A NONLINEAR HEAT CONDUCTION MODEL FOR LASER IRRADIATED PRINTING PLATES

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Abstract: A nonlinear heat conduction model was developed on the basis of the FINITE DIFFERENCE METHOD to calculate temperature distributions in laser irradiated printing plates. The basis of the numerical model is the assumption that the laser heated zone in the material represents a semi infinite slab subdivided in finite spatial cells. By employing a step by step iteration algorithm referring to space and time, the temperature dependent character of the thermophysical parameters of the laser exposed material such as specific heat capacity, heat conduction and density as well as the optical material parameters such as absorptivity and reflection are taken into account. Particularly in the case of gravure or flexography, melting and vaporization processes can be included in the numerical calculation.

Introduction

Aside from the general technology in which diazo or photopolymer coatings are exposed by laser wave lengths in the ultraviolet region, the shift in research to date has focused on the use of plates irradiated by IR laser diodes / 1 /.

It seems that thermal imaging could be a viable alternative to existing technologies for direct - to - plate applications. The evolution of IR laser diodes with more powerful imaging meets the requirements regarding lower costs and improved longevity and could become a driving force in the offset copy.

A theoretical in - depth understanding of laser initiated thermal reactions requires the understanding of the heating regime in the sandwich coating / base during the irradiation process.

This is also true in the field of laser irradiated plates in gravure as well as in flexography.

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This paper suggests a numerical model for the calculation of the temperature distribution along the depth direction of laser exposed plates in various branches of printing technology.

For a precise thermal analysis it is essential to investigate the thermal chemical and physical processes in the printing plate which take place during the heating process.

The primary distribution of absorbed energy follows the light distribution. In the case of thermal imaging the absorbed light energy is immediately transferred into thermal energy and leads to a heating at the location of photon absorption. The rise in temperature depends on the heat capacity of the plate material.

These processes are accompanied by energy consumption triggered by diverse chemical or physical processes, representing a crucial factor influencing the 'thermal sensibility' of the irradiated material.

Due to the temperature gradient, heat conduction takes place spreading the temperature distribution.

Physical parameters such as heat conduction λ, specific heat capacity c, density ρ, light absorptivity and reflection coefficients α and R show frequent alterations in temperature dependence and consequently are considered variables. Hence the classical equation of heat transport (1) with F as energy source and γ as energy loss, does not offer a realistic temperature profile.

$$\rho \cdot c \cdot \frac{\partial T}{\partial t} = div\left(grad\, T(z,t)\right) + F - \gamma \qquad (1)$$

In short we are faced with a nonlinear light absorption and heat conduction problem which can not be solved by means of classical analytical methods.

In this case, employing **FINITE DIFFERENCE METHOD** can provide a numerical solution. The use of the iterative procedure allows one to factor in the temperature dependence of the thermophysical as well as of the optical parameters. In this way an adapted approach to the problem is feasible.

The following term for the heat diffusion parameter results from the more or less arbitrary choice of the laser beam Gauss radius ω:

$$t_p = \frac{\omega^2}{4 \cdot K} \qquad (2)$$

with

$$K = \frac{\lambda}{\rho \cdot c} \quad (3)$$

The one dimensional consideration of the heat conduction problem remains valid up to a heat diffusion length range

$$d = \sqrt{4 K t_p} \quad (4)$$

This relation also involves the restriction that the heat diffusion period must be greater than the duration of the laser exposition, i. e.

$$t_D > t_p$$

Due to the very short laser exposition period necessary to realise a halftone dot into the offset coating or to vaporize holes in gravure and flexo plates, it is sufficient to limit the consideration to the heat conduction towards the material depth.

The model

The basis of the numerical calculation procedure is the assumption that the heated bulk phase in the laser irradiated material has the shape of a semi infinite slab (comp. the figure 1). This particular model supposes that the end of the slab remains the same temperature throughout the whole laser irradiation process. A further prerequisite of the model is the assumption of a rotation symmetrical laser beam and a perpendicular irradiation angle. Diffraction and scattering phenomena are not considered.

A decisive simplification of the problem is reached (comp. the figure 1) by subdividing the slab in cylindrical shaped cells with radius r_0 and high Δz.

Similarly, the laser exposition duration is subdivided in finite time steps with a constant span Δt.

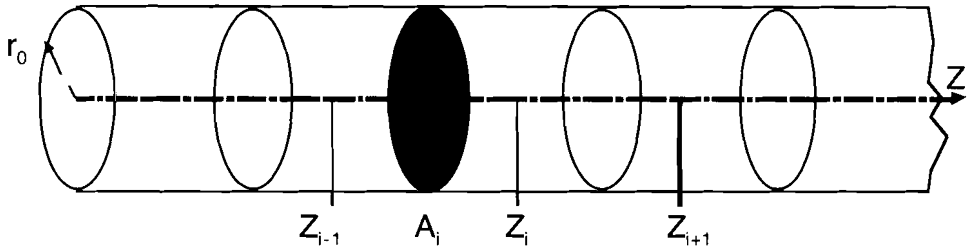

Fig. 1 : Cell subdivided slab modeling the laser heated material zone

Limiting restrictions are chosen in order to assure that there is no energy exchange between the cell subdivided slab and the surrounding nonirradiated material (adiabatic regime).

In the case of a sandwich system like a printing plate consisting of two layers the FOURIER law should be taken into account at the border of the coating and the base.

The temperature T_i is supposed unique inside the cells and remains constant through the time span Δt. T_i is changing regarding the well known laws of heat conduction at the time $t = l\,\Delta T$ (with $l = 1,n.$)

Thus the flowing heat across the area A_i during the time step Δt is

$$\Delta q_i = \lambda_i \cdot A_i \cdot \Delta t \cdot \frac{\Delta T_i}{\Delta z} \tag{5}$$

$\Delta T_i = T_{i-1} - T_i$	temperature difference beween the $(i - 1)$th and i th cell
$\Delta z = z_i - z_{i-1}$	length of the cell
$\lambda_i = \lambda(T_i)$	temperature depending heat conduction parameter

Considering the heat source F as well as the heat sink γ the heat energy in the ith cell has the form

$$\Delta Q_i = (\lambda_i \cdot T_i - \lambda_{i-1} \cdot \Delta T_{i+1}) \cdot \frac{\Delta A_i}{\Delta z} \cdot \Delta t + F_i - \gamma_i . \tag{6}$$

The rise in temperature in the i th cell follows the equation

$$\Delta T_i = \frac{\Delta Q_i}{c_i \cdot \rho_i \cdot V} \tag{7}$$

with

$c_i = c(T_i)$	temperature depending heat capacity
$\rho_i = \rho(T_i)$	temperature depending density
$V = \pi \cdot r_0^2 \cdot \Delta z$	cell volume

From equation (6) and (7) one can derive the relation for the temperature in the *i* th cell at the time $t = l\ \Delta t$ (l denotes the number of the respective time step).

$$T_i = \frac{\lambda_{i,l-1} \cdot \Delta t}{c_{i,l-1} \cdot \rho_{i,l-1} \cdot \Delta z^2} \cdot T_{i-1,l-1} - \frac{(\lambda_{i,l-1} + \lambda_{i+1,l-1})}{c_{i,l-1} \cdot \rho_{i,l-1} \cdot \Delta z^2} \cdot T_{i,l-1} + T_{i,l-1} + \\ + \frac{\lambda_{i+1,l-1} \cdot \Delta t}{c_{i,l-1} \cdot \rho_{i,l-1} \cdot \Delta z^2} \cdot T_{i+1,l-1} + \frac{F_i}{c_{i,l-1} \cdot \rho_{i,l-1} \cdot V} - \frac{\gamma_i(T^*)}{c_{i,l-1} \cdot \rho_{i,l-1} \cdot V} \tag{8}$$

with $\gamma_i = \begin{cases} 0 & T < T^* \\ \gamma & T = T^* \\ 0 & T > T^* \end{cases}$.

Equation (8) is a recursive formula for the calculation of the temperature distribution in the slab for each time step.

Energy consuming conversion processes such as thermal chemical reactions, melting, etc. are simulated so that after the cell has reached the conversion temperature T^* heat energy is absorbed in the cell by maintaining the same temperature as long as the chemical or thermophysical reaction does not come to a close.

In vaporization processes (gravure plates, flexo plates) the vaporized cell no longer exists in the model and the next cell is regarded as top cell of the slab.

Furthermore, alterations of the thermophysical variables are neglected through the time span Δt and consequently treated as constants in each time interval.

The expression

$$Q = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^{2\pi} f_1(t) \cdot f_2(z) \cdot f_3(r,\varphi) \cdot r \cdot dr \cdot d\varphi \cdot dz \cdot dt. \tag{9}$$

results for the absorbed laser radiation energy in the slab volume, i. e., Q is a function of time and depth.

The function f_1 describes the time dependent behavior of the laser energy flux and f_2 represents a form function for the laser energy absorption in the bulk phase of the irradiated material.

C is a normalization factor remaining constant through the time span Δt.

To describe the time dependent behavior of f_1 the following choice of formulaes is appropriate:

$$f_1 = \begin{cases} 1 - exp\left(\frac{-t}{\tau_1}\right) & 0 \le t \le \tau_p \\ \left(1 - exp\left(-\frac{\tau_p}{\tau_1}\right)\right) \cdot exp\left(-\frac{t-\tau_p}{\tau_2}\right) & \tau \ge \tau_p \end{cases} . \qquad (10)$$

The values of the time parameters as τ_1 (switch on period), τ_2 (swich off period) and τ_p (swich off time) of the light source determine the time dependent behavior of the emitted laser energy by using pulsed lasers (comp. the figure 2).

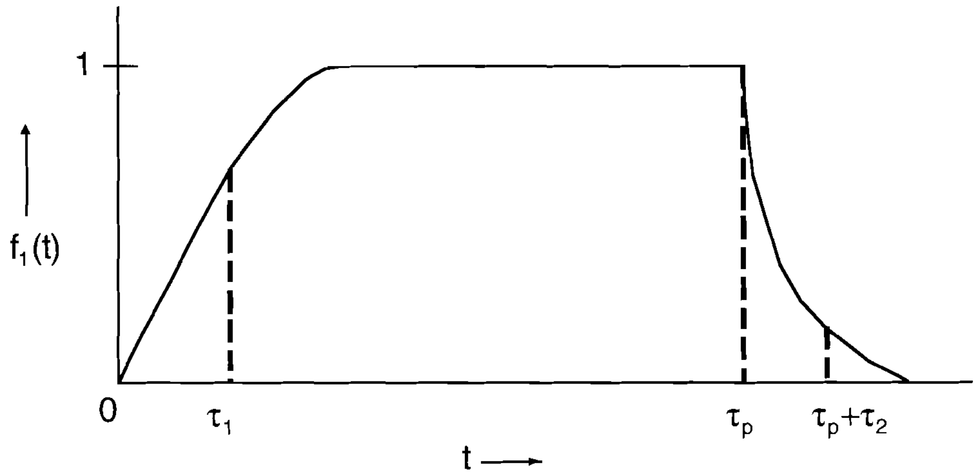

Fig. 2 : Time dependent shape of a laser pulse

Supposing temperature dependent reflection and / or absorption of the laser energy also the spatial function f_2 alterates.

Consequently, the temperature dependent behavior of the optical material parameters and the time dependent laser power require a recursive step by step calculation of the normalization factor C (the calculation procedure for the factor C is showed in the appendix).

By carrying out the successive iteration in (8), the actual temperature dependent values of the thermophysical parameters employed in the l th time step are determined by using the temperature calculated in the previous time step (l - 1). The approximation includes the assumption that the deviation of the temperature dependent parameter values is neglectable between two time steps. The data required for the calculation of the actual cell temperature are retrieved from diverse subroutines of the computer program after each time step.

To reach numerical stability one must take care to estimate the time span by employing the formula by J. von Neumann

$$\Delta t = \frac{c_{min}(T) \cdot \rho_{min}(T) \cdot \Delta z^2}{2 \cdot \lambda_{max}} . \qquad (11)$$

c_{min}, ρ_{min} and λ_{max} denote the lowest as well as the highest value of the temperature dependent physical parameters which are fitted by using appropriated polynom approximations.

For the choice of Δz the relation $\Delta z < \frac{1}{\alpha_{min}}$ is useful.

Conclusion

The knowledge of the time dependent spatial temperature distribution in laser irradiated plate materials enable us to adjust the set of laser parameters according to the thermophysical and optical parameters of the plate material. In the case of temperature dependent alterations of the thermophysical and optical data due to physical or chemical processes taking place in the laser exposed material zone, the heat conduction equation can not be solved by employing classical analytical methods.

Using the **FINITE DIFFERENCE METHOD** the calculation of the temperature according to the material depth and time can be carried out numerically.

The numerical procedure includes a step by step computation of the temperature in the cells of the spatial subdivided slab. The temperature dependence of the optical and thermophysical parameters is fitted by means of selected polynom approximations.

Appendix

Starting with the absorbed laser energy

$$Q = \iint D(r, \varphi, z, t) \cdot dV \cdot dt . \tag{12}$$

the energy density can be written as a product of the form factors f_1, f_2, f_3 and the normalization factor C :

$$D(r, \varphi, z, t) = C(t) \cdot f_1(t) \cdot f_2(z, t) \cdot f_3(r, \varphi) . \tag{13}$$

For the factor C (t) follows the expression:

$$C(t) = \frac{Q}{\int_0^{\infty} f_1(t) \cdot dt \cdot \int_0^{2\pi} \int_0^{\infty} f_3(r, \varphi) \cdot r \cdot dr \cdot d\varphi \cdot \int_0^{\infty} f_2(z, t) \cdot dz} . \tag{14}$$

The absorption of the laser light obeys the Lambert law:

$$f_2(z, t) = exp\left(-\alpha\left(z, T(z, t)\right) \cdot z\right) . \tag{15}$$

$\alpha(z, T(z, t))$ denotes the temperature dependent absorption coefficient which remains constant throughout the time interval Δt.

The successive iteration procedure to calculate the temperature in the cell divided slab requires a time subdivision of the total laser energy.

Considering the absorption behavior of the i th cell of the irradiated material in the l th time step, the intensity loss of the laser radiation after running across the cell is described by the function

$$f_2(z) = Q \cdot \underset{i=1}{\overset{n}{\pi}} \cdot exp\left(-\alpha_i \cdot z_i\right) exp\left(-\alpha_n\left(z - z_n\right)\right) . \tag{16}$$

with $z_n \leq z \leq z_{n+1}$

and $\alpha_i = \alpha \cdot \left(T_{l-1}(z_i)\right)$.

Furthermore taking into account the temperature dependent reflection coefficient $R(T)$ the definitive shape of f_2 is

$$f_2(z) = \left(1 - R\left(T_{l-1}(z_1)\right)\right) \cdot Q \cdot \underset{i=1}{\overset{n}{\pi}} \cdot exp\left(-\alpha_i(T_{l-1}) \cdot z_i\right) . \tag{17}$$

According to the thermophysical parameters the absorption α as well as the reflection coefficient R remain constant through the time step Δt.

For the l th time step the heat energy can be represented by the equation

$$Q_{\Delta t_i} = \int_0^{\infty}\int_{t_{i-1}}^{t_i}\int_0^{2\pi}\int_0^{\infty} C_{\Delta t_i} \cdot f_1(t) \cdot f_2(z) \cdot f_3(r,\varphi) \cdot r \cdot dr \cdot d\varphi \cdot dt \cdot dz. \tag{18}$$

By spatial subdividing of the slab the first integral in (18) can be approximated by a sum of integrals extending over the respective length of the

i th cell Δz:

$$\Delta Q_{\Delta t_i} = \sum_{i=1}^{m} \int_{i \cdot \Delta z}^{(i+1)\cdot \Delta z} \int_{t_{i-1}}^{t_i}\int_0^{2\pi}\int_0^{\infty} C_{\Delta t_i} \cdot f_1(t) \cdot f_2(z) \cdot f_3(r,\varphi) \cdot r \cdot dr \cdot d\varphi \cdot dt \cdot dz. \tag{19}$$

with $m = \frac{L}{\Delta z}$ (L - length of the slab).

For the constant C results the term

$$C_{\Delta t_i} = \frac{\Delta Q_{\Delta t_i}}{\sum_{i=1}^{m} \int_{i \cdot \Delta z}^{(i+1)\cdot \Delta z} \int_{t_{i-1}}^{t_i}\int_0^{2\pi}\int_0^{\infty} C_{\Delta t_i} \cdot f_1(t) \cdot f_2(z) \cdot f_3(r,\varphi) \cdot r \cdot dr \cdot d\varphi \cdot dt \cdot dz} \tag{20}$$

which is valid for the time step Δt.

Supposing a Gauss shaped laser beam profile the equation of $f_3\ (r,\varphi)$ is

$$f_3(r,\varphi) = exp\left(\frac{-2r}{\omega^2}\right). \tag{21}$$

ω denotes the Gauss radius.

From integration over the half space and over the whole angle follows the term

$$\frac{\pi \cdot \omega^2}{2} \cdot \sum_{i=1}^{m} \prod_{i=1}^{n} \cdot exp\left(-\alpha_i(T_{i-1}) \cdot z_i\right) \cdot \left(z_i - z_{i-1}\right)$$

n = 1,m

and leads to the equation

$$C_{\Delta t_l} = \frac{2 \cdot \Delta Q_{\Delta t_l}}{\left(1 - R\left(T_{l-1}(z_1)\right)\right) \cdot \pi \cdot \omega^2 \cdot \int_{t_{l-1}}^{t_l} f_1(t) \cdot dt \cdot \left(\sum_{i=1}^{m} \left(\prod_{i=1}^{n} \cdot exp\left(-\alpha_i(T_{l-1}) \cdot z_i(z_i - z_{i-1}\right) \right) \right)} . \qquad (22)$$

The values of the optical parameters α and R in equation (22) correspond with the respective cell temperature of the previous time step Δt_{l-1}.

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