EVALUATION OF A LIGHT DIFFUSION MODEL FOR DOT GAIN

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Abstract: In two previous TAGA papers [1][2], we have presented a model for optical dot gain based on a physical model of light diffusion within the paper sheet. The model centered around a point spread function for diffuse reflec-tion, which we calculated by simulations on a supercomputer. We now relate the parameters of our simulation model to the traditional Kubelka-Munk parameters K and S, and we also present a few ideas for obtaining the point spread function by digital image analysis. Finally, we extend the model to color printing, and show that the optical dot gain has a considerable impact on the color gamut of the reproduction process. A large optical dot gain yields a larger color gamut. For comparison, a simple model of physical dot gain is presented, and it is shown that a large physical dot gain also expands the color gamut. This could well account for reports that stochastic screening yields a larger color gamut than conventional screening.

An image processing model for optical dot gain

For clarity reasons, we will begin by briefly restating our model for optical dot gain. We assume that the print substrate (the paper) is flat, smooth and reasonably uniform, that the ink is placed in a thin layer entirely on top of the substrate, and that the ink is properly characterized by its absorption properties only. The pattern of ink on the surface can then be described by a two-dimensional absorption function, or, more conveniently, a transmission function T(x, y) taking on values between 0 and 1, inclusive. Light that enters the paper is diffused by the Yule-Nielsen effect before it is reflected. This can be described by a convolution operation with a point spread function (PSF) for diffuse reflection, P(x, y). The total integral of this PSF is the diffuse reflectance R_0 of the print substrate, and the spatial extent of the PSF describes the amount of lateral spreading of light in the Yule-Nielsen effect. The attenuated, diffused and reflected light then has to pass

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once more through the ink layer to reach he viewer, which is described by a final point-wise multiplication with the transmission function. If we denote the incident light intensity with I, the reflected image $R(x, y)$ is thus described by:

$$R(x, y) = I\,(T(x, y) * P(x, y))\,T(x, y) \qquad (1)$$

This is a non-linear model for optical dot gain expressed in image processing terms. It allows for direct simulation of the reflected image from an arbitrary halftone pattern, provided that the PSF is known. We have shown that it is possible to calculate the PSF by direct simulation of the multiple scattering optical system of a model paper sheet, using modern computers. A typical simulated PSF for diffuse incident light is closely approximated by a simple exponential function:

$$P(x, y) \approx R_0 \frac{a}{2\pi r} e^{-ar} \qquad (2)$$

The shape of the PSF, and of course therefore also the parameters R_0 and a in the approximation above, depend in a non-trivial way on the scattering and absorption cross sections and the thickness of the substrate. The cross sections are in turn related to the K and S parameters of the famous Kubelka-Munk theory, although not in such a simple way as one would have hoped.

Kubelka-Munk revisited

The original formulation of the Kubelka-Munk (K-M) theory, which has been in widespread use for a long time in the papermaking business, contains two parameters K and S, which describe the absorption and scattering power of the medium under consideration. Within the approximations of the K-M theory, the relations between K and S and the absorption and scattering cross sections σ_a and σ_s, are:

$$K = 2\,\sigma_a \quad S = \sigma_s \qquad (3)$$

It is important to remember, though, that although the solution to the K-M differential equations is exact, the equations themselves are only an approximation of the physical scattering system, and in some respects that approximation is very crude.

It is commonly recognized that K-M theory breaks down for media with strong absorption in relation to the scattering power, and also for fairly transparent or translucent media. This is in fact pointed out in the original article by Kubelka and Munk [3], but it is a limitation that has proven to be easy to forget.

Another less known and less studied property of K-M theory is that there is an inherent assumption of perfectly diffuse light everywhere inside and outside the medium. Upon closer scrutinization, we find that this assumption does not correspond to any physically relevant, or even achievable, illumination geometry. Diffuse light *externally* incident on a surface, which is used by instruments measuring the K-M parameters, is not equivalent to the peculiar *internally* incident diffuse light of the K-M model.

Using our simulation model, which is able to get closer to the physical truth than K-M theory, we can show that this difference in fact has quite a large influence on the predictions from the models. Figure 1 shows two sets of curves for the predicted reflectances of isotropically scattering media with different optical properties. It is clearly seen that the results from K-M theory differ significantly from our simulations. However, if we change the initial conditions for the incident light in our simulation to mimic the physically incorrect assumption of K-M theory, our simulations agree well with K-M theory, except for strongly absorbing or translucent media, where the K-M theory is not valid. This is illustrated in figure 2.

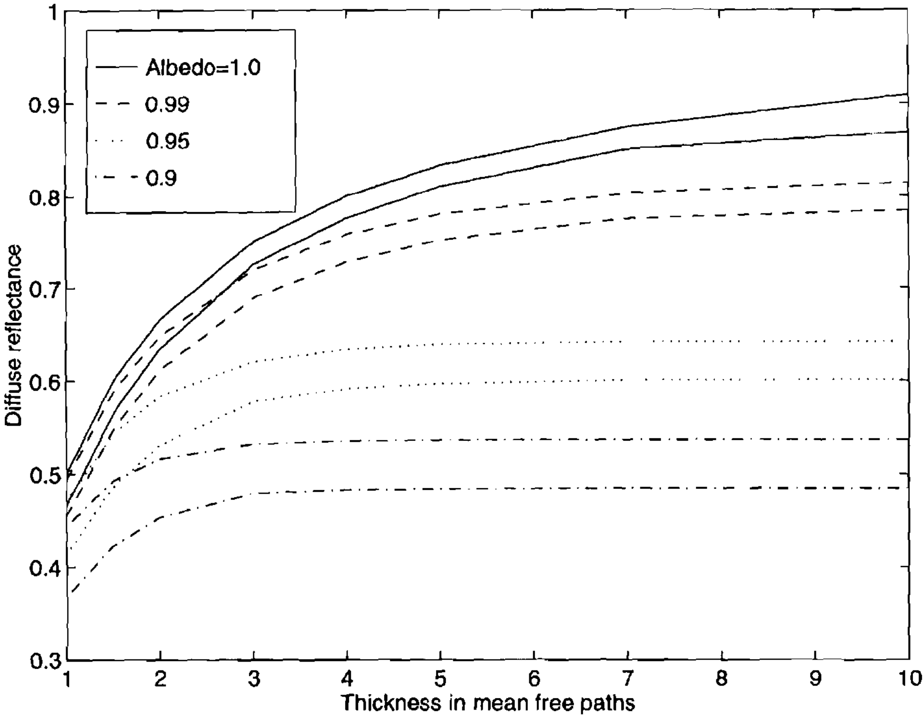

Figure 1: Pair-wise comparisons between K-M theory and our physical model Top curves in each pair K-M, bottom curves our model.

So, if we want to measure the scattering and absorption cross sections of a material, the two K-M parameters are not the final answer. The K-M parameters *are* in fact related to the physical cross sections σ_a and σ_s, but not in any linear, separable or otherwise simple and obvious way. By using a simulation like ours as a guidance, it is possible to fit a curve to experimental data and calculate the true cross sections from the same amount of measurements as is used for K-M meas-

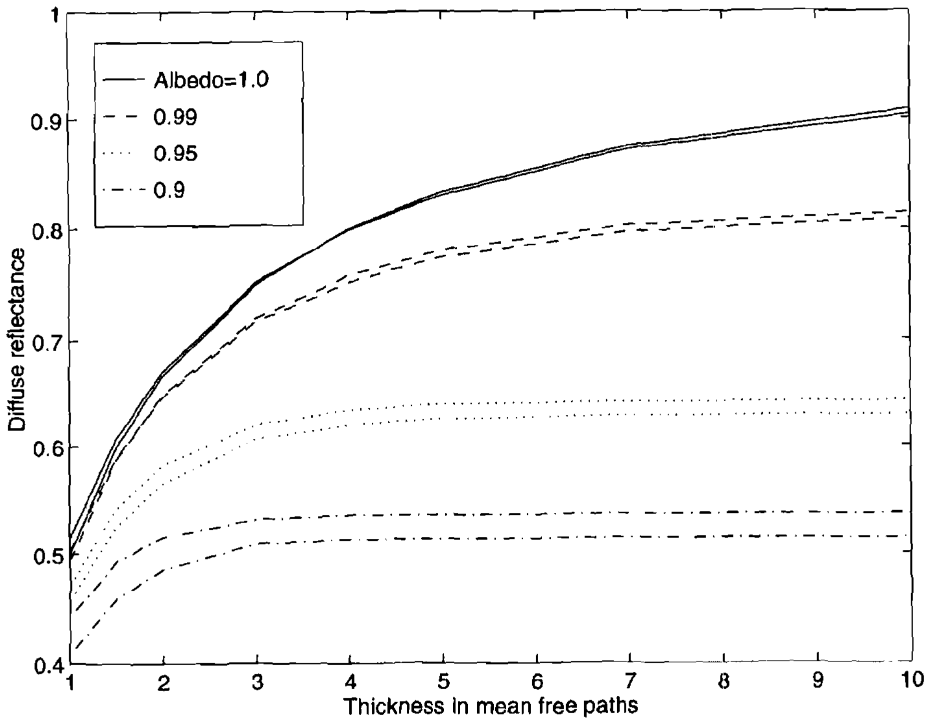

Figure 2: Comparison between K-M and our simulation with comparable assumptions. Top curves K-M, bottom curves our model.

urements, but it is not possible to derive a theoretical and explicit closed-form expression for the exact relation between either K and S or the experimental reflectance measurements and the cross sections σ_a and σ_s

The virtue of K-M theory is that the parameters lend themselves very well to experimental measurement. The drawback is that the assumptions made are only valid within a quite limited region, and even then only approximately so. The reason why K-M theory still works reasonably well is that normally no inferences are made from the two parameters of the model to actual optical properties of the material. They are merely treated as two experimentally measured parameters with no particular physical meaning. For detailed modelling of the optical behavior of turbid media like paper, a physical approach like ours is more accurate and more rewarding in terms of understanding, although somewhat more difficult to undertake.

Unfortunately, it is impossible to go into full detail on our model and our simulations on the limited space available here. Further reading and can be found in a recent licentiate thesis from our group [4].

Previous work on the spread function

Quite a few authors have recognized the existence of a "spread function" that is responsible for the optical dot gain, and some measurements have been presented in the past [5][6][7]. However, the measurements have all suffered from a high level of noise, and previous authors have not taken into account the dependence of the spread function on the illumination and detection geometry, or at least they have failed to mention it. Measurements have been performed with two different approaches. The first approach measures the reflection from a small illuminated spot. The second approach measures on a sharp edge, either a physical edge placed on the surface by printing or other means, or a projected edge from a special illumination setup. Since the measurements are quite noisy, no detailed comparison can be made to our simulations, but our model shows a qualitative agreement with all of the previously published results.

Purely theoretical work in the field is very sparse and quite limited, due to the complexity of multiple scattering problems. Only one mildly relevant study has been found, made by Giovanelli in 1956 [8], but that one assumes the same kind of internally incident diffuse light as the K-M theory, which we have shown to be incorrect. Nevertheless, if we adjust our model to use this kind of incident light, it agrees well with the theoretical findings of Giovanelli.†

Direct measurement methods

For direct measurement of the PSF, one could imagine illuminating a small spot on an unprinted paper surface and acquiring an image of the reflected light. That reflected image would be the PSF itself. The problem is that the spot would have to be very small, only a few micrometers across, and that the illumination geometry would be more or less directional. It is hard to imagine a practical illumination setup that could project diffuse light onto one point only. Furthermore, a point measurement of the PSF would be very different for different points in the surface, and a lot of measurements would have to be averaged to reduce the influence of such fluctuations.

Indirect measurement

Instead of trying to measure the PSF directly, indirect methods could provide a better path to the goal. The most natural way of measuring the lateral spread of light is at a reasonably sharp and straight printed or projected edge. A projected edge has the advantage of that the spread function can be observed directly on both sides of the edge. A printed edge, or any kind of physical edge placed directly on top of the paper, blocks the dark side of the edge from observation,

† It should be noted that even though Giovanelli's approach is theoretical, the solution presented is not exact, but an approximation.

but on the other hand, with a physical edge there is a free choice of illumination geometry, which is not the case for a projected edge.

The edge spread function (ESF) is a one-dimensional property $E(x)$, which gives us the opportunity to acquire a reflected two-dimensional image and perform an averaging along the edge to smooth out the local variations in our measurement. The experimental workload is thus much smaller for such a measurement.

From an observation of the ESF we can calculate the line spread function (LSF), denoted by $L(x)$, by a differentiation:

$$L(x) = \frac{d}{dx} E(x) = E'(x) \tag{4}$$

The LSF is in turn a projection of the PSF onto one dimension:

$$L(x) = \int_{-\infty}^{\infty} P(x, y) dy \tag{5}$$

If we assume that the PSF is circularly symmetric, i.e. that we can express it as:

$$P(x, y) = P\left(\sqrt{x^2, y^2}\right) = P(r) \tag{6}$$

The transformation from $P(r)$ to L(x) is the Abel transform:

$$L(x) = 2\int_{x}^{\infty} \frac{P(r)r}{\sqrt{r^2 - x^2}} dr \tag{7}$$

Using a few tricks (see for example [9]), this formula may be inverted to:

$$P(r) = -\frac{1}{\pi} \int_{r}^{\infty} \frac{L'(x)}{\sqrt{x^2 - r^2}} dx \tag{8}$$

Where

$$L'(x) = \frac{d}{dx} L(x) = \frac{d^2}{dx^2} E(x) = E''(x) \tag{9}$$

If we cannot assume that the PSF is circularly symmetric, we need some other clue to its shape, or else an inversion from the ESF or LSF is not even possible. The PSF is only circularly symmetric if the illumination is circularly symmetric and the paper is isotropic, neither of which may be the case in practice.

Another approach for measuring the PSF that does not assume any particular symmetry is a direct optimization method using the model equation (1) and measurements of $R(x, y)$ and $T(x, y)$ to fit a model PSF to experiments. If R and

T are both known, it is a standard optimization problem to find P. Using some of our knowledge gathered from simulations, it is possible to reduce the dimensionality of the optimization to a few parameters, for example the parameters R_0 and a if equation (2). The main problem here is to acquire a proper measurement of the transmission function $T(x, y)$. We have made attempts to extract it from look-through images of a print, but with only limited success. To make that method work properly, we need to remove the influence of the varying transmittance of the paper, perhaps by immersing the paper in some kind of fluid. This has not yet been tried. There are also non-optical methods that could be utilized for measuring the actual physical ink distribution in the surface and calculate a transmission image from that [10][11].

The optimization approach has one other clear advantage: there is no need for any particular printed test pattern. A standard print with any reasonable amount of small scale detail will do nicely, e.g. a halftone pattern.

Extension to color printing

At TAGA 1995 in Paris, we presented plots of predicted optical dot gain from different halftone geometries [2]. More detail on this can be found in another paper this year [12.]. For color prints, the situation is a bit more complicated. In order to properly model the physical aspects of color, we need to incorporate a wavelength dependence for every component of equation (1):

$$R(x, y, \lambda) = I(\lambda)\,(T(x, y, \lambda) * P(x, y, \lambda))\,T(x, y, \lambda) \qquad (10)$$

The dependence on wavelength is most obvious in the ink film transmission $T(x, y, \lambda)$, since we have more than one color of ink on the surface, but the wavelength dependence of the illumination and the PSF are also important. Since the K-M parameters are known to depend heavily on wavelength, we might expect that the PSF is also different for different wavelengths of light. However, we have no experimental evidence to support this yet.

Implications for the color gamut

By performing a simulation in the spectral domain, sampled at 5 nm intervals in our case, we can evaluate the qualitative effects of optical dot gain for color printing. As input data, we used spectrophotometric measurements on actual offset prints on coated paper printed with standard European CMYK inks. The transmittance of the primary colors were calculated as the square root of the reflectance divided by the reflectance of the unprinted paper:

$$R_C(\lambda) = \sqrt{\frac{R_C(\lambda)}{R_0(\lambda)}},\; T_M(\lambda) = \sqrt{\frac{R_M(\lambda)}{R_0(\lambda)}},\; T_Y(\lambda) = \sqrt{\frac{R_Y(\lambda)}{R_0(\lambda)}},\; T_K(\lambda) = \sqrt{\frac{R_K(\lambda)}{R_0(\lambda)}} \qquad (11)$$

A number of ideal color halftone patterns were simulated, assigning each pixel in the digital image either of the sixteen possible full-tone color combinations, and the transmission of each pixel was calculated as the product of the transmissions for each of the primaries:

$$T_0(\lambda) = 1,\ T_{CM}(\lambda) = T_C(\lambda)T_M(\lambda),\ T_{MYK}(\lambda) = T_M(\lambda)T_Y(\lambda)T_K(\lambda) \qquad (12)$$

and so on. We can thus calculate simulated transmission images for color halftone printing and evaluate the effects of optical dot gain using our model. For simplicity and for lack of concrete evidence to the contrary, we used a PSF that was not dependent on wavelength for the time being.

The result was quite surprising to us. It turns out that the optical dot gain can actually expand the color gamut.

For reference, we consider the extreme cases where there is either no optical dot gain at all (the PSF is an impulse function and no light is diffused sideways) or maximum optical dot gain (the PSF has a very large spatial extent, which in effect yields a complete diffusion of the incident light before reflection). The former corresponds to a Yule-Nielsen parameter of 1, the latter to a Yule-Nielsen parameter of 2. With stochastic screening, it is perfectly possible to come close to the latter extreme case, since the lateral spreading of light is on the same scale as the size of the individual halftone dots.

All calculations were performed in the spectral domain. For presentation, we transform the spectral color data into the CIE L*a*b* color space. The result is shown in figure 3. It is clearly seen that the color gamut expands considerably in light and middle tones under the influence of optical dot gain, even though the extremes of the gamut, the full-tone colors, stay exactly the same.

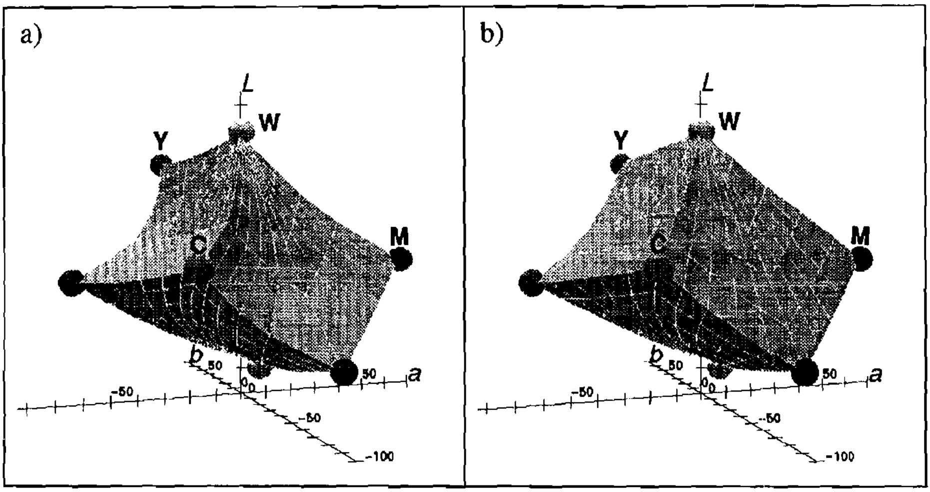

Figure 3: Color gamut for a) no dot gain, b) maximum optical dot gain

Physical dot gain

Given that the optical dot gain has an effect on the color gamut, it comes natural to also investigate the effect of physical dot gain. A simple model of physical dot gain can be easily incorporated into our image processing model framework. Instead of assuming perfect halftone dots with sharp edges, we model a smearing of the ink by first calculating a perfectly sharp simulated halftone image $H(x, y)$ which takes on the values 0 or 1 only. To this image we apply a linear blurring (low-pass) filter $B(x, y)$. If the blurring filter kernel is properly normalized, this operation does not change the total amount of ink on the surface, but merely redistributes it by smearing out sharp edges. After the smearing, we exponentiate the result to get our final transmission image $T(x, y)$ for the model:

$$T(x,y) = 10^{-D_{max}(H(x,y)*B(x,y))} \tag{13}$$

In this equation, D_{max} is the full-tone density. For color halftone images, a dependence of wavelength needs to be incorporated, and we also need to calculate one transmission image for each primary ink, but the basic model remains the same:

$$\begin{aligned}
T_C(x,y,\lambda) &= 10^{-D_C(\lambda)(H_C(x,y)*B(x,y))} \\
T_M(x,y,\lambda) &= 10^{-D_M(\lambda)(H_M(x,y)*B(x,y))} \\
T_Y(x,y,\lambda) &= 10^{-D_Y(\lambda)(H_Y(x,y)*B(x,y))} \\
T_K(x,y,\lambda) &= 10^{-D_K(\lambda)(H_K(x,y)*B(x,y))}
\end{aligned} \tag{14}$$

Once again, we show the extreme case for reference. If we have a very large physical dot gain, the halftone pattern is effectively blurred out into a smooth layer of ink of varying thickness, and we approach the characteristics of a true subtractive process. The color gamut of such a process is shown in figure 4. For this extreme case, subsequent optical dot gain has no effect, so this is really the largest color gamut achievable using inks with these particular spectral properties.

It is clear that the physical dot gain also expands the color gamut. It should be noted, however, that at least for offset printing, the extreme physical dot gain corresponding to the color gamut of figure 4 is not encountered in practice, whereas the extreme optical dot gain of figure 3 is actually quite close to what we might expect from an offset printed stochastic screen with a small spot size.

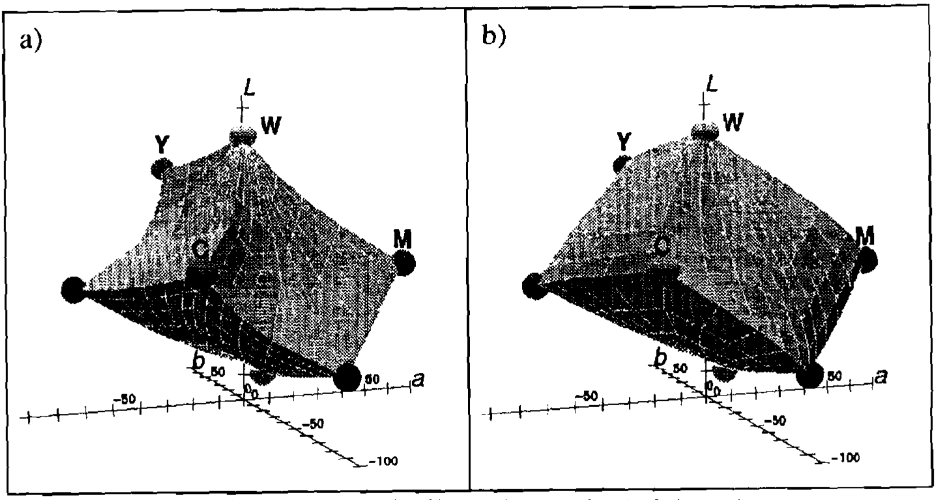

Figure 4: Color gamut for a) no dot gain, b) maximum physical dot gain

Conclusion

Our previously presented model for optical dot gain contains a point spread function for diffuse reflection, which was calculated by computer simulations. For practical purposes, this point spread function may also be calculated from measurements identical to those used for determining the Kubelka-Munk coefficients, although with somewhat more complicated calculations. The point spread function could also be measured, either directly or indirectly, by digital image analysis methods. Previously published results and the agreement between our model predictions and common experience support the validity of our model, at least qualitatively.

By extending our model and applying it to color printing, we have shown that optical dot gain is not merely an unwanted distortion. It actually expands the color gamut for color printing. Physical dot gain expands the color gamut in much the same way. Our image processing model framework has proven very convenient for simulation and modelling of optical and physical dot gain alike.

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