AERA-CORRECTED NEUGEBAUER EQUATIONS

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Abstract: The negative effects of printing media on the light propagation of color reproduction, first surface reflection, absorption and multiple internal reflection etc, can be attributed to the defect or extension of dot areas. These effects break down the linearity between the tristimulus values (X,Y,Z) and the dot areas, in original Neugebauer Equations. A nonlinear form of the Equations, with modification of dot areas, have been presented. The modified Equations have been sucessfully applied to 3- and 4-color processes. In addition, blacktype presetting for 4-color process has been described in detail.

Introduction

Analytical and/or numerical relations between the measurable quantities and the dot areas are the ground of color printing. Neugebauer Equations (Neugebauer,1937), a group of linear algebrical equations of tristimulus values X, Y, Z and the dot aeras, are the most well known and

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widely used in pratice. The light emerging from the printing can be divided into two parts, the main path and residual path of light(Yule(1967),Yang et al(1996)). The former is caused by the ideal light diffusing from paper and dye layers, and the later results from some residual effects, such as the first surface reflection, penatration of light into paper, light multiple internal reflections in the dye layers, and ink opacity etc. To account for the residual effects, several attempts of modification to the original Neugebauer Equations have been made (Kotera(1986), Pobboravsky and Pearson(1972), Zhou and Dong(1989)). Among them, Pobboravsky and Pearson proposed modification by introducing the Yule-Neilson's *n* factors (Yule and Neilson,1951) into Neugebauer Equations (Pobboravsky and Pearson,1972), which were used to modify the so-called Murray-Davis Equation (Murray,1936). As pointed by themself, this modification worked well in some cases (Pobboravsky and Pearson,1972), some cases not(Pobboravsky,1976). Thus further study for modification of these equations is needed.

Modified Neugebauer Equations

In single color printing, if we neglect the contributation from residual pathes of light, the flux of reflected light can be writen into two parts,

$$dF(\lambda)d\lambda = R_a(\lambda)a\,d\lambda + R_0(\lambda)(1-a)d\lambda \tag{1}$$

where $R_a(\lambda)$ and $R_0(\lambda)$ are monochrome reflectances of the dye layer and the paper, a and $(1-a)$, their percentage of aeras. Substitute the left side of the equation by $R(\lambda)d\lambda$ (unit area is assumed), multiply both sides by $kS(\lambda)x(\lambda)$, and integrate over the band of the visable light, we get,

$$k\int_{Vis} S(\lambda)x(\lambda)\,R(\lambda)d\lambda = ak\int_{Vis} S(\lambda)x(\lambda)\,R_a(\lambda)d\lambda + (1-a)\,k\int_{Vis} S(\lambda)x(\lambda)\,R_0(\lambda)d\lambda \tag{2}$$

where k is the normalization factor, $S(\lambda)$ the relative energy distribution of the standard CIE light source, and $x(\lambda)$ the monochrome X-stimulus value. Eq.(2) can be written as,

$$X=aX_a+(1-a)X_0 \tag{3}$$

or

$$X=X_0-(X_0-X_a)a \tag{4}$$

where X, X_a and X_0 are the X-stimulus values of the printing, the primary dot (ie. the solid of ink) and the white paper, respectively. Thus in the ideal case, the tristimulus values is linearly decreased as the primary aera increases. For multiple colors case, Eq.(3) can be readily generalized to the so-called Neugebauer Equations,

$$X = \sum_{i=1}^{n} f_i X_i$$

$$Y = \sum_{i=1}^{n} f_i Y_i \tag{5}$$

$$Z = \sum_{i=1}^{n} f_i Z_i$$

where (X, Y, Z) is the tristimulus values of the print and (X_i, Y_i, Z_i) of the i'th dot (pure layer of ink or white paper). n equals to 8 or 16, for 3-, respectively, 4-color processes. The combined dot area f can be expressed by the primary percentages c(cyan), m(pink), y(yellow), and bk(black) (Demichel,1924).

When the residual light exists, the linearity between the tristimulus values and the geormatric aeras of primary dots break down. As discussed in previous (Yang et. al,1996), the contributation of the residual light can be attributed to the defect and/or extension of the primary dots. For example, the flux due to the first-surface reflection increases the amount of white light, which is equivalent to a area extension of white paper, the opacity of ink, on the contrary, to a defect of its dot area. Then Eq.(1) should be replaced by a nonlinear form,

$$dF(\lambda)d\lambda = R_a(\lambda)a^{\alpha_1}d\lambda + R_0(\lambda)(1-a)^{\alpha_0}d\lambda \qquad (6)$$

where α_1 and α_0 are modification parameters, and a^{α_1} and $(1-a)^{\alpha_0}$ have the meaning of effective dot areas. Then Neugebauer Equations (Eqs.5) can be modified as

$$X = \sum_{i=1}^{n} f_i^{\alpha_i} X_i$$

$$Y = \sum_{i=1}^{n} f_i^{\alpha_i} Y_i \qquad (7)$$

$$Z = \sum_{i=1}^{n} f_i^{\alpha_i} Z_i$$

Set $\alpha_i = 1 + \delta_i$ and notice that $f_i = 1 - \sum_{j \neq i} f_i$, Eqs.(7) can be expanded as

$$X = \sum_{i=1}^{n} f_i X_i + \sum_{i=1}^{n} \sum_{j \neq i}^{n} \delta_i f_i f_j X_i$$

$$Y = \sum_{i=1}^{n} f_i Y_i + \sum_{i=1}^{n} \sum_{j \neq i}^{n} \delta_i f_i f_j Y_i \qquad (8)$$

$$Z = \sum_{i=1}^{n} f_i Z_i) + \sum_{i=1}^{n} \sum_{j \neq i}^{n} \delta_i f_i f_j Z_i$$

Clearly, the original form of Neugebauer Equations is the zero-order approximation of the equations.

The area modification factors α_i are not directly measurable, but can be determined by matching a number of colormetrically specified colors.

3-COLOR PROCESS. There are 8 kinds of dots, corresponding to zreo through 3 order of color. As a reasonable approximation, we can assume the color of the same order has the identical aera modification parameter. Then the parameters are contracted into 4. They are, $\alpha_w=\alpha_0$ (w is white), $\alpha_c=\alpha_m=\alpha_y=\alpha_1$ (c,m and y mean cyan,pink and yellow), $\alpha_{cy}=\alpha_{ym}=\alpha_{mc}=\alpha_2$, and $\alpha_{cym}=\alpha_3$. By matching to about 300 pieces of specified colors with known (X,Y,Z) values and primary percentages (*c*,*m*,*y*)(Shi etal, 1992), we have determined the parameters, which have been listed in Tab.1. it shows that the effective dot aeras of zero and first order of color are slightly bigger than their geormetric aeras, but slightly smaller for the third order color. Significant influence on the second order color occurs, which is in qualitatively agreement with other work(Kotera,1986).

After determination of the α-parameters, Eqs. (7) can be used to predict dot areas of the printing with known tristimulus values (X,Y,Z). Application to two pieces of color which are randomly selected from a map (Shi etal, 1992) has been made. The calculated and measured dot percentages of them, have been listed in Tab. 2. The measurement was performed by the DM-500 density detector, for the separate films of the map. The calculations are in good agreement with the measurements.

4-COLOR PROCESS, Almost all above discussions can be directly generallied to the 4-color process, where there exists 16 primary dots. The α-parameters can be contrated into 5 parameters, which correspond color with zero to four orders. As listed in Tab.1, no area defect (or extension) occurs in the fourth order of color. The rest parameters are a little different from those in 3-color process. The calculation of the dot percentages, however, is rather more difficult. The difficulty results from the fact that there are 4 primary percentages (*c,m,y,* and *bk*) in 4-color process, but only 3 equations (Eq.7). Thus, one of the primary percentages, *bk* (blacktype) for example, has to be preset, in advance.

In principle, neutral gray can be obtained by certain amounts of cyan, pink, and yellow inks printing. It can be reached, also, by printing with amount of black ink. Thus from both economic and technical points of view, it is convinent to replace the multilayers of neutral gray, by one black layer. This consideration is exactly the starting point of our blacktype presetting. As shown in section 2, the X-stimulus of black, is propotional to its black dot *bk*. Taking the residual paths of light into account, we add a *quadratic* term in it, ie,

$$X=X_0+\eta_{1x}bk+\eta_{2x}bk^2 \qquad (9)$$

where η_{1x} and η_{2x} are constants which can be determined by fitting to a number of specified neutral gray of black printing. For a specified neutral gray, by Eq.(9) and Neugebauer Eqs, one can establish correspondences, on the ground of colormetrical balance, between dot area of black (*bk*) and those of cyan, pink and yellow inks (we note it as ``*bk-(c,m,y) relation*" in abbreviation). Now the blacktype presetting can be achieved by the follwing steps.

(1) For a specified color (X, Y and Z), we calculate its dot areas by 3-color Neugebauer Eqs, at first.

(2) Picking the neutral gray component (c_n,m_n,y_n) from the calculation.

(3) According to the ``*bk-(c,m,y)* relation", the black dot aera bk_n can be found out and then used as the blacktype presetting.

After preseting the blacktype, the dot areas of other 3 color inks can be calculated by the Neugebauer Eqs. Algorithm and computer package for determination of the α-parameters and dot areas calculation have been established in our Institute. Further study for 4-color process is undertaken.

Table 1. α parameters of 3- and 4-color processes

Process	α parameters				
	α_0	α_1	α_2	α_3	α_4
3-color	0.94	0.96	1.36	1.06	
4-color	1.10	1.10	1.43	1.12	1.00

Table 2. Calculated and measured dot areas

No.	Calculation			Measurement		
	c	m	y	c	m	y
1	26	0	3	27	7	3
2	13	28	55	16	28	54

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