An Investigation of Rubber Roll Distortion in a Web Offset Press

by

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SUMMARY

The paper describes two finite element modelling approaches to represent the nip junction in a printing press. The dry contact model may be used to determine the stress levels which are present in the roller system and may be built using commercially available software. The wet contact model combines the fluid flow through the nip and the elastic behaviour of the rubber covering. Stable solutions were achieved in which the pressure field compared favourably with experimental data. The analysis also showed the very high shear rates which are present in the nip due to the pressure gradient and how this 'works' the ink reducing its viscosity as it passes through the nip. The subsurface stresses in the elastomer are controlled principally by the level of engagement, whereas the energy dissipation rate in the elastomer is controlled by the frequency of cyclic stressing and the stress levels induced.The flow through the nip was found to increase with the level of engagement due to the increased width of the contact and was also found to increase nearly linearly with speed due to the dominance of Couette flow through the junction.

Introduction

Printing processes use a series of rollers arranged in a train to carry either ink or font solution from their supply duct to the substrate. The latter can vary from different weight paper for newsprint and catalogues to films for the food wrapping industry. In all cases, the fluid is usually transferred by a series of hard and soft rubber covered roller systems. Then, as the ink passes down the train, it is 'worked' by means of a shearing action in the nip contacts and this will have an effect on its viscosity characteristic at the printing plate. Research work [1] has shown that ink viscosity is one of the most important parameters in achieving high quality and consistent print and its control may also lead to a reduction in wasteage during the press startup period to achieve 'good copy'.

A numerical model may be developed in order to understand more fully the behaviour in the printing press nip with regard to ink viscosity, flow and the stresses developed within the roller systems. These are important since the viscosity of the working fluid affects print quality, the quantity of ink which is transferred to the plate influences the colour of the printed copy and the stress levels in the rubber roller are important with regard to its life (or failure). The purpose of this paper is to describe the development of models and their validation which may be used to investigate the nip details and to demonstrate their applicability to a printing press nip.

Analysis Background

The analysis of the nip contact can be achieved under the assumption of either a dry or wet junction. In a dry junction analysis, the effect of ink is neglected and approximates a roller running with a very thin ink film corresponding to an area of low print coverage. Under this circumstance, only the stresses in the roller contact are of importance and the analysis may be used to establish them. This type of analysis may also be used to optimise roller layup using different compounds to give appropriate mechanical characteristics, such as low internal mechanical hysteresis and stiffness to achieve the correct mechanical behaviour. Dry contact analysis is relatively straightforward to complete and can be based on a Hertzian model [2] where only a single material is concerned, in the case of composite structures, more complex numerical techniques are necessary to model the mechanical engagement and the consequent stress evolution.

In a wet contact, it is well known that when a fluid flows through a rolling nip, the entrained fluid develops a pressure field within the nip and maintains a small separation between the surfaces. Where one of the rolling surfaces is a soft elastomeric material, it permits mechanical engagement in the contact since the rigid cylinder indents the elastomer covered counterpart. When combined with the entrained fluids, this results in a soft elastohydrodynamic line contact where the hydrodynamic contribution is made by the entrained fluid and the elastic contribution arises from the deformation of the elastomer covered roller. In the case of printing and coating processes, the engagement is significant, however, the pressures developed are likely to be relatively small due to the very soft nature of the elastomer where 35 shore (A) hardness is typical. Finally, the fluids used in either printing or coating processes are usually non-Newtonian and generally exhibit a shear-thinning behaviour where the viscosity decreases as the shearing of the ink becomes more extreme. These aspects need to be taken into consideration in any modelling work which focuses on the nip interaction.

Numerical Model Development

From the available literature on numerical modelling applied to elasto-hydrodynamic lubrication [1-10], it may be concluded that little work has been presented which focuses on the non-Newtonian lubrication of a soft elastohydrodynamic contact. This has been identified earlier as a requirement for investigating the behaviour of the nip between rollers in a printing unit. Two models to describe this interaction will be considered in this study and these will be derived in the theoretical section of this paper.

In this study, two numerical models for the nip junction will be considered. The first will focus on a dry nip junction, whereas the second will include both elastic and fluid interactions.

Dry Contact Model

In the dry contact model, the focus of concern is the mechanical interaction between the rigid cylinder and the elastomeric counterpart with which it is in contact. The physics of a small displacement linear elastic analysis of such a contact is described fully in [11]. This physical model is appropriate in this case since the roller mechanical engagements are small (typically less than 0.5mm). The solution of this equation set may be achieved by means of the finite element method [12] and the adoption of a virtual work formulation leads to a matrix of equations given by:-

$$[K]_s\{\delta\} = \{f\}_s$$

where $[K]_s = \int_\Omega [B]^T [D][B] d\Omega$

In this equation set, the matrix [B] captures the strain displacement relationship and the matrix [D] the material properties which relate stress to strain. This provides the route for incorporating the plain strain model for the contact between two rollers. The force vector {f} reflects the loading which is imposed as the two roller surfaces come into engagement.

A schematic view of the dry nip model is shown in Figure 2 which includes also the elements which are used to represent the contact between the roller surfaces. In this study, line gap type elements are used. In the calculation, the surfaces are brought progressively into contact and the reaction at each point is activated automatically as the engagement develops. The stiffness matrix for this linear type of element is described in [13].

This form of analysis was completed using a finite element analysis package [14] and the roller pair was represented using the utilities which are available in the modelling system.

Wet Contact Model

In the wet contact then as explained previously the solution of the equations describing pressure generation in the fluid film and elastic deformation in the rubber layer covering the roller core need to be solved simultaneously. Such an analysis approach is necessary since the film thickness profile which is modified by the rubber layer deformation affects the pressure calculation and the pressure field imposes the load on the elastomeric surface. Thus the equations are coupled and highly nonlinear and are consequently extremely difficult to solve. This elastohydrodynamic lubrication analysis combines the solution for film pressure which is based on the generalised pressure equation and the equations of elasticity in the roller covering. The generalised pressure equation can be written as [15],

$$\frac{d}{dx}\left[G\frac{dp}{dx}\right]=U\frac{dh}{dx} \quad \text{.......... (1)}$$

where

$$G=\int_o^h \frac{y}{\mu}(y-F)dy$$

$$G=\int_o^h \frac{y}{\mu}dy;\ F_0=\int_o^h \frac{1}{\mu}dy;\ F=\frac{F_1}{F_0}$$

Then, if the variation of viscosity (μ) due to local shear effects is known over the film thickness, the integrals can be evaluated and the pressure equation (1) solved.

The viscosity field can be established most conveniently by means of a power law equation [16] where the shear stress is related to velocity gradient via the equation,

$$\tau=m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} \quad \text{.......... (2)}$$

The term $m\left|\frac{du}{dy}\right|^{n-1}$ represents a viscosity coefficient and for a Newtonian fluid when $n=1$, m is the dynamic viscosity and where $n<1$, the fluid assumes a pseudoplastic form and shear thins and which is appropriate for many inks. Under conditions of steady flow, equilibrium is expressed via the equation.

$$\frac{dp}{dx}=\frac{d\tau}{dy} \quad \text{.......... (3)}$$

or

$$\frac{dp}{dx}=\frac{d}{dy}\left[m\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}\right] \quad \text{.......... (4)}$$

When there is no slippage between rollers, the boundary conditions for equation (4) can be written as,

$$u=U \text{ for } y=O \text{ or } h \text{ and } \frac{du}{dy}=O \text{ for } y=\frac{h}{2}$$

The latter reflects the result that shearing of the fluid occurs due to the pressure gradient only and that there is symmetry about the film mid-thickness, roller rotation only introduces a bulk movement of ink through the nip. Integrating (4) for $y \geq \frac{h}{2}$ and $\frac{dp}{dx}>0$ which is appropriate in the nip upstream region gives,

$$\frac{du}{dy}=\left(\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}}\left[y-\frac{h}{2}\right]^{\frac{1}{n}} \quad \text{.......... (5)}$$

and

$$u(y)=\left(\frac{n}{n+1}\right)\left(\frac{1}{m}\frac{dp}{dx}\right)^{\frac{1}{n}}\left[\left(y-\frac{h}{2}\right)^{\frac{n+1}{n}}-\left(\frac{h}{2}\right)^{\frac{n+1}{n}}\right]+U$$

A similar set of equations may be derived in the region where $y \geq \frac{h}{2}$ and $\frac{dp}{dx}<0$ which corresponds to the nip outlet zone.

The velocity gradients in equation (5) may be used in the equation,

$$\mu=m\left|\frac{du}{dy}\right|^{n-1}$$

to establish the cross-film viscosity variation. In the case of a pseudoplastic fluid $n<1$, the overall value of the exponent becomes negative. Then, where velocity gradients are close to zero, the viscosity becomes large and physically unrealistic. To overcome this singularity, whenever the velocity gradient fell below a threshold value $\left(\frac{du}{dy}\right)_T$, the viscosity was fixed at a Newtonian value derived from experimental measurement. This approach gives a fall-off in viscosity at the roller surface while retaining a core section of uniform viscosity.

In contrast to the dry contact model, this coupled set of governing equations cannot be analysed using commercially available software and therefore a dedicated analysis code was developed during the research project. The following paragraphs outline the strategy which was adopted to solve these equations.

The equation (1) was solved by the finite element method and the governing equations were derived using the Galerkin weighted residual method [17].

The fluid matrix can be written,

$$\left\{ \int_{\Omega} \left[G_i \frac{dW_i}{dx} \frac{dN_j}{dx} \right] d\Omega \right\} \left\{ p_j \right\} = \int_{\Omega} W_i \left[U \frac{dh_i}{dx} \right] d\Omega$$

or

$$[K]_p \{p\} = \{f\}_p$$

The application of the finite element method to the elastic analysis has been described previously and leads to the matrix of equations

$$[K]_s \{\delta\} = \{f\}_s$$

This permits the direct calculation of displacement when the domain is subjected to traction loading. In this case, the load is restricted to the pressure component only since shear stresses are negligible by comparison. The loading was assembled into the vector $\{f\}_s$.

Closure of the equation set is facilitated by approximating the film as a cylinder-plane contact problem where the elastomer forms part of the plane surface as shown in Figure 3, then

$$h = h_o + \frac{x^2}{2R'} + \mathrm{v}(x)$$

and for two rollers of radii R1 and R2 an effective radius can be written

$$R' = \frac{R_1 R_2}{R_1 + R_2}$$

This parabolic approximation is a good fit over comparatively narrow nip widths and for the purpose of the present study is very appropriate since a typical nip subtends an included angle of up to about 10 degrees. The nominal film thickness for a rigid contact (h_0) can assume a negative value and under this circumstance represents the mechanical engagement of the rollers. The distortion v(x) is derived from a solution of the elasticity equations in the elastomeric layer.

Dry Contact Analysis

For this model, the calculation domain and boundary conditions are shown in Figure 2 which includes also the elemental division in both the roller and elastomer domains. In this model, the actual roller geometries are used (as opposed to the cylinder-plane geometry). The boundary conditions at the elastomer edges permit radial motion only and the contact between the rollers is frictionless and so only normal loading was applied at the contact. The material properties used for the rubber were Poissons ratio of 0.45 and elastic modulus of 2MPa whereas the opposing roller was assumed to be completely rigid by assigning to it the material properties of steel.

Typical results from this analysis are shown in Figure 4 in the form of radial stress contours through the elastomeric layer and rubber blanket deformation. These have been computed for a roller engagement of 0.3mm. This result shows clearly the high level of radial stress at the elastomer surface and the contact width which will be adopted as a starting point in the wet contact analysis. The results from the other case studies completed are listed in Table 1 and the values can be compared directly with the results from experiment conducted on an instrumented roller as depicted in Figure 5 where the pressure at the roller surface and at the back of the elastomer are presented. The material properties adopted were determined in separate experiments and in comparing the results in Figure 5 and Table 1, the predicted values are lower than those measured experimentally, however, the trend agreements are seen to be good. This shows clearly how the model can be used to estimate stresses in the body of the elastomer and provides the path for future exploitation to optimise rubber blanket performance.

Engagement (ho) (mm)	Surface stress (MPa)	Subsurface stress (Mpa)	Load (N/m)	Contact width (mm)
0.1	0.098	0.029	186.6	3.21
0.2	0.156	0.063	475.6	5.13
0.3	0.202	0.107	850.5	6.75

Table 1 Summary of maximum stress levels, roller load and contact width from the dry contact analysis

Wet Junction Analysis

The calculation domain and boundary conditions are shown in Figure 3. Inlet and outlet pressures are zero and the latter also satisfies the zero gradient condition by setting negative pressures to zero as they occur in the solution back substitution. This approach satisfies the continuity constraint on fluid flow and provides an automatic determination of the film splitting boundary. To perform the analysis also requires the specification of a contact width. The extent of the region upstream of the nip dictates whether the junction is either starved or flooded (see Figure 1). Thus the initial nip width was estimated from the experimental investigation and the separate numerical study on the dry contact In the elastomer domain shown in Figure 3, circumferential straining was allowed and perfect bonding to the steel core was assumed. Material parameters for the rubber were identical to those in the dry contact study and the rubber thickness was 8 mm being typical for rubber covered rollers as used in the printing process.

The solution method employed iteration between the hydrodynamic and elastomer models where it was necessary to apply relaxation to obtain convergence. Also, the engagement was introduced over the first 40 iterations and the solution was assumed to be complete when the maximum relative pressure change between consecutive iterations had reduced within 0.01% and the maximum relative change in the elastic deformation between consecutive iterations had reduced to less than 1%. This usually required about 500 iterations, however benefit was derived by using converged solutions as a starting point for the subsequent similar type of calculation. By using this approach it was possible to simulate the contact up to linear speeds of 10m/s which are typical for high speed printing and coating processes.

Figure 6 illustrates a comparison between measured and predicted film pressure for a roller surface speed of 3.0 m/s and an engagement of 0.2mm. In this calculation, the ink was characterised to be non-Newtonian and the coefficients m and n were derived from experiments on a typical ink and assigned values of 27 and 0.78 respectively. The cut-off for shear rate was set at 500 sec^{-1}. The agreement achieved between the computed film pressure and experimental measurement is close over part of the loaded film, but underpredicts the maximum pressure. The reasons for this are not immediately apparent and may be attributed to one of two causes in that either local stiffening of the elastomer may occur as described in [18] or there is a local increase in ink viscosity and that the power law representation is not appropriate. The figure also includes the result when the elastomer modulus is assigned a value of 3MPa from which it can be seen that the agreement with the experimental pressure is improved in both magnitude and form and this points towards the occurrence of a stiffening phenomenon as described in [18]. The results in Figure 6 also shows the film thickness profile through the nip where the form exhibits a tapered shape as opposed to a flat profile which is associated with a simple Hertzian model of the junction. No experimental film thicknesses were measured since at this time there are no suitable sensors which may be used with a rubber target surface, so no comparison with experiment has been presented.

In this analysis, the subambient pressure which was measured on film cavitation was not captured by the basic numerical model. Even when the algorithm was modified to allow negative pressure, the excursions measured exceeded those which were predicted. Further improvement in the predicted subambient pressure field could be achieved by extending the calculation domain and retaining the subambient pressure in the solution. This will also lead to a reduction in the maximum film pressure and at the same time violate flow continuity at the film outlet. Therefore a film rupture condition which includes surface tension effects could be considered to be more appropriate [19]. As explained in [20] models may be developed to predict the number of filaments which are formed and is based on the physics of balance between pressure and surface tension at the nip outlet. These ink filaments eventually break up to form 'misting' which has been observed at the contact outlet when running the press. This rupture behaviour was found to be notably present with the non-Newtonian behaviour in the film, it was not so evident when a simple Newtonian oil was used in the experimental programme. Such behaviour can not be captured using a simple power law equation, in this case more complex representations of the fluid behaviour need to be considered [21].

The detailed shear rate and viscosity fields throughout the film are depicted in Figure 7 for the conditions associated with Figure 6 for an elastomer modulus of 2MPa. The contours have been presented in a parametric geometrical frame over half the film only since they are imaged about the mid-height.

The shear rate contours depict clearly the high rates which are present at the inlet and cavitation points. This is particularly evident at the inlet, close to the roller surface and in each case this arises from the steep pressure gradients which are present in these zones. The shear rate on the film mid-height (i.e. y/h =0.5) reflects the cut-off value of 500 s^{-1} defined as input to the analysis. This figure shows clearly the magnitude of the shear rates which are present in the nip where they can be seen to achieve extremely high values of 4500 to 5000 s^{-1} and these will be commensurately larger at higher running speeds and at higher engagement where the pressure gradients in the film will be even more severe (see Figure 12). This points to the clear need for a viscometer which is capable of characterising the inks at these high shear rates. Corresponding viscosity fields from the power law model are also shown in Figure 7 where shear thinning is clearly evident in the areas of high shear rate close to the roller surface and at the film inlet and outlet regions. These viscosity fields have been captured in the solution of the generalised pressure equation.

The analysis also yields the stress field generated in the elastomer and as an example, the maximum principal stress field corresponding to the condition in Figure 6 is shown in Figure 8. This shows clearly the high compressive stresses which are generated at the inner and outer rubber layer surface and the high tensile stresses which are generated in the rubber subsurface. Clearly the stress gradients in the elastomer are severe and the subsurface stresses contribute to the failure of the rubber rollers when they are operated at high speed, particularly when there is more than one rigid roller in contact with the elastomeric partner. Methods for dealing with this are described in [22], but this may only be used when the generation rate internal to the elastomer is known and which may only be derived from an analysis of this type.

Figure 9 illustrates the variation of flow rate through the nip junction for three levels of engagement up to a linear speed of 3m/s. This graph shows a surprising result in that the ink flow through the nip appears to increase as the engagement increases. The reason for this is not immediately obvious, but may be traced to the broadening of the contact strip with its attendant increase in load carrying ability. The dominant flow contribution arises from the convected component as opposed to the pressure term, however the overall flow through the nip was found to be constant, conforming the accuracy of the solution and the ability to satsify the governing equations.

Calculations were also carried out to illustrate the applicability of the model to high speed printing processes. Therefore the principal factor investigated was printing speed. Calculation was completed for linear speeds up to 10m/s at an engagement of 0.2mm. Pressure profiles in the film and the film thickness profiles in Figure 10. These figures show clearly the increase in film pressure and thickness which occur as the speed is increased. The contact width also increases by the delay of the rupture due to the higher pressures which are generated in the contact, thus pumping a larger amount of ink through the nip. In examining the effect of speed, the initial changes which occur in both pressure and film thickness are most significant. With further calculation it would be possible to develop equations relating film thickness to roller design, material properties and operating conditions. This is seen as a natural development of this work and may be tackled at a later date.

Shear rate and viscosity results are shown in Figures 11 and 12 and the former depicts clearly the severe nature of the shear rate which may be encountered in the film, being typically up to 12,000 s^{-1} and which occurs in the nip entry region. The low level of shear rate at the central region of the nip is a consequence of the null pressure gradient in the nip which is not affected by the linear speed. The resultant viscosity field is shown in Figure 12 and this depicts the way in which the viscosity changes in the contact. It shows also how the nip 'works' the ink and changes its viscosity and which is most clearly eveident at the higher speed.

The levels of maximum principal stress in the nip are shown in Figure 13 where they can be seen to be independent of speed, principally since the change in film thickness with speed is minimal in comparison with the initial engagement as shown in Figure 10. Thus the subsurface stresses in the roller are determined by the initial mechanical engagement when the press is set up and not by running conditions. However, when running speeds are high, the frequency of stress cycling is increased and this appears as a proportional increase in the heat generation term in the roller subsurface and it is this which eventually leads to failure of the roller at the subsurface.

Finally, the ink flow through the nip is shown in Figure 14 for a linear speed up to 10 m/s where it can be seen to be nearly linear in form. This confirms the dominant nature of the Couette component of flow through the nip and which has been discussed previously in connection with Figure 9.

Conclusion

A non-Newtonian model to predict the elastohydrodynamic behaviour in a soft rolling contact has been developed and tested against experimental data. The work shows that good comparison may be achieved in the positive pressure zones, however, the power law model is not capable of capturing the details in the rupture zone. The model has also depicted the form of the stress contours in the elastomer and the severe stress gradients which exist through the rubber layer thickness. The model also suggests that thicker films are generated in the case of larger engagement of the rollers due to the broadening of the contact width. In running the model over a wide speed range, it showed how shear thinning of the ink occurs as it is worked in the nip and the that the stresses in the rubber are determined by the level of engagement used in setting up the press. The flow through the junction is dominated by the Couette component and is therefore a direct function of film thickness and roller surface speed.

Notation

$[B]$	strain matrix
c	half-contact width
$[D]$	elasticity matrix
E_R	elastic modulus of elastomer (rubber)
$\{f\}$	loading matrix
h	film thickness
h_o	roller engagement/separation at $x = 0$
$[K]$	stiffness matrix
m	power law consistency index
N_j	element shape function for node j
n	power law exponent
p,	pressure
R	equivalent roller radius
u	fluid velocity
U	roller surface velocity
v	elastomer deformation due to pressure
W_i	weighting function for nodal equation i
$\{\delta\}$	displacement matrix
μ	viscosity of fluid
τ	shear stress
Ω	element integration domain

References

[1] Bohan Mark F.J., Claypole Tim C. and Gethin David T.; The Application of Taguchi Methods to the Study of Ink Transfer in Heat Set Web Offset Printing: *Proc TAGA 1995*, p513-530.

[2] Dowson, D and Higginson, G.R.; Elastohydrodynamic Lubrication; Published by Pergammon Press, 1977.

[3] MacPhee, J., Shieh, J. and Hamrock, B.J.; The Application of Elastohydrodynamic Lubrication Theory to the Prediction of Conditions Existing in Lithographic Printing Press Roller Nips; *Advances in Printing Science and Technology*, 21, 1992, p 242-276.

[4] Hannah, M.; Contact Stress and Deformation in a Thin Elastic Layer; *Quarterly Journal of Mechanics and Applied Maths*, 4, 1951, p 94-105.

[5] Miller, R.D.W.; Some Effects of Compressibility on the Indentation of a Thin Elastic Layer by a Smooth Rigid Cylinder; Applied Scientific Research, 16, 1966, p 405-424.

[6] Hooke, C.J. and O'Donoghue, J.P.; Elastohydrodynamic Lubrication of Soft, Highly Deformed Contacts; Proc. I.Mech.E., Journal of Mechanical Engineering Sciences, 14, 1972, p 34-48.

[7] Jacobson, B.O. and Hamrock, B.J.; Non-Newtonian Fluid Model Incorporated into Elastohydrodynamic Lubrication of Rectangular Contacts; Trans. ASME (JOLT), 106, 1984, p 275-284.

[8] Hall, R.W. and Savage M.D. Two Dimensional Elastohydrynamic Lubrication Part 2: Solution of Line Contract Problem; Proc. I. Mech. E., 202, 1988, p354-360.

[9] Jaffar, M.J.; Estimation of Minimum Film Thickness for Line Contacts in the Transition Region Between the Isoriscous-Elastic and the Piezoviscous-Elastic Lubrication Regimes; Proc. I.Mech.E., Journal of Mechanical Engineering Sciences, 203, 1989, p379-386.

[10] Khonsari, M.N. and Hua, D.Y.; Generalised Non-Newtonian Elastohydrodynamic Lubrication, Tribology International, 26, 1993, p405-411.

[11] Timoshenko, S. P. and Goodier J. N.; Theory of Elasticity, published by McGraw Hill, 1970.

[12] Zienkiewicz, O.C.; The Finite Element Method, Edition 3; Published by McGraw Hill, London, 1977.

[13] Mazurkiewitz, M. and Ostachowitz, W.; Theory of Finite Element Method for Elastic Contact Problems of Solid Bodies; Computers and Structures, 17, 1983, p51-59.

[14] PAFEC Ltd., Strelley Hall, Strelly, Nottingham, UK.

[15] Dowson, D.; A Generalised Reynolds Equation for Fluid Film Lubrication; International Journal of Mechanical Engineering Sciences., 4, 1962, p 159-170.

[16] Bird, R.B., Stewart, W.E. and Lightfood, E.N.; Transport Phenomena, published by Wiley, 1960.

[17] Finlayson, B.A.; The Method of Weighted Residual and Variational Principles; Published by Academic Press, 1972.

[18] Lindley, P.B. ;Engineering Design with Natural Rubber, NR Technical Bulletin, Kuala Lumpur, Malaysian Rubber Producers' Research Association, 1992.

[19] Coyne, J.C. and Elrod, H.G.; Conditions for the Rupture of a Lubricating Film; Part 1: Theoretical Model; Journal of Lubrication Technology, 92, 1970, p451.

[20] Full, C.; A Theoretical Model of Striated Film Rupture Applied to the Cylinder-Plane; Trans ASME (JOT), 107, 1985, p419-422.

[21] Wilkinson, W.L.; Non-Newtonian Fluids - Fluid Mechanics, Mixing and Heat Transfer; Published by Pergamon Press Ltd, 1960.

[22] Gecim, B. and Winer, W.O.; Effect of a Surface Film on the Surface Temperature of a Rotating Cylinder; Trans ASME (JOT), 108, 1986, p92-97.