AN EVALUATION OF FRACTAL TRANSFORMS FOR IMAGE APPROXIMATION

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Abstract
After a controversial start, the use of fractal approximations to achieve image compression has begun to attract serious attention as a technology for picture archiving and transmission. Fractal transforms are competitive in a number of ways with other compression techniques. This paper describes the trade offs available between complexity (cost) of fractal transforms and their fidelity. Some comparisons with JPEG compressed images will be given, and the question of achievable compression ratios will be discussed.

1. Introduction

The Bath Fractal Trandform (BFT) is a general strategy for finding least squares approximations to data in any number of dimensions (including time) by contraction mappings of fractal functions of arbitrary complexity. In two dimensions this includes as low order cases fractal transforms previously reported by Jacquin [1], by Monro and Dudbridge [2] and as patented by Barnsley [3]. It is possible to search for these mappings, or to define them on domains which are predetermined.

We consider polynomial instances of the BFT, in which case the complexity of coding by the BFT of any order is linear with the number of pixels. We pay particular attention to instances where the coding is done by non-overlapping domains without searching. A minimal plotting algorithm (MPA) is then known for rendering the image, also of linear complexity. The assymetry of coding and decoding is then greatly reduced compared with earlier fractal methods, and fidelity can be gained at reasonable computational cost.

2. Background

The term 'Fractal Transform' has been used to describe codes for image compression which work by contraction mappings of regions of an image onto itself. In principle any segmentation of an image can be used, but in practice the method has mainly been applied

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to block coding, in which the segmentation is a division of the image into rectangular or square blocks of pixels. In principle the mappings of the image segmentation can be any approximation of the image segments according to Barnsley's Collage Theorem [4], but in practice a non-overlapping tiling of the image by reduced copies of the block segmentation is used. Figure 1 illustrates the usual form of a fractal transform. It is fractal in the sense of its self similarity, and also because it has scale independent properties; zooming in on the image indefinitely will reveal new detail. Barnsley also introduced a quite elegant method of representing gray scale or colour as an invariant measure of the fractal, i.e. based on the density of points in the fractal. Unfortunately this has proved to present numerical difficulties which may be insurmountable, and many workers have been diverted into futile attempts to code images in this way.

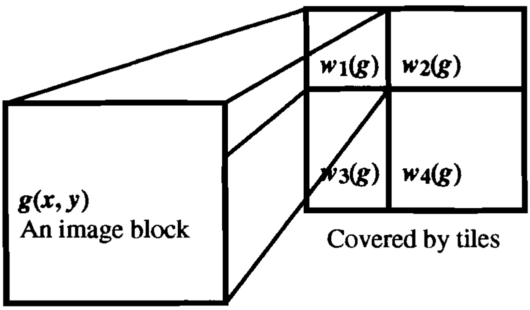

Figure 1. Fractal block coding by contraction mappings of blocks onto a tiling of the image.

Two successful implementations of fractal block coding techniques have been described. In Jacquin's ITT-coding [1], each "range block" is encoded by mapping from a larger "domain block". Monro and Dudbridge [2] encoded a block by tiling it with reduced copies of itself, using a least-squares criterion to derive an optimal mapping. Both of these methods are particular cases of the more general Bath Fractal Transform (BFT), introduced recently [5]. Those interested in the mathematical theory are referred to references [2] and [5].

Our original coding method [2] tiled a block by four copies of itself ($N = 4$), using a bilinear BFT with four parameters ($M = 4$). The reader may wish to refer to [2] where the BFT is worked out more fully for this case, which is particularly important for fast evaluation as we shall see. In ITT-coding, the image is tiled by adjoint "range blocks". For each of these a larger "domain block" is mapped onto it, which is selected by searching the domain blocks. The mappings may include rotation, reflection or greylevel scaling, and always shrink the domain onto the range. The ensemble of mappings is an Iterated Function System [4] (IFS) of high order. To study the combined complexity options, the range block is here called a "tile", and a "parent" is the same as the domain block.

3. Searching and Complexity Options

In this evaluation, 4 by 4 pixel tiles and 8 by 8 pixel parent blocks are used. The parents align with tile boundaries, hence they overlap one another. A level zero search chooses the parent that a tile is inside without searching, as in [2] and [4]. A level-one search considers each of the four parent blocks which overlap the tile. The BFT is derived for each parent, and thus the best parent for each tile is identified. A level-two search includes all the parents which overlap those in level one, and so on. Figure 2 shows the 16 parent blocks in a level 2 search, the 4 in level 1 and the single parent of the level 0 search, all for the shaded tile.

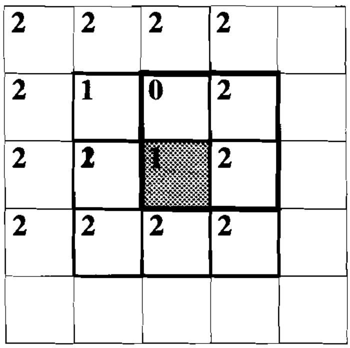

Figure 2. Parents labelled, upper left, by search level.

A parent block might be rotated to any of four orientations, and there are five possible reflections, giving 20 combinations of both. However only 8 of these are distinct. With the BFT, various orders of polynomial fractal functions can be used, and we are interested in comparing the effect of searching and order of approximation on fidelity.

4. Results Of Cost/fidelity Evaluation

A 128 by 128 pixel fragment of the intensity (Y) component from the standard test image "Gold Hill" was chosen for investigation. Table 1 shows the rms errors measured over the image fragment for combinations of complexity at two searching levels. Either rotations or reflections alone give a similar reduction in rms error, which from experience we know will be noticable in the picture quality. Searching over all 8 combinations provides little additional benefit, for both the searches shown. More is gained by increasing the search level than by consideration of rotations and reflections. A noticeable feature is that in all cases but one the rms error is reduced further by using a higher order approximation than it is by considering rotations and reflections.

Table 1 (a). RMS Error, Search Level 0, Gold Hill Fragment				
BFT Order	NoRots or Refls	4 Rots Only	5 Refls Only	8 Rots & Refls
0	16.54	15.48	15.44	15.26
1	11.19	10.63	10.68	10.47
2	9.51	9.17	9.20	9.08
3	9.06	8.79	8.83	8.73

Table 1 (b). RMS Error, Search Level 2, Gold Hill Fragment				
BFT Order	NoRots or Refls	4 Rots Only	5 Refls Only	8 Rots & Refls
0	13.25	11.86	11.80	11.58
1	9.74	9.06	9.08	8.85
2	8.33	7.90	7.91	7.75
3	8.18	7.72	7.73	7.58

The rms errors have also been evaluated for different orders of approximation over a wider range of searching levels. Here the entire Gold Hill image is coded, and the rms errors obtained were generally smaller because the fragment used above contains more detail than other areas. Figure 3 is a graph of the results. A feature of these graphs is the flatness of all the curves.

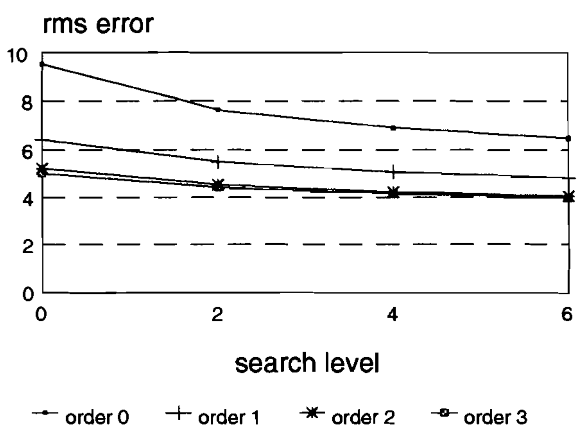

Figure 3. RMS errors as a function of BFT order and search level.

Figure 4 shows the monochrome "Gold Hill" image and approximations of it and a (different) fragment using various BFT combinations, all without rotations or reflections. The rms errors over the fragments are given in each case.

Figure 4 (a). Original Gold Hill image.

Figure 4 (b). BFT order 0, no search, e_{rms} 12.89

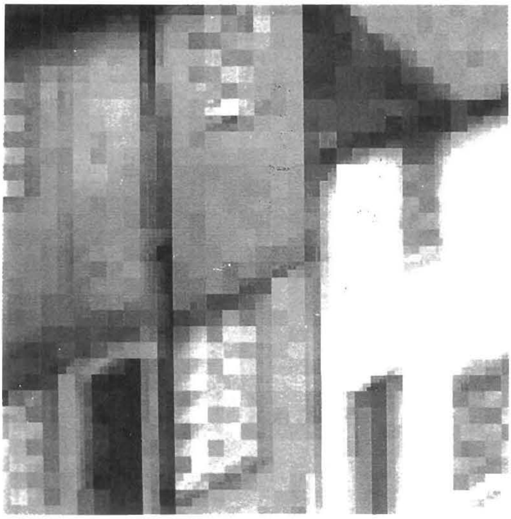

Figure 4 (c). BFT order 0, search level 0, enlarged, e_{rms} 12.89

Figure 4 (d). BFT order 0, search level 6, e_{rms} 8.98.

Figure 4 (e). BFT order 3, search level 0, e_{rms} 6.81

Figure 4 (f). BFT order 3, search level 6, e_{rms} 5.36

5. Discussion

Unlike many transforms, fractal transforms involve a loss of fidelity, so understanding of these losses is necessary before attempting to use them for compression. We have quantized the results of the bilinear BFT on monochrome images with virtually no additional loss of fidelity at a compression ratio greater than 8:1. Work continues to study the trade of compression against fidelity.

Because of the rich choice of cost/fidelity combinations, there are a variety of potential applications of this technology. Of most interest to the TAGA audience is probably their use in picture archiving systems. In these, high cost coding methods can be used to achieve the best fidelity, because pictures will be coded once and decoded often. The very fast decoding algorithms remain an advantage, since remote access to these systems for browsing at low resolution is a growing requirement. The multiresolution attributes of fractals make them ideally suited to this environment.

For real time video applications such as playback in multimedia systems, video mail or digital television, it is important to consider the speed of the methods used. The generalization of fractal transforms afforded by the BFT opens up a range of implementation options. Significantly, increasing the order of the transform gives greater error reduction than the searching or rotation options at lower order. Because of this, variants with little or no searching make fractal transforms viable for real time coding. Also in relation to cost, the optimal Minimal Plotting Algorithm (MPA) [6] for decoding of fractal functions can only be applied to the level zero case in which no pixel from a parent is mapped into a different parent. This favours zero searching, which also produces the most symmetrical cost between coding and decoding.

6. References

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