FRACTAL ANALYSIS ON SELECTED MATERIAL SURFACES

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Abstract:

The collapse of concepts such as "length of a curve" or "area of a surface" in cases where the object is irregular over many scales of length leads to the concept of dimension to nonintegral values. Findings show that by using the technique of Fractal Analysis the fissured peripheral structures of several "chaotic objects" in offset printing such as peripheries of halftone dots or surface properties of printing plates can be characterized quantitatively by means of the term "fractional dimension".

It is the purpose of this paper to introduce the concept of fractal geometry as a means for the study and quantification of the inking capabilities of printing plate surfaces without subjecting them to a printing operation.

More pointedly, this paper teaches the use of unsophisticated means, using fractal dimensions, to study and quantify how the dry edge behavior of ink puddles, can be used to describe the texture properties of printing plate surfaces.

Introduction:

In nature, most structures are not composed of straight lines, forming a polygon, but are rough and irregular. They exhibit a degree of irregularity that surpasses the descriptive power of classical Euclidean geometry. The classic example, most often, quoted is the problem of measuring the length of Great Britain. The coast of Great Britain cannot be represented by means of a straight line, an ellipse ore any other classical curve.

We are faced with the same "pathological" problem in attempting to analyze the outer edge of copied and printed dots, respectively, and to access the quality of the halftone transfer properties of the offset printing process. Magnified halftone dots show what has been described as "fringes", "islets" and "lakes" (Figure 1). Halftone dot

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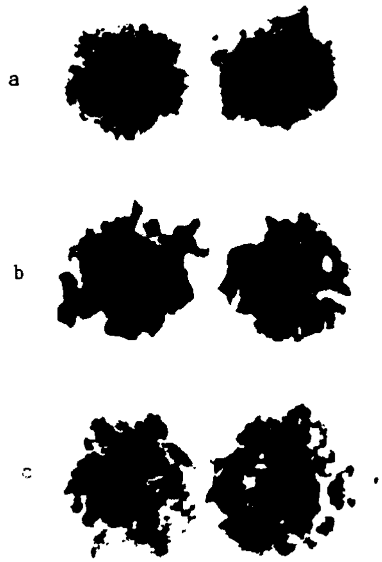

Figure 1: Halftone Dots printed in an offset printing process on paper of graded quality (1)
a coated paper P 1
b, c woodfree paper P 2, P 3 (1)

contours and offset printing plate surfaces have one common feature, their chaotic character. Often those outlines represent, in the mathematical sense, non rectifiable and non differential curves having infinite length in each finite range. In most cases they consist of vertices and so no point has a tangent.

In recent years experiments, in both physics, surface science and chemistry, have been described using the concept of Fractals. It, especially, appears that disordered structures can follow geometric principles which are just as simple as those followed by straight lines and planes.

"Order" and "disorder" can be interpreted as the presence or absence of reference points in a structure. The Fractal Model popularized by Mandelbrot (2) has had considerable success when used for modelling such phenomena in a wide variety of scientific and technical discipline. The fractal dimension characterizes a symmetrical property of the structure (3). Fractal surfaces are rough at all magnifications, in other words, they are scale invariant. They are invariant in the same manner as molecules are rotation or reflection invariant, and pure crystals and homogeneous metallic surfaces are space translation invariant. These objects are invariant under a surface translation which replaces a small part, that is, under a change in scale of the picture. Scale invariant structures are called Fractals.

The generalization and extension of the concept of dimension to non integral values by Hausdorff-Mass has existed, in mathematics, for over 50 years. That concept also provides an excellent description of the character of outer edge of halftone dots in offset printing (1), (4). In that sense the property irregularity and corresponding structure function relations of halftone dot outlines can often be described in terms of simple exponential terms including a constant called "fractional dimension" (2). The formal meaning of the dimension is the slope of a plot of the length of the perimeter of an item measured as a function of the length of the measuring tool or our image resolution. For surfaces the same definition is possible (5). This technique has been used by Russ (6, 7, 8) to calculate the roughness of metal and ceramic surfaces etched by chemical means.

In an analysis of selected copied and printed halftone dot outer edges proved (1, 4) the scale invariant character of these outlines. Using mechanically grained offset plates the outer edge of copied halftone dots showed fractional dimension values of 1.2 in contrast to fractional values of 1.1 fromthe same halftone dots on electrolytically grained plates (4). Higher values indicate an increase in roughness for the dimension studied.

The Fractal concept has also been used for evaluating halftone dots printed on three grades of paper, (in order to vary the "noise level" paper differences were accomplished by using art, printing paper and woodfree paper) (1) and different offset plate surfaces, (mechanically, electrolytically and anodically oxidized substrates). The results show a drastic increase in the fractional dimension of the halftone dot peripheral structures which can be attributed to a decrease in paper quality (Figure 2).

Halftone dot structures on electrolytically grained plate images showed no change in fractional dimension compared to halftones on film. Mechanically grained plate halftone image fractional dimensions differ from those of electrolytically grained plates. Surprisingly, the mechanically grained plate halftone fractional dimensions were close to those of the fractional dimension values of printed halftone images from electrolytically grained plates.

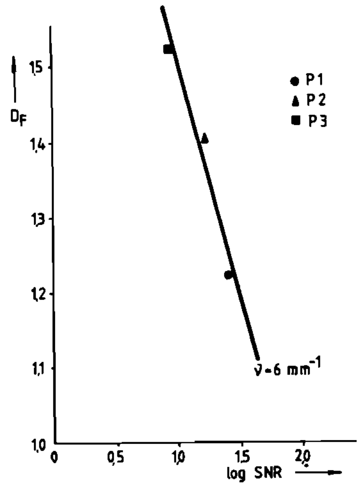

Figure 2: Dependence of the fractionary dimension D_f of the peripheral structures of halftone dots at a dot area on the signal - to - noise ratio S N R. at a spatial frequency of 6 mm^{-1} (1)

Conventually, a profilemeter (a device to physically track a stylus across the surface) is used to scan the surface of printing plates for experimental or quality control data. Although this technique reveals an obvious irregular structure, unfortunately, the values obtained do not provide a satisfactory quantifiable description of the irregularity. The surface texture properties of grained offset plates, however, can also be evaluated by the use of Fractal techniques.

Experimental:

It is the purpose of this paper to introduce the concept of fractal geometry as a means for the study and qualification of printing plate surfaces (without subjecting them to a printing operation) using unsophisticated means. This paper explores how the dry edge behavoir of ink puddles, made by drops of ink, applied with an eye dropper, onto various non printed printing plate surfaces, can be used to describe the texture properties of the respective sample surfaces. To pursue this work outlines of ink blots on several sample surfaces were analyzed by means of the concept of Fractals.

In contrast to the exhaustive spatial and temporal techniques reported above (3) a simple radial distribution method was used. In this "hands on" method the test ink was merely dropped onto the plate a section of the blot edge chosen arbitrarily and n arcs of known radii R were drawn from the origin using a pair of compasses. (Figure 3).

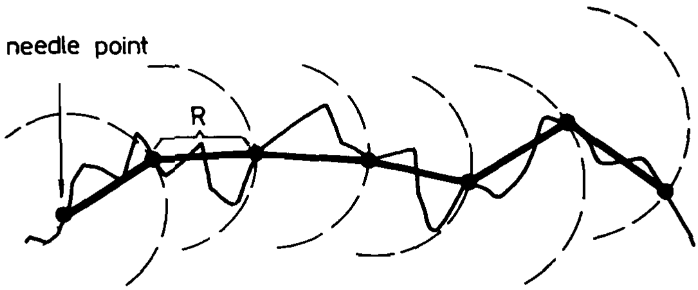

Figure 3: Construction of a polygon into an ink blot periphery to measure the scaling behaviour

The effective length (L) of the open polygon in a fixed length range was counted by cumulating the i single lengths of the polygon segments R yielded:

$$L = \sum_i R_i \quad , \qquad i = 1 \; n. \tag{1}$$

The scaling relationship

$$L(R) \propto R^{D_f} \tag{2}$$

was applied to obtain the fractional dimension D_f.

By plotting the terms Log L/cm versus Log R/cm and measuring the slopes m of the straight lines computed by regression analysis of the data the fractional dimension D_f followed from the expression

$$D_f = 1 - m \tag{3}$$

Fractal Analysis:

A collection of 6 anodic aluminum plate samples were analyzed using the "ink blot technique". Before the ink drop was applied to the uncoated plates the plate surfaces were treated by moistering with an aqueous solution which included a tensid detergent. After drying the deposited blot, parts of the ink boundary between the blot and the uncovered metallic plate surface were magnified, using a microscope, and photographed. Figure 4 shows 6 frames of the parts of the ink blot edge/clean metal surface studied. In order to determine the ink blot outlines are fractal parts of the enlarged outer edges were framed by means of line segment.

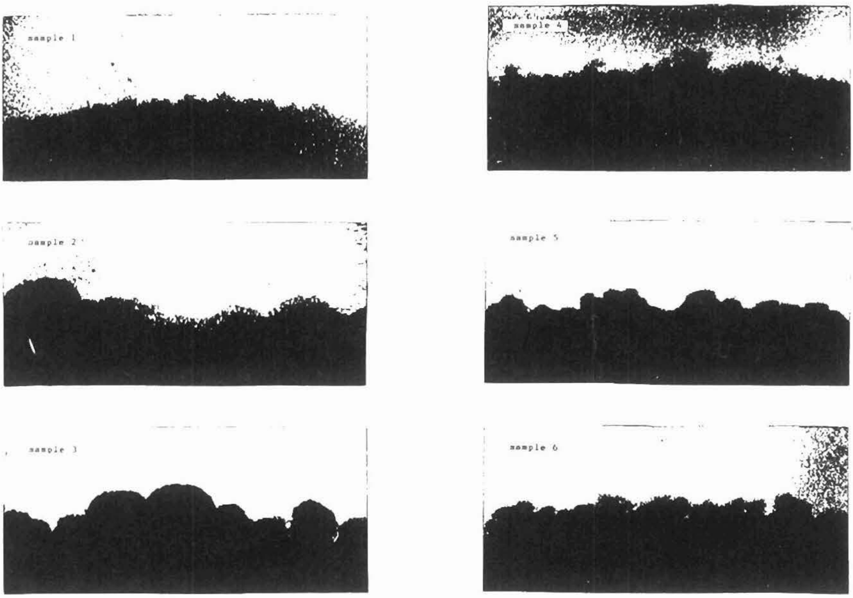

Figure 4: Selected parts of ink blots dropped on various aluminum samples

Depending on the choice of different lengths of R different values of the segment length L of the open polygons were obtained. Log Log plots of the total length of the open polygon L versus the respective side length R show nearly straight line shapes in the measuring range considered (Figure 5). The linear character of the curves in the Log Log plots prove that the contour of the ink blots on the anodic oxidized aluminum surfaces are scale invariant and, consequently, behave like Fractals in the scale range investigated. The D_f values of the periphery structures of the ink blots computed from the slopes of the regression straight lines and the respective correlation coefficients r of the corresponding Log L/cm - Log R/cm plots are shown in the table below (Table 1).

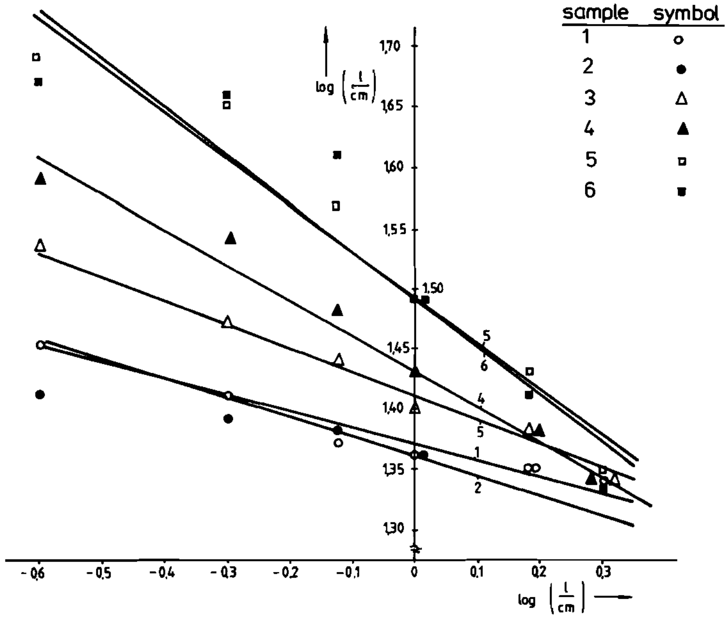

Figure 5: Double logarithmic plot of the total length L of polygons in ink blot peripheries magnified versus the respective side length R of the polygon

Table1: Fractionary dimensions D_f of ink blot peripheries and correlation coefficients r of the straight lines fitting the scale data by means of regression

Sample	D_f value	r
1	1.12	- 0.98
2	1.17	- 0.97
3	1.2	- 0.99
4	1.29	- 0.99
5	1.4	- 0.97
6	1.4	- 0.94

Principle Component Analysis of the Data Set:

The application of Principal Component Analysis, PCA (9) on the scaled data set allows one to answer the question of the how sample's ink blot outer edges differ from one another. From P original features (6 Log L/cm values) p new non-correlated variables are constituted, the eigenvector (first principle component) describes the main part of the variance of the dot matrix, the second one explains the main part of the residual variance, and is orthogonal to the first eigenvector, etc. If it is possible to characterize the main part of the data variability by means of one, two or three eigenvectors, the position of the features in a one, two, or three dimensional subspace (the basic vectors of which are the determined eigenvectors) will be describable without considerable loss of information. One gets qualitative statements with regard to sample similarity or dissimilarity concerning the characteristic vector by means of the so-called "score" coordinates which determine the position of the objects (in this work, the ink blot periphery structures on various sample surfaces).

The matrix L forms the starting position of the eigenvector analysis which had to be carried out.

$$L = \begin{pmatrix} 1.45 & 1.41 & 1.37 & 1.36 & 1.35 & 1.34 \\ 1.41 & 1.39 & 1.38 & 1.36 & 1.35 & 1.30 \\ 1.51 & 1.47 & 1.44 & 1.40 & 1.38 & 1.34 \\ 1.59 & 1.54 & 1.48 & 1.43 & 1.38 & 1.34 \\ 1.69 & 1.65 & 1.57 & 1.49 & 1.43 & 1.34 \\ 1.67 & 1.66 & 1.61 & 1.48 & 1.41 & 1.34 \end{pmatrix}$$

The 6 columns L of the table presented contain the Log L/cm values of the ink blot polygon outer edges with an ordering obtained by means of polygon segment grading. The n=6 row vectors of the matrix represent the measuring data set. The relation

$$C_{ij} = \frac{X_i}{\left(\sum_j (L_i - \bar{L}_j)^2\right)^{1/2}} \tag{4}$$

$i = 1 \ldots n$
$j = 1 \ldots k$

transforms the covariance matrix $X^T X$ into the correlation coefficient matrix

$$R = C^T C \tag{5}$$

The matrix X follows from the transformation

$$X_{ij} = L_{ij} - \bar{L}_j \ . \qquad (6)$$

L is the respective mean value of the j th column.

The 6 row quadratic and symmetric coefficient matrix is given by

$$R = \begin{pmatrix} 1 & 0.99 & 0.96 & 0.99 & 0.96 & 0.61 \\ 0.99 & 1 & 0.99 & 0.99 & 0.96 & 0.54 \\ 0.96 & 0.99 & 1 & 0.98 & 0.94 & 0.47 \\ 0.99 & 0.99 & 0.98 & 1 & 0.98 & 0.52 \\ 0.96 & 0.96 & 0.94 & 0.98 & 1 & 0.51 \\ 0.61 & 0.54 & 0.47 & 0.52 & 0.51 & 1 \end{pmatrix}$$

With the help of the iteration algorithms, after, Hotelling (9, 10) the eigenvector problem

$$(R_{ij} - \lambda \delta_{ij})\ u_1 = 0 \qquad (7)$$

can be solved where

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \qquad (8)$$

is called the Kronecker symbol.
Being a homogeneous equation system (7) has a non-trivial solution u=0 if the coefficient determinant of the system becomes 0 for each value of u.

$$\left| R_{ij} - u_1 \delta_{ij} \right| = 0 \qquad (9)$$

The first principle component is

$$u_1 = (0.996, 0.993, 0.973, 0.992, 0.969, 0.618)$$

and the eigenvalue referring to it becomes $\lambda_1 = 5.2$.

From the relation

$$\eta_i = \left(\frac{\sum_{i=1}^{6} \lambda_i}{tr\, C^T C} \right) \qquad (10)$$

we get the respective variance percentage which is described by the first principle component. For the vector u_1 $(i = 1)$ is

$$\eta_1 = 87\ \%.$$

From the so called score matrix

$$Y = \sqrt{\frac{n}{k}}\ C\ U \tag{11}$$

follows a discrimination of the samples according to the score values computed. Figure 6 shows the distribution of the samples investigated along the score axis considering the first principle component.

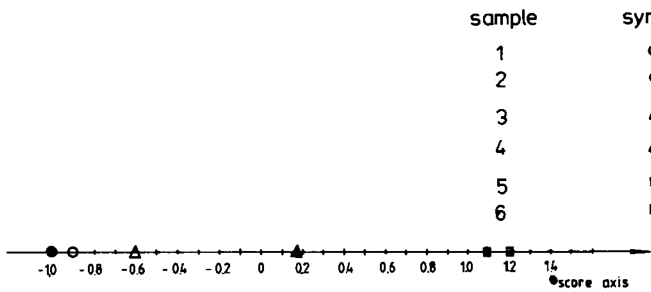

Figure 6: Discrimination of the scale behaviour of the ink blot peripheries according to the scores considerably the first component

Following from the data compression reached by PCA the impact of the surface irregularity on the flow behavoir of the ink blots becomes clearly obvious. Three kinds of surface classes governing the fractal properties of the respective ink blot outer edges are found when one applies the "unsupervised learning method". This finding corresponds with the results of the regression analysis of the scale data which revealed straight lines with three slope amounts essentially, it seems that the flow behavoir of the dried ink blots edge contour obey the irregular character of the surface geometry.

Conclusion:

A simple hands on method to determine the ink acceptance on grained anodic offset plate surface structure has been proposed.

The technique employes Fractal analysis of the peripheral borders of hand applied ink samples.

Fractional dimension values are determined by means of straight line fit in a Log Log plot of total length versus side length of the polygon determined by bordering the ink periphery.

Mathematical models in simple exponential terms including the dimensionless parameter "fractional dimension" have been illucidated to provide numerical values that describe irregularities of the ink/ plate border edge zone in a finite scale.

A spectral decomposition of the scale data field obtained by the use of pattern cognition method of Principal Component Analysis (PCA) provided a classification of the ink blot samples.

The fractional character of the structure of the dried ink blot outer edge may be an indication of the surface irregularity.

Further work comparing this "static" parameter fractional dimension and the dynamic parameter fractional dimensions obtained on press is needed.

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