A Study of Roller Deformation Using Finite Element Analysis

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Abstract: A formula is derived for calculating stripe width versus load for a rubber covered press roller. The results of nine new sets of measurements of stripe width versus load together with three earlier ones, and the derived formula, are used to determine the degree of correlation between Young's modulus and various measurements of durometer. The correlations are then compared with one derived by others, from an analysis of durometer gauge indentation. Conclusions are reached regarding the most accurate method both for measuring durometer and for assessing Young's modulus of a rubber roller. Calculations of stress and strain in a typical rubber offset press roller at its conjunction with a rigid flat plate, using the finite element analysis (FEA) method, are also presented. The results obtained, of stripe width versus load over a stripe width range of 0.13 to 0.46 inch for a 3 inch diameter roller, are shown to support the value of Young's modulus inferred from stripe versus load measurements. It is also shown that finite element analysis is a powerful tool for gaining insight into nip behavior.

Introduction

Compliant or rubber-like rollers are key elements of both the inking and dampening systems used on lithographic presses. Therefore, it would seem that

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much could be gained by carrying out calculations of their behavior using fnite element analysis (FEA) methods. The mechanical properties of such rollers are usually defined in terms of a single parameter: Type A (or Shore A) durometer rending (ASTM, 1992) which is a measure of hardness. In order to carry out these calculations of roller performance, as for example load at a given stripe width, it is necessary to know the value of either Young's modulus, E, or the shear modulus of elasticity, G, of the rubber. Unfortunately, the known correlations between Young's modulus and durometer reading differ by as much as a factor of two. To address this uncertainty, additional experiments, and analyses were carried out.

The main objective of this paper is threefold: to present an appropriate formula for calculating stripe width versus load in a roller nip, to identify a reliable method for obtaining a measure of roller hardness that can be used to assess the value of Young's modulus, and to demonstrate that finite element analysis method calculations provide confirmation of the derived formula and method, and insight into nip behavior.

The three succeeding sections describe the work carried out to achieve the stated objectives. These are followed by a section that contains a discussion and a list of conclusions drawn from this work.

Closed Form Calculation

It might appear that it is a straightforward task to calculate the stresses and strains existing in the nip formed by the conjunction of a rigid steel-like roller and a roller with a thin compliant cover, using well known formulas. There are, however, two complications. The first is the inappropriateness of the equations based on the theory of Hertz for stresses resulting from contact between curved bodies (Hertz, 1881). The second of these is the above mentioned uncertainty in what value of Young's modulus should be assigned to a roller of given durometer. The first of these complications is taken up in the following paragraphs.

It was frst pointed out by Hannah (Hannah, 1951) that the equations based on Hertz's theory, for two cylinders in line contact, do not apply if one of the cylinders is a composite one consisting of a rigid steel-like core covered by a thin compliant or rubber-like material. Subsequently, Deshpande (Deshpande, 1978) derived specific equations, based on a solution to the non-Hertzian problem by Meijers (Meijers, 1968) for calculating both stripe width and squeeze versus load for the case where the composite cylinder has a very thin cover, as for example, a blanket cylinder. For a typical press roller pair, where the compliant roller has a much thicker cover, Deshpande's more general equation (5) can be used as the basis for obtaining the appropriate relationship between stripe width and load. This is done by rearranging Deshpande's equation (5) as follows:

$$\left(\frac{FR}{Eh^2}\right) = P_1\left(\frac{c^2}{h^2}\right)^{P_2} \qquad (1)$$

where:

F = load (pounds per inch of roller length)
E = Young's modulus of compliant roller cover (pounds/inch^2)
h = thickness of compliant roller cover (inch)
c= one half width of contact (inch)

$$R = \left(\frac{R_r R_c}{R_r + R_c}\right)$$

R_r = radius of rigid roller
R_c = radius of compliant roller
Poisson's ratio of compliant roller cover = 0.5
P_1 and P_2 are functions of the value of the ratio c_{max}/h

Equation (1) in turn can be rearranged to obtain an expression for stripe width as follows:

$$S = 2\left(\frac{R}{EP_1}\right)^m h^n F^m \qquad (2)$$

where:

S = 2c = stripe width
m= $1/P_2$
n = $(P_2-1)/P_2$

It has been found that, for given ranges of the ratio c/h, reasonably accurate results are obtained if P_1 and P_2, and hence m and n, are assumed to be constant. To find the appropriate constant values the following procedure is used:

1. Calculate values of the expression on the left side of equation (1) versus the ratio c/h using Deshpande's equation (5).
2. Plot the calculated values versus the ratio c/h for values of c/h ranging from zero up to various maximum values of c, defined as c_{max}.
3. Fit the various plots to equation (1) to obtain values of P_1 and p_2 for each value of c_{max}/h.
4. Calculate the corresponding values of m, n, and S_{max}/h.

The relationships of m and n so obtained are shown in Figure 1 as a function of the ratio S_{max}/h, over the range $0 < (S/h) < 1.5$. The value of P_1 over this same range varies from 1.047 to 2.017.

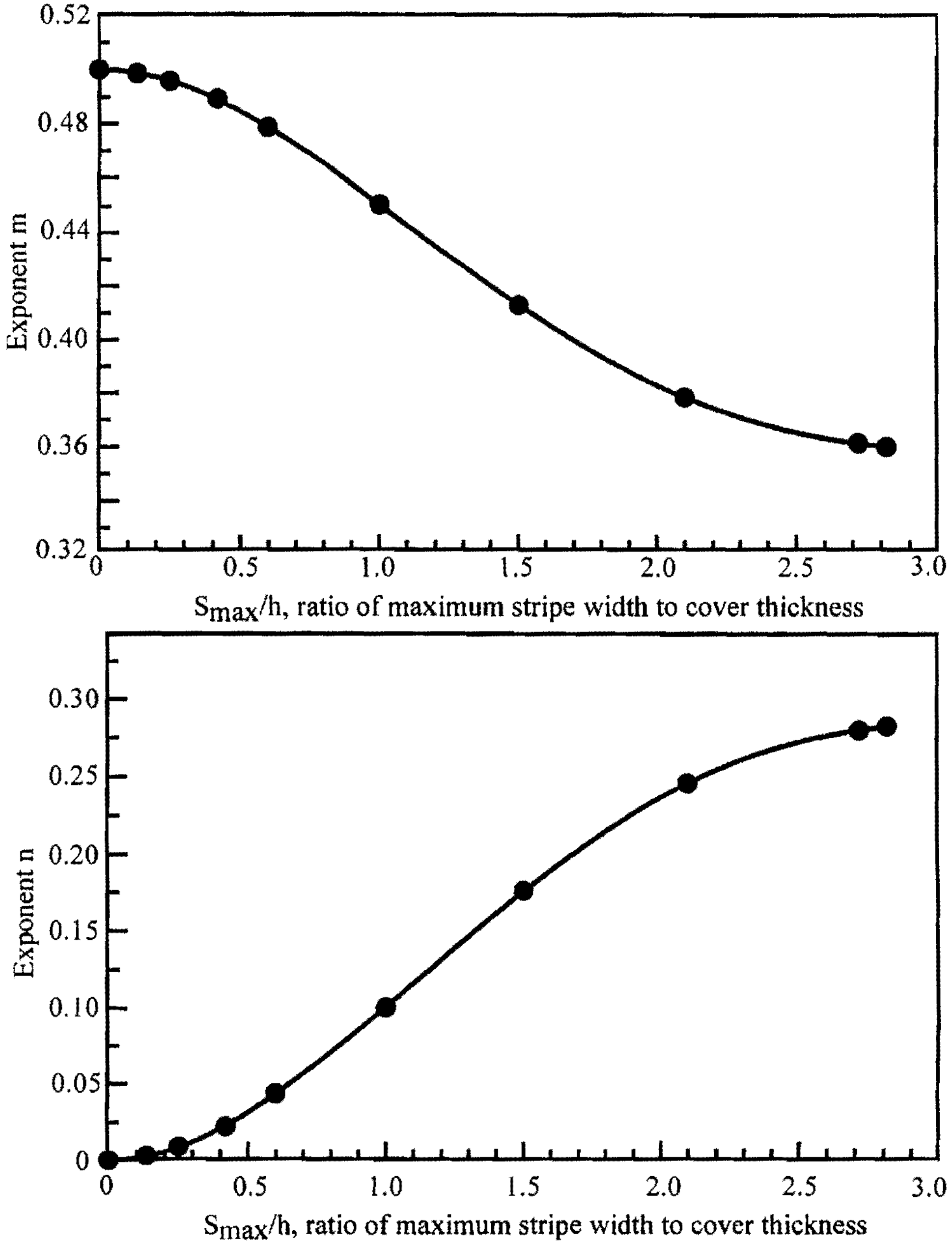

Figure 1 Plots of relationships between the exponents m and n in equation (2) and the ratio of maximum stripe width to cover thickness.

Examination of the two plots in Figure 1 reveals that for very small values of S_{max}/h, i.e. small indentations of the compliant cover, the exponent m approaches 0.5 and the exponent n approaches zero. Equation (2) then takes on the following form where stripe width is independent of cover thickness:

$$S = 2\left(\frac{R}{1.047E}\right)^{0.5} F^{0.5} \qquad (3)$$

This is Hertz's equation for parallel cylinders that is to be found. in standard handbooks (Roark, R.J. and Young, W.C., 1975), for the case where Poisson's ratio is 0.5, i.e. for rubber. For very large values of S_{max}/h, i.e. very deep indentations, m and n both approach 0.333 and equation (2) takes on the final specific form derived by Deshpande as follows:

$$S = 2\left(\frac{Rh}{2.017E}\right)^{0.333} F^{0.333} \qquad (4)$$

This equation is appropriate for blanket cylinders where stripe width is several multiples of blanket thickness. (The reader is cautioned that the applicability of this equation for values of S_{max}/h greater than 12 has not been explored.)

For rollers, it has been found in the course of this work that values of m corresponding to a S_{max}/h ratio of about 1.5 generally produce a good fit of equation (2) to measured values of stripe width versus load. Thus the equation to be used for relating stripe width to load in press roller nips is as follows:

$$S = 1.572\left(\frac{R}{E}\right)^{0.412} h^{0.176} F^{0.412} = CF^{0.412} \qquad (5)$$

where:

C= a constant for a given roller and maximum stripe width

The above equations show that the relationship between stripe width and load depends not only on geometry but also on the stiffness of the compliant roller cover, as expressed by the value of Young's modulus, E, for the cover material.

Experimental data obtained for two different types of rollers have been used to verify the correctness of Deshpande's approach. The first, plotted in Deshpande's paper, showed good agreement between measured values (Miller and Poulter, 1962) and calculated values of penetration, or squeeze, versus load for a 6 inch diameter cylinder with a relatively hard (approximately 60 durometer) cover, only 0.13 inch thick. The value of E (436 pounds/square inch) used in the calculations was derived by Miller and Poulter from measurements of rubber cover hardness. This first set of measurements is representative of blanket cylinders.

The experimental data obtained for the second type of rollers, stripe versus static load for typical press rollers of varying hardnesses and geometry are described in the section that follows.

Correlation of Durometer and Young's Modulus

Two entirely different methods have been used to develop a correlation between durometer reading, the property employed to specify rubber hardness, and Young's modulus of elasticity, the mechanical property needed in stress-strain calculations. In Method 1 (Briscoe and Sebastian, 1993) traditional elasticity theory was used to derive the equation that describes the compliance curve of an indented rubber sample for the indenter geometry used in the Type A durometer gauge, in terms of load, depth of penetration, and modulus of elasticity, E, of the sample. This equation was then used to calculate compliance curves for seven different values of E. When plotted, the intersections of these

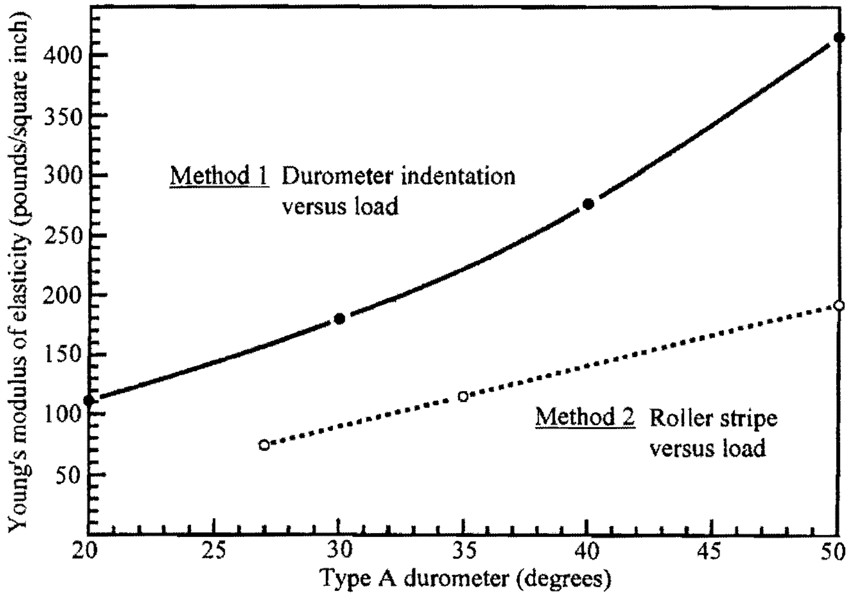

Figure 2 Comparison of two different correlations of Young's modulus and Type A durometer hardness.

curves with the compliance curve of the spring-loaded indenter of the durometer gauge produced a correlation between Type A durometer reading and E. The validity of the derived relationship was then checked by comparing measured and calculated compliance curves for three different samples of rubber having Type A durometer readings of approximately 28, 50, and 85. These comparisons showed good agreement.

In Method 2, measured data of stripe width versus load for three different rollers of 27, 35, and 50 Type A durometer, 2 7/8 inch in diameter, indented by a flat plate, (MacPhee and Wirth, 1989), were fitted to the 1/3 power law curves of Deshpande and the corresponding values of E (74, 115, and 193 pounds/square inch respectively) were calculated using equation (4).

Figure 2 shows that the correlations obtained from these methods differ by over a factor of two, with the load versus durometer indentation method yielding a higher value of E for a given durometer reading. The low slopes of the curves in Figure 2 seemed to rule out the possibility that this difference could be attributed to errors in measuring durometer hardness. To resolve this anomaly, a series of additional stripe versus load measurements were undertaken in parallel with the FEA calculations that are reported on in the next section.

The three rollers used to obtain the stripe versus load measurements plotted in Figure 2 were no longer available. Therefore, the additional measurements of stripe versus load and durometer were made on nine different rollers of varying geometry and cover composition. These are identified as Rollers 4 - 12 in Table 1, while the rollers used to obtain the earlier data in Figure 2 are identified as Rollers 1 - 3.

Table 1 Characteristics of rollers used in stripe versus load measurements.

Roller No.	Diam. (inch)	Cover thickness (inch)	Type A Durom.	Best fit value of m in equ. (2)	Best fit value of C in equ. (5)	Derived value of E (psi)
1	2.875	0.312	27	0.327		74
2	2.875	0.250	35	0.368		115
3	2.875	0.250	50	0.345		193
4	3.57	0.513	26	0.418	0.3540	50
5	2.56	0.440	25	0.413	0.2528	76
6	2.75	0.438	30	0.406	0.2329	99
7	3.22	0.396	46	0.417	0.2300	115
8	2.00	0.375	24	0.416	0.2184	79
9	2.00	0.375	24	0.404	0.1979	100
10	2.00	0.375	40	0.406	0.1607	168
11	2.00	0.375	25	0.416	0.2663	49
12	2.00	0.375	28	0.414	0.1989	99
Mean value for Rollers 4 to 12				0.412		

In making these new measurements, the previous procedure (MacPhee and Wirth, 1989) was used with the following improvements:

1. The hinged plate was equipped with frictionless bearings.
2. For each measurement, a strip of transparent tape was fastened to the plate so as to obtain an imprint of the stripe formed with the inked rubber roller.
3. Following, the imprint, the tape was removed, fastened to a piece of cardboard, and stripe width was measured in three places using a vernier gauge.

It should also be noted that the new measurements reported on here are for the case where the weighted plate was left on impression for ten seconds, so as to emulate the procedure used to set roller stripes on press.

Figure 3 shows plots of the data obtained on Rollers 8, 9, and 11. The good fits exhibited by these three data plots to equation (2) are typical of all the data for Rollers 4 - 12.

The values of the exponent m in equation (2) obtained from the fitted curves are listed in Table 1. The mean value of 0.412 is in remarkable agreement with the value obtained in the course of deriving equation (5).

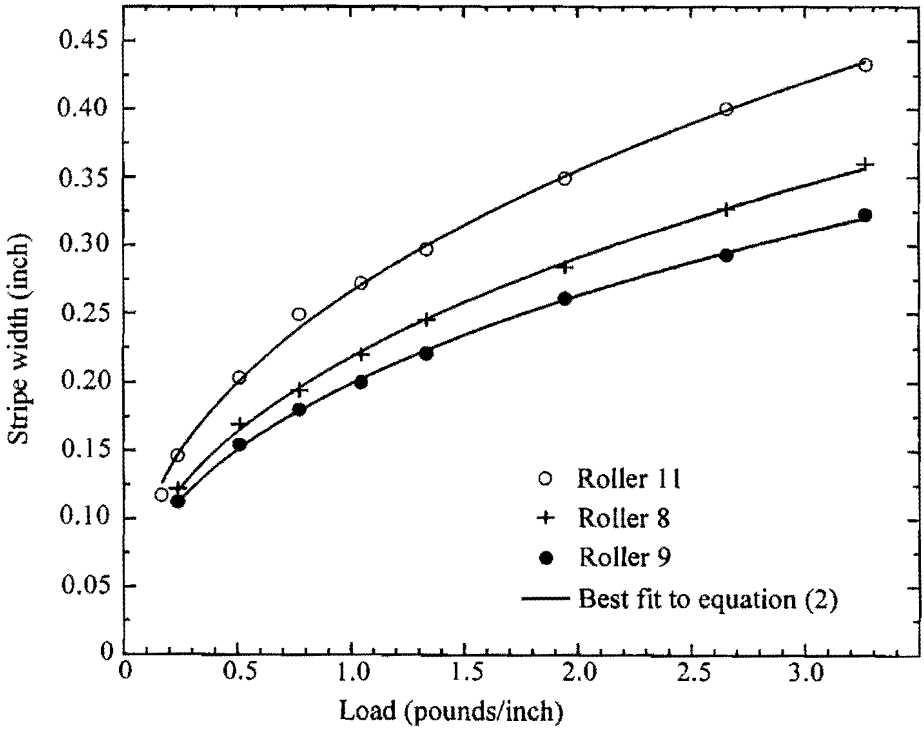

Figure 3 Plots of typical measured values of stripe width versus load.

The estimates of Young's modulus E derived from these measurements were obtained in accordance with the following procedure:

1. A best fit of equation (5) was obtained for each set of measurements using a non-linear-least-squares method. This yielded the values of C given in Table 1.
2. E was calculated from C using equation (5) and the roller properties R and h given in Table 1.

The Type A durometer hardness of each roller was also measured to enable the plot of Young's modulus versus durometer, given in Figure 4, to be generated. These measurements were disappointing in that they disclosed a very poor correlation between Young's modulus and durometer. For example, almost identical durometer readings of 24, 24, and 25 were obtained for three rollers with identical geometry, numbered 8, 9, and 11, that displayed values of E that ranged over a factor of two. That this difference in E is real can be seen from the data in Figure 3, that shows that twice the load is required to produce a given stripe width on Roller 9, compared to Roller 11, i.e. that Roller 11 is clearly

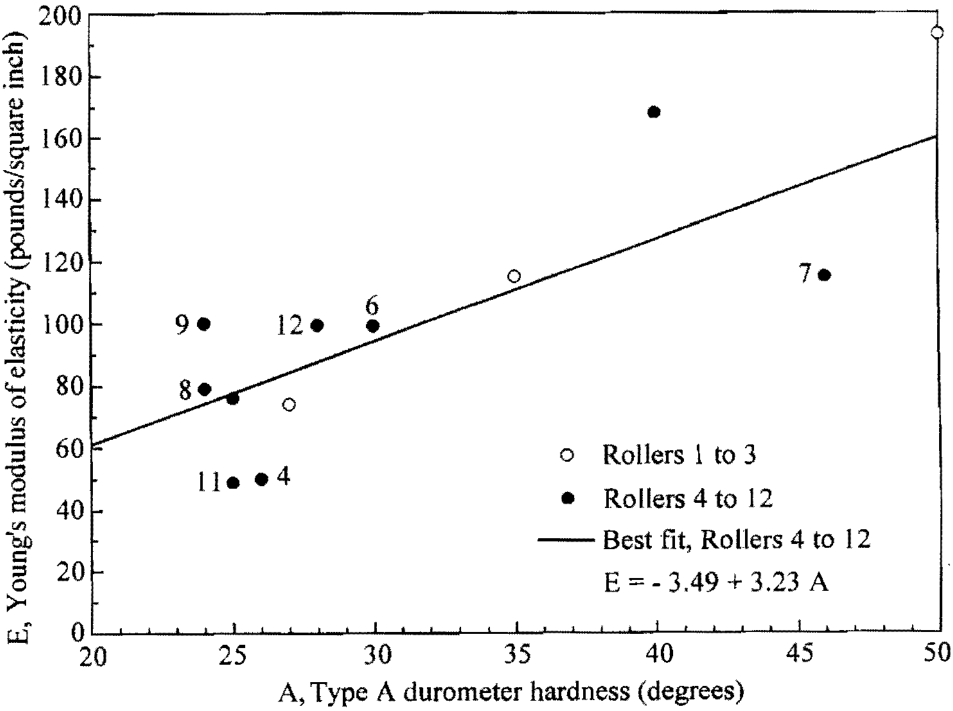

Figure 4 Correlation of Young's modulus and Type A durometer based on current measurements. Durometers obtained using rolling technique. Numbers identify rollers as listed in Table 1.

twice as soft or compliant as Roller 9. Thus the anomaly exhibited by the data on these two rollers in Figure 4 is clearly due to the inability of the durometer gauge, as used, to distinguish between the two rollers. Conversely, the durometers obtained for Rollers 6, 7, 9, and 12 ranged from 24 to 46 even though the corresponding range in Young's modulus was only from 99 to 115 pounds per square inch.

Based on these findings, it was decided to explore the use of different durometer gauges and different techniques for making measurements with them. Three different gauges, a Type A, a Type O, and a non-standard gauge were employed; and two different measuring techniques, dubbed rolling approach and fixed travel approach, were used. In addition, the effect of lubricating the contact area of the durometer indenter was investigated.

The Type O durometer gauge is identical to the Type A except that a 3/32 inch diameter spherical indenter is used in place of a truncated cone. The non-standard gauge had a spherical indenter with a diameter of 0.20 inch and a 1300 gram spring force when fully compressed, compared to 822 grams for the Types A and O (ASTM, 1992).

In the rolling approach technique, an edge of the gauge foot was placed on

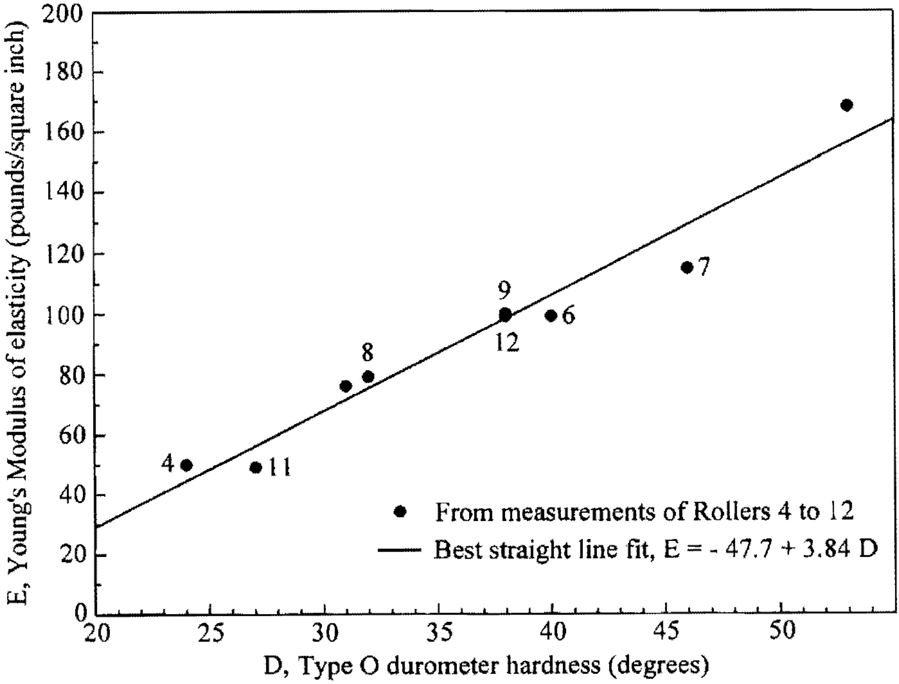

Figure 5 Correlation of Young's modulus and durometer hardness obtained using a Type O durometer gauge mounted in the quill of a milling machine so as to prevent deformation of the roller by the gauge foot. Numbers identify rollers as listed in Table 1.

the roller, parallel to the roller axis. The gauge was then rolled over the roller surface to obtain a reading. In the second technique, the gauge was mounted in the quill of a milling machine and moved into contact using the quill lever. Quill travel was limited by an adjustable stop to insure that the roller was deformed only by the gauge indenter and not by the gauge foot.

In all, eight different sets of measurements were made. The combination that resulted in the least scatter in plotted data was that of the Type O durometer gauge and the fixed travel approach measuring technique. The much better correlation between Young's modulus and durometer hardness thus obtained can be seen in the plot given in Figure 5, as compared to Figure 4. Tn comparing Figures 4 and 5 it is strikingly evident that the Type O gauge, as used, is much better able to discriminate roller cover compliance.

The results from all eight combinations were quantitatively assessed by calculating the standard deviation of the errors between the values of Young's modulus obtained from the measurements and the corresponding values obtained from the respective best straight line fits of the data. For example, the standard deviation for the data in Figure 4 is 27.5 while that of Figure 5 is 8.5 pounds per square inch. Figure 6 shows how the various combinations performed, as assessed by this method.

These results indicate that the poor performance of the Type A durometer is not improved either by technique or by lubricating the sample. Conversely, the

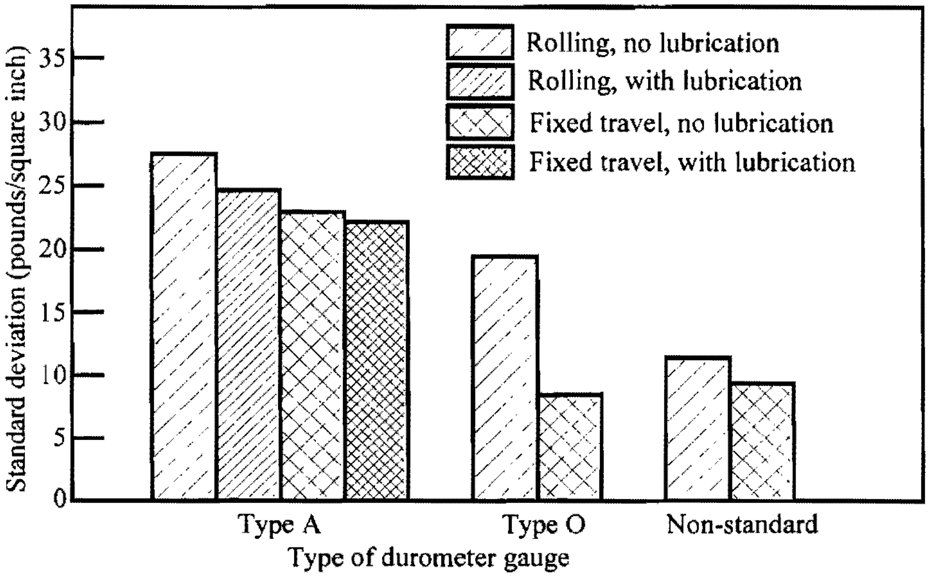

Figure 6 Relative performance of different durometer gauges and measuring techniques. Standard deviation is of the errors between the values of Young's modulus obtained from a given set of measurements and the corresponding values obtained from the respective best straight line fit of the data.

fixed travel technique produces a big improvement in the Type O gauge measurements, which otherwise are not much better than the Type A. The non-standard gauge, with its relatively large spherical indenter, performs very well independent of technique. A common characteristic of all of the correlations derived from measurements of Rollers 4 to 12 is that they are consistent with the Method 2 curve given in Figure 2.

Finite Element Analysis Calculations

The primary reason for carrying out calculations based on the finite element analysis method was to obtain a picture of the deformation of a typical rubber press roller in the region of the nip formed with an adjoining rigid roller. Secondary reasons were to confirm both the closed form method of calculation represented by equation (5) and the method of deriving values of Young's modulus from durometer measurements. Accordingly, calculations were carried out for two different configurations as follows:

Configuration 1 A 3 inch diameter roller with a 5/16 inch thick rubber cover, indented by a flat steel plate.
Configuration 2 A 3 inch diameter roller with a 3/8 inch thick rubber cover, indented by a 3 inch diameter steel roller in parallel.

Configuration 1 was selected because it corresponds closely to the conditions

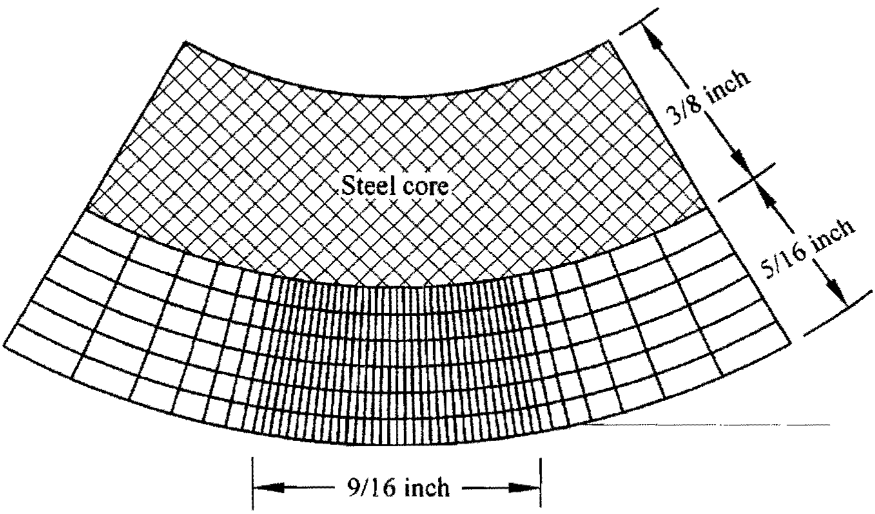

Figure 7 Mesh geometry used in calculation of Configuration 2.

under which the measured data for Roller 1 in Table 1 was obtained. Configuration 2 was selected as being representative of the roller nip in the inking system of a medium size press. In all of the calculations the modulus of elasticity used was 74 pounds per square inch, corresponding to that of Roller 1 in Table 1, based on the Method 2 correlation between modulus and durometer shown in Figure 2. The bulk modulus of the rubber was assumed to be 200,000 pounds/square inch The stresses and strains were calculated in a 60 degree sector of the rubber cover, centered under the nip. The mesh geometry used to simulate the rubber cover in Configuration 1 is shown in Figure 7.

For Configuration 1, calculations were carried for six different indentations ranging from 0.0021 to 0.021 inch.. Each set of results was then used to evaluate stripe width and load. Figure 8 is a plot of the six calculated stripe widths versus load. Also shown are the measured data and a plot of equation (5). For all practical purposes, the slightly larger radius of 1.5 inch used in the finite element analysis calculations vis-à-vis the 1.438 inch radius of the roller used in obtaining the measured data is of no significance. (Equation (5) predicts a corresponding difference in stripe width of only 1.4 percent at a given force.)

As can be seen, all three sets of data are in reasonably good agreement, except for the next to the highest value obtained from the finite element analysis

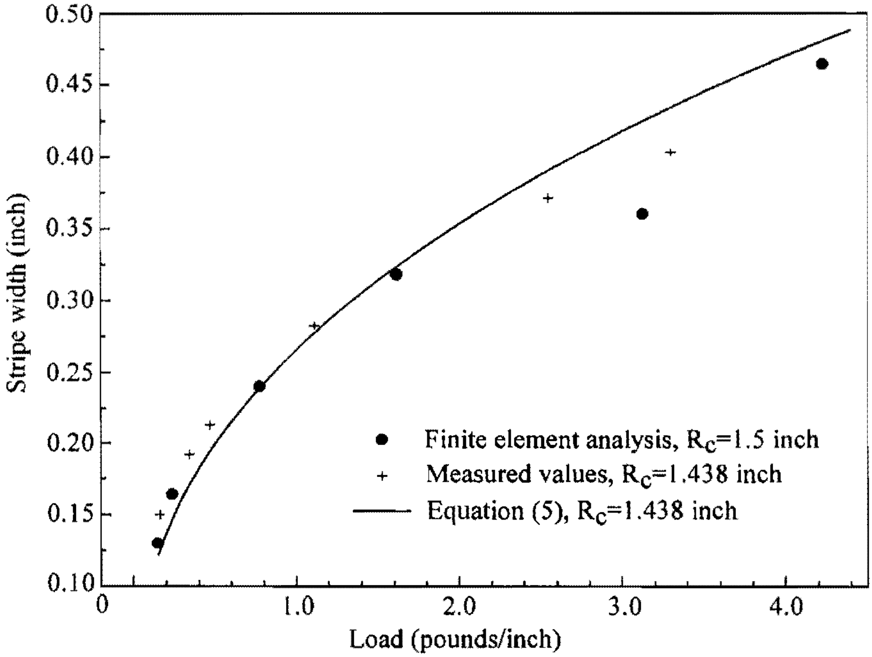

Figure 8 Data for rubber covered Roller 1 in Table 1, indented by a flat plate. Roller cover thickness is 5/16 inch, modulus of elasticity is 74 pounds/square inch, and measured Type A durometer is 27.

calculations. One possible explanation is that at this stripe width (0.360 inch), stripe width begins to approach the width of the volume occupied by the closely spaced mesh points used in the finite element analysis calculations, as shown in Figure 7. It is also to be noted that these stripe widths greatly exceed the stripe width of 3/16 inch recommended for this size roller (based on U.S. practice). Therefore it is believed that it can be argued that the conformity of the data in Figure 8 lends credence to both the correlation of Young's modulus and durometer obtained using Method 2 and the finite element analysis method of calculating rubber roller deformation.

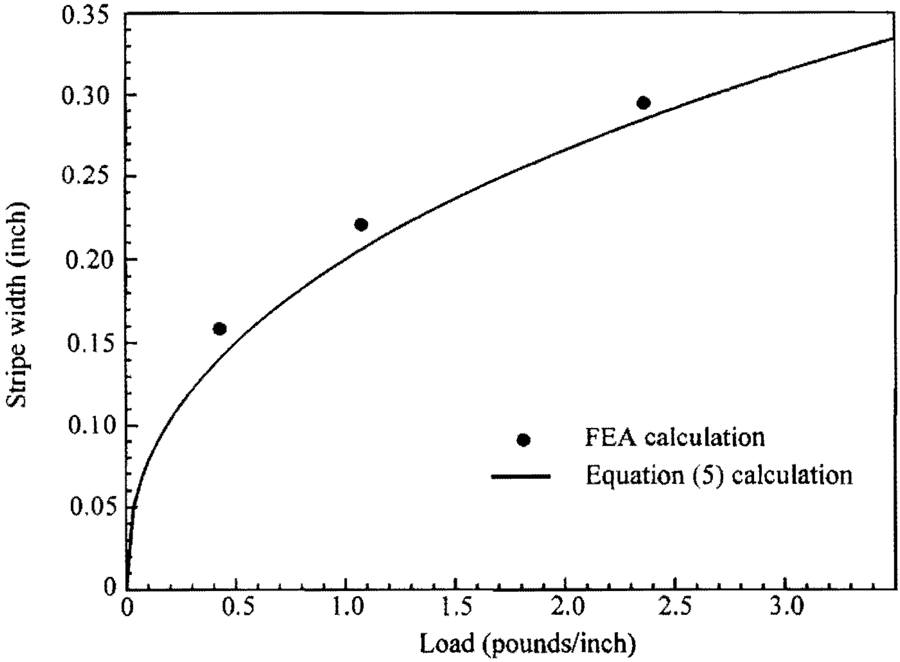

Figure 9 Data for hypothetical rubber roller indented by a parallel rigid roller. Roller diameters are 3 inch, thickness of rubber roller cover is 0.375 inch, and modulus of elasticity is 74 pounds/ square inch.

For Configuration 2, calculations were carried out for indentations of 0.006, 0.012, and 0.021 inch. Figure 9 shows a plot of the corresponding stripe widths versus load, along with a curve of data calculated using equation (5). The reasonably good agreement between the two methods of calculation provides further verification of both Deshpande's method and equation (5).

The results of the finite element analysis calculations were also used to extract mach useful information about the real-life roller nip represented by this configuration. For example, one surprising result is the character of gross roller deformation, illustrated in Figure 10. Contrary to popular belief, the reduction in volume caused by the indentation does not appear as a pronounced bulge at the nip entrance and exit. Rather, the expansion in rubber is of a global nature in that the rubber cover spreads out.

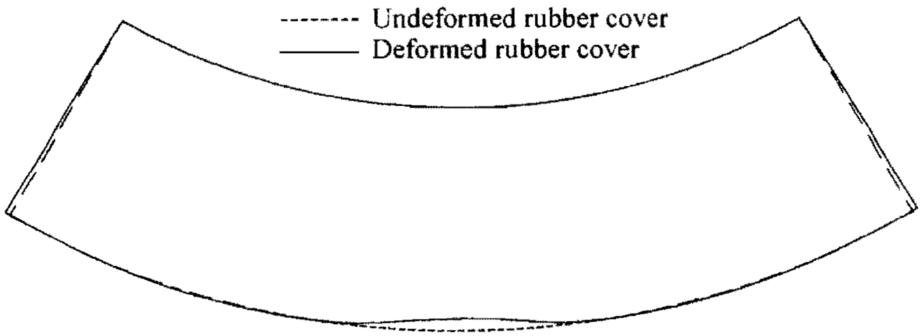

Figure 10 Gross deformation of 3 inch diameter rubber covered roller indented by a 3 inch diameter steel roller in parallel. Roller indentation is 0.021 inch, stripe width is 0.294 inch, and thickness of rubber cover is 3/8 inch.

Discussion and Conclusions

It should be recognized that no account has been taken here of temperature effects or the non-linear elastic and viscoelastic characteristics of rubber. Temperature differences, which could not be controlled, might account for some of the data scatter, although the effect is thought to be small. Similarly, non-linear effects were not thought to be signifcant because the strains involved in these problems are quite small. For this reason, it was assumed that the elastic behavior of rubber could be represented by Hooke's law. In contrast, viscoelastic effects may well have been a major cause of the observed scatter in measured values, since values of Young's modulus, derived from measurements of stripe widths at shorter times after load application, exhibited increases of 10 to 15 percent for some rollers. Similar changes in durometer readings were observed when using the fixed travel technique for making measurements.

It is quite clear that the Type A durometer does a poor job in discriminating hardness or compliance of identical geometry rollers covered with different rubber compounds. Although it has been shown here that durometer gauges with spherical indenters, like the Type O, perform better in this regard, it is not understood why. The author suspects that the response of the Type A durometer is affected by differences in surface roughness of the sample being measured, due to its relatively sharper indenter, and that this could explain the shortcomings of this type gauge. That is, it is suspected that the relatively sharper indenter of the Type A gauge may become caught in crevices in the surface being measured, and thus undergo greater deflection than if it had slid over the expanding surface.

One anomaly that has not been explained is the lower than expected values of m obtained for Rollers 1 - 3, given in Table 1. One possible reason is that the procedure used for measuring stripe width of these rollers is considered to be less accurate. That is, since stripe width was obtained by measuring the imprint left on the inked roller, a relatively large error can be made because the edge of this imprint is difficult to discern. In contrast, the edge of the imprint obtained using the current improved procedure stands out clearly. Unfortunately, the earlier measurements could not be repeated using the current procedure because these rollers were no longer available. It is also possible that the relatively thin cover thicknesses on the first rollers were a contributing factor, although the respective values of the ratio S_{max}/h of 1.3, 1.3, and 1.1 would belie this.

Another unexplained anomaly is the very large difference (factor of two) in the two correlations between Young's modulus and durometer harduess, described in this paper as Methods I and II, and illustrated in Figure 2. It is conceivable that this can be explained by the fact that the measurements used in Method 1 were of samples with flat surfaces whereas the measurements in Method 2 were of ones with cylindrical surfaces. That is, a durometer indenter pressed against a cylindrical surface may encounter less resistance (and hence produce a lower reading) compared to a flat surface of the same material, due solely to the differences in geometry.

Beyond the uncertainties just discussed, the following conclusions are drawn, based on the results obtained during the course of this work:

1. A closed form method, in the form of equation (5), has been derived for calculating stripe width versus load (or force) in the nips formed by typical rubber covered press rollers.

2. For calculations of stress and strain in a rubber covered roller, the Method 2 correlation, based on measurements of stripe versus load, should be used to establish the value of Young's modulus from readings of durometer hardness. Three such correlations are given in Figures 2, 4 and 5 of this paper.

3. The Type A durometer gauge was shown to exhibit very poor discrimination in measuring the hardness of rubber rollers covered with different elastomers. Durometer gauges with spherical indenters, such as the Type O, are capable of performing much better in this regard, depending on the measuring technique used.

4. Measuring technique had little effect on the results obtained with a Type A durometer gauge and a non-standard gauge having a 0.20 inch diameter spherical indenter. In contrast, mounting a Type O durometer gauge in a milling

machine quill, so as to prevent roller deformation by the gauge foot, reduced data scatter significantly.

5. The results of finite element analysis calculations provide independent support for Conclusions 1 and 2 above. This method of calculation also affords a powerful tool for gaining insight into the conditions existing in printing press roller nips.

Acknowledgments

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