2}\right)d\tau+C\right] \\ &= e^{-\frac{V_2}{L_2}\tau}\left[\frac{\dot{q}_1}{V_2}e^{\frac{V_2}{L_2}\tau}-\frac{\dot{q}_1}{L_2}+C\right] \end{aligned} \tag{A.6}$$

$$\Rightarrow \quad U=\frac{\dot{q}_1}{V_2}-\frac{\dot{q}_1}{V_2}e^{-\frac{V_2}{L_2}\tau}+Ce^{-\frac{V_2}{L_2}\tau}$$
$$\Rightarrow \quad \frac{1}{1+T_2\epsilon}=\frac{\dot{q}_1}{V_2}-\left(\frac{\dot{q}_1}{V_2}-C\right)e^{-\frac{V_2}{L_2}\tau} \tag{A.7}$$

The above derivation leads to the general solution:

$$T_2=\frac{1}{\epsilon}\left[\frac{1}{\frac{\dot{q}_1}{V_2}-\left(\frac{\dot{q}_1}{V_2}-C\right)e^{-\frac{V_2}{L_2}\tau}}-1\right] \tag{A.8}$$

2. The General Solution of Eq. (14) - For The Downstream Span (L_3)

Repeat Eq. (14) here.

$$\begin{aligned} \frac{dT_3}{d\tau} &= \frac{V_3(1+T_3\epsilon)}{L_3\epsilon}-\frac{V_2(1+T_3\epsilon)^2}{L_3\epsilon(1+T_2\epsilon)} \\ &= \frac{V_3(1+T_3\epsilon)}{L_3\epsilon}-\frac{(1+T_3\epsilon)^2}{L_3\epsilon}\left[\dot{q}_1+\frac{gV_1}{1+T_1\epsilon}e^{-\frac{V_2}{L_2}\tau}\right] \\ &= \frac{V_3(1+T_3\epsilon)}{L_3\epsilon}-\frac{1}{L_3\epsilon}(1+T_3\epsilon)^2\dot{q}_1\left(ge^{-\frac{V_2}{L_2}\tau}+1\right) \end{aligned} \tag{A.9}$$

Let $U^{-1}=1+T_3\epsilon$,

$$\frac{dU}{d\tau}=\frac{d}{d\tau}\left(\frac{1}{1+T_3\epsilon}\right)=\frac{-\epsilon\frac{dT_3}{d\tau}}{(1+T_3\epsilon)^2} \tag{A.10}$$

$$\therefore\quad \frac{dT_3}{d\tau}=-\frac{1}{\epsilon}(1+T_3\epsilon)^2\frac{dU}{d\tau}=-\frac{1}{\epsilon}U^{-2}\frac{dU}{d\tau} \tag{A.11}$$

By combining Eq. (A.9) and (A.11),

$$-\frac{1}{\epsilon}U^{-2}\frac{dU}{d\tau}=\frac{V_3}{L_3\epsilon}U^{-1}-\frac{\dot{q}_1+g\dot{q}_1e^{-\frac{V_2}{L_2}\tau}}{L_3}\frac{U^{-2}}{\epsilon} \tag{A.12}$$

By eliminating the common factor U^{-2}/ϵ, Eq. (A.12) is simplified to a first order differential equation in U.

$$\frac{dU}{d\tau}+\frac{V_3}{L_3}U=\frac{\dot{q}_1+g\dot{q}_1e^{-\frac{V_2}{L_2}\tau}}{L_3} \tag{A.13}$$

The solution for U is

$$\begin{aligned}U&=e^{-\int\frac{V_3}{L_3}d\tau}\left[\int e^{\int\frac{V_3}{L_3}d\tau}\left(\frac{\dot{q}_1+g\dot{q}_1e^{-\frac{V_2}{L_2}\tau}}{L_3}\right)d\tau+C\right]\\&=e^{-\frac{V_3}{L_3}\tau}\left[\frac{\dot{q}_1}{L_3}\int e^{\frac{V_3}{L_3}\tau}d\tau+\frac{g\dot{q}_1}{L_3}\int e^{\left(\frac{V_3}{L_3}-\frac{V_2}{L_2}\right)\tau}d\tau+C\right]\\&=\frac{\dot{q}_1}{V_3}+\left(\frac{\dot{q}_1}{V_3}+\frac{g\dot{q}_1}{L_3\left(\frac{V_3}{L_3}-\frac{V_2}{L_2}\right)}+C\right)e^{-\frac{V_3}{L_3}\tau}+\frac{g\dot{q}_1}{L_3\left(\frac{V_3}{L_3}-\frac{V_2}{L_2}\right)}e^{-\frac{V_2}{L_2}\tau}\end{aligned} \tag{A.14}$$

It is concluded that

$$T_3=\frac{1}{\epsilon}\left[\frac{1}{\frac{\dot{q}_1g}{L_3\left(\frac{V_3}{L_3}-\frac{V_2}{L_2}\right)}e^{-\frac{V_2}{L_2}\tau}+\left(C-\frac{\dot{q}_1}{V_3}-\frac{g\dot{q}_1}{L_3\left(\frac{V_3}{L_3}-\frac{V_2}{L_2}\right)}\right)e^{-\frac{V_3}{L_3}\tau}+\frac{\dot{q}_1}{V_3}}-1\right] \tag{A.15}$$

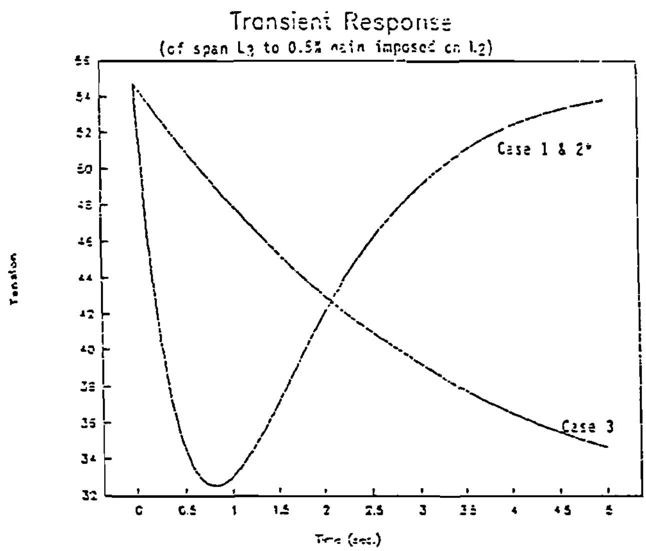
B. The Computed Transient Responses

Eq. 11 and 17 illustrate the complete web tension response to a step speed gain for the spans L_2 and L_3 respectively. The transient response is simply the early part of the complete response. Figure A.1, A.2, and A.3 compare the transient response of T_3 to a speed gain of 0.5% imposed on a upstream span (L_2) for various combinations of speed and span. As mention in the text, the duration of transient period is not determined by the speed alone, but by the ratio of the speed and span.

In Figure A.1 the responses for Case 1 and 2 are identical and therefore overlap as both cases have the same ratio of speed/span (300/200 & 30/20).

Figure A.2 compares the responses for L_3= 20, 200, and 2000' while V and L_2 remain the same.

Figure A.3 shows the differences in the transient period for three cases: L_2=30, L_3=20 for Case 1; 300, 200 respectively for Case 2; 3000 and 2000 respectively for Case 3 as the speed remains the same in three cases.



* Curves for Case 1 and 2 are overlapped.

Figure A.1 Case 1 = L_2 = 300, L_3 = 200, V = 300
Case 2 = L_2 = 30, L_3 = 20, V = 30
Case 3 = L_2 = 3000, L_3 = 2000, V = 300

All units are inch and second.

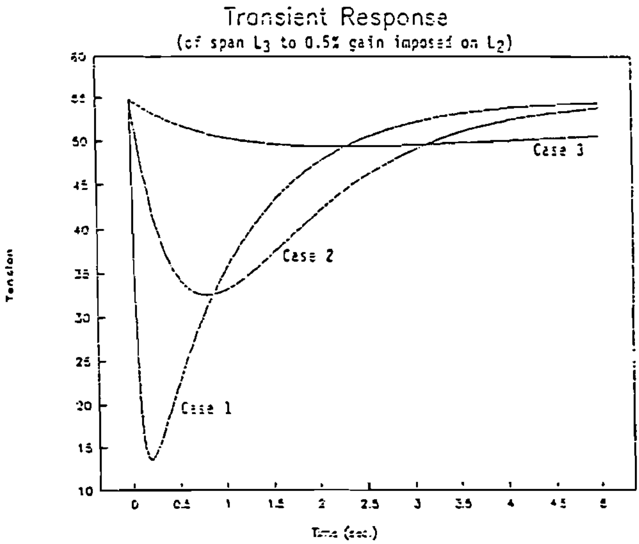


Figure A.2 L_2 = 300, V = 300 for all 3 cases
L_3 = 20, 200, and 2000 for Case 1, 2 and 3 respectively.

Figure A.3

V = 300 for all 3 cases.

Case 1:
L_2 = 30, L_3 = 20

Case 2:
L_2 = 300, L_3 = 200

Case 3:
L_2 = 3000, L_3 = 2000