Spectral Reflectance Prediction of Ink Overprints by Kubelka-Munk Turbid Media Theory

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Keywords: Overprints, Neugebauer-primaries

Abstract

An algorithm was tested for predicting the spectral reflectance factor of ink overprints, known as the Neugebauer primaries, using Kubelka-Munk turbid media theory. Two ink optical constants, spectral absorption and scattering coefficients, ink thickness, and the paper substrate's spectral reflectance factors are the key parameters to estimate the spectral reflectance factors of a translucent ink printed on top of an opaque support. This article shows the processes to determine the two optical constants and thickness, i.e., the characteristic parameters of an ink. Once all characteristic parameters of each ink are determined, the prediction of overprints is simply exercising the general form of the Kubelka-Munk equation. High colorimetric and spectral accuracy was obtained using this approach when predicting 100% overprints produced using the DuPont Water Proof proofing process.

Introduction

Development of a spectral-based color reproduction system at the Munsell Color Science Laboratory has been studied by several researchers under Professor Roy S. Berns (1999) in order to minimize metamerism between originals and printed reproductions. A multi-spectral imaging system requires a multi-spectral input system, used to acquire spectral information of original objects, and a multi-spectral output system, used to reproduce the captured spectral information onto different media such as paper. The research efforts for input systems are multi-spectral color acquisition (Burns,1997; Burns and Berns,1996, 1997a, 1997b)

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and high-resolution multi-spectral imaging archives (Imai and Berns, 1998). The research efforts for output systems include building color management modules using linear optimization (Iino and Berns, 1998a, 1998b) and spectral-based color separation algorithm development for multiple-ink printing (Tzeng, 1997). The multi-spectral output system is devised with an ink-set selection module and a spectral printer model. The ink-set selection module statistically uncovers a set of colorants that best represents a spectral image in terms of colorimetric and spectral accuracy, followed by a search through an existing ink database, such as the Pantone fourteen basic colors, to obtain the most spectrally similar ink set. The spectral printer model utilizes the Yule-Nielsen (1951) modified spectral Neugebauer (1937) equation to characterize the halftone printing process.

Consider the microscopic structure of ink on paper as delivered by a typical halftone printing process, shown in Figure 1; a three-color halftone print is shown for demonstration. The total spectral reflectance factor over a square area, which is assumed to be the area of interest, is a summation of the individual spectral reflectance factors of each color inside the area.

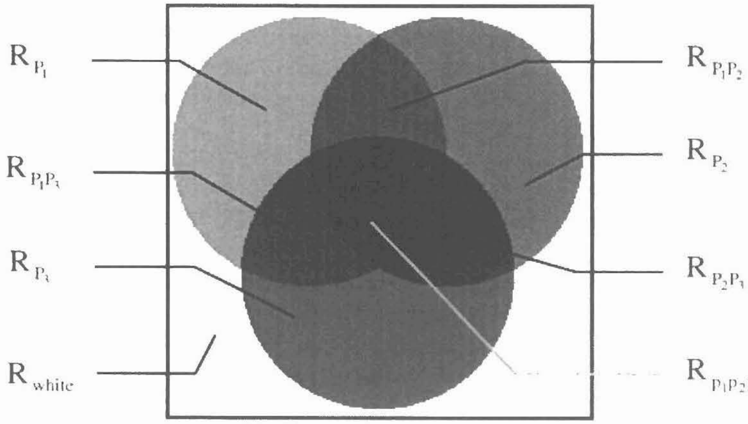

Figure 1: The microscopic structure of color formation by a halftone printing process where $R_{\lambda,color}$ represents the spectral reflectance factor of a color appearing in the square area.

As we can see, the colors appearing inside the area of interest for reproduction are not only the primary colors, white, primary one (P_1), primary two (P_2), and primary three (P_3), but also the overprints of the primaries, primary one on primary two (P_1P_2), primary one on primary three (P_1P_3), primary two on primary three (P_2P_3), and the three-primary overprint ($P_1P_2P_3$). These colors are usually called the Neugebauer primaries. Intuitively, the total spectral reflectance factor, $R_{\lambda,mix}$, over the area of interest is the linear sum, known as the spectral Neugebauer equation, of each spectral reflectance factor of the Neugebauer primaries modulated by their corresponding probability of occurrences. Yule-Nielsen further introduced an empirical n-factor to modify the spectral Neugebauer equation in order to account for light scattering within the paper, usually referred to as optical dot gain. The Yule-Nielsen modified spectral Neugebauer equation for a three-color (P_1, P_2, and P_3) printing process is defined as

$$\begin{aligned} R_{\lambda,mix} = [& a_{P_1} R_{\lambda,P_1}^{1/n} + a_{P_2} R_{\lambda,P_2}^{1/n} + a_{P_3} R_{\lambda,P_3}^{1/n} + \\ & a_{P_1P_2} R_{\lambda,P_1P_2}^{1/n} + a_{P_1P_3} R_{\lambda,P_1P_3}^{1/n} + a_{P_2P_3} R_{\lambda,P_2P_3}^{1/n} + a_{P_1P_2P_3} R_{\lambda,P_1P_2P_3}^{1/n} + \\ & (1 - a_{P_1} - a_{P_2} - a_{P_3} - a_{P_1P_2} - a_{P_1P_3} - a_{P_2P_3} - a_{P_1P_2P_3}) R_{\lambda,white}^{1/n}]^n \end{aligned}, \quad (1)$$

where a (indexed by P_1, P_2, P_3, P_1P_2, P_1P_3, P_2P_3, and $P_1P_2P_3$) represents the fractional dot area of a Neugebauer primary.

In order to use the Yule-Nielsen modified spectral Neugebauer equation, the spectral reflectance factor of each ink overprinted at 100% dot area coverage (known as secondary, tertiary, quaternary primaries, and so forth) are required as *a priori* knowledge for predicting the fractional dot area coverage of given spectra from an input spectral image. There are 2^j-j-1 overprints for a j-color halftone printing system. For example, a seven-color halftone printing process needs to print and measure 120 (2^7-7-1) overprints. As the number of colors used for halftone printing increases linearly, the number of overprints increases exponentially. Hence, an analytical method for predicting the spectral properties of overprints can avoid the necessity of exhaustively printing and measuring each overprint upon using different ink and paper materials.

Previous Research

Previous research for this task had been performed by Allen (1969). He proposed a three-layer model using Kubelka-Munk turbid media theory for translucent inks printed on top of a highly scattering support (Kubelka and Munk, 1931; Kubelka, 1948). Somehow, Allen abandoned the complex model proposed in 1969 in favor of a simpler approach (Allen and Hoffenberg, 1973). Basically, they applied a thin ink film on a Mylar film and backed the film with both black and white supports in optical contact in order to determine two surface reflectance measurements over the printed Mylar film. The two optical constants known as absorption, K, and scattering, S, coefficients of an arbitrary ink were numerically estimated by using the weight of the ink film to calibrate the thickness. According to Kubelka-Munk theory, the surface reflectance factor is a function of K, S, the thickness of the ink film alone, and the reflectance factor of the background.

Van De Capelle and Meireson (1997) took a different approach in which the surface reflectance factor of an ink printed on an opaque support is a function of three parameters, conceptually similar to the absorption and scattering coefficients and an additional interaction term. The determination of these three parameters requires printing an arbitrary ink onto white, gray, and black surfaces for setting up three simultaneous equations in order to solve for the three unknowns.

Another type of approach was recently exercised by Stollnitz, et al. (1998). It was claimed that the surface reflectance factor of multiple ink layers sitting on top of an opaque support is a function of the transmission factor of each ink layer, multiple internal reflections at each interface, and the reflectance factor of support. The scattering of each ink layer is not considered.

Finally, Viggiano and Hoagland (1998) used the additivity of ink density to predict ink overprints.

Unfortunately, the colorimetric and spectral accuracy for the Allen, Stollnitz, and Viggiano studies were not disclosed. The approach by Van De Capelle has been patented by Barco Co. Thus, it was of interest to explore whether Kubelka-Munk theory could predict these overprints with sufficient colorimetric and spectral accuracy for spectral-based color reproduction.

Technical Approach

The famous Kubelka-Munk basic equation (1931) is shown in Eq. (2):

$$R_\lambda = \frac{1 - R_{\lambda,g}(a_\lambda - b_\lambda \coth(b_\lambda S_\lambda X))}{a_\lambda - R_{\lambda,g} + b_\lambda \coth(b_\lambda S_\lambda X)}, \tag{2}$$

where λ is a wavelength within the visible spectrum, $R_{\lambda,g}$ is the spectral reflectance factor of an opaque support, K_λ is the absorption coefficient, S_λ is the scattering coefficient, X is the thickness of the layer of colorant, a_λ is equal to $1+(K/S)_\lambda$, b_λ is equal to $((a_\lambda)^2 - 1)^{1/2}$, and coth( ) is the hyperbolic cotangent function. The determination of K and S is carried out by drawing down or printing, for example, a thin ink film on black and white contrast paper, depicted as Figure 2. Contrast paper whose surface is covered by a transparent plastic layer or resin coating is recommended to use. This layer prevents the ink from submerging into the paper fiber. The process utilizing contrast paper with plastic or resin coating is similar to that of Allen and Hoffenberg's preparation using Mylar film back-coated by black and white paints.

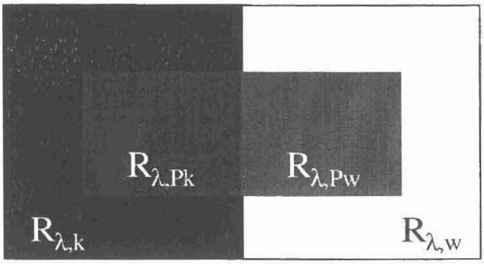

Figure 2: An ink film applied on black and white contrast paper.

Four spectra can be attained by this technique to set up the two nonlinear equations using Eq. (2) by assuming the ink thickness is unity and homogenous where $R_{\lambda,Pw}$ and $R_{\lambda,Pk}$ are the spectral reflectance factor of an primary ink (P) printed over white and black support, respectively; and $R_{\lambda,w}$ and $R_{\lambda,k}$ are the spectral reflectance factor of the white and black areas of the contrast paper, respectively. First, the surface reflectance factor, $R_{\lambda,Pw}$, of the ink printed on top of the white background is described by

$$R_{\lambda,Pw} = \frac{1 - R_{\lambda,w}(a_{\lambda,P} - b_{\lambda,P}\coth(b_{\lambda,P}S_{\lambda,P}))}{a_{\lambda,P} - R_{\lambda,w} + b_{\lambda,P}\coth(b_{\lambda,P}S_{\lambda,P})}, \tag{3}$$

where X is assumed to be unity. Second, the surface reflectance factor, $R_{\lambda,Pk}$, of the same primary ink printed on top of black background is described by

$$R_{\lambda,Pk} = \frac{1 - R_{\lambda,k}(a_{\lambda,P} - b_{\lambda,P}\coth(b_{\lambda,P}S_{\lambda,P}))}{a_{\lambda,P} - R_{\lambda,k} + b_{\lambda,P}\coth(b_{\lambda,P}S_{\lambda,P})}. \tag{4}$$

Notice that the difference between Eqs. (3) and (4) is the term of $R_{\lambda,g}$ in Eq. (2) which is substituted with the spectral reflectance factor of the white support in Eq. (3) and substituted with the spectral reflectance factor of the black support in Eq. (4). Thus, Eqs. (3) and (4) construct a nonlinear system with two equations and two unknowns, K_{λ} and S_{λ}. To solve this system of nonlinear equations, a numerical method based on the techniques of operational research can be employed to estimate these two optical constants. This is repeated for each ink of interest.

Once all the optical constants related to the assumed thickness are numerically determined, the prediction of overprints depends on the thickness of inks actually printed on a medium such as the SWOP specified standard paper. The effective thickness for each ink can be estimated using Eq. (2) by the known optical constants for each ink, measured surface reflectance factors of each ink printed on a specific paper, and the measured reflectance factor, $R_{\lambda,paper}$, for the specific paper. The equation for estimating the thickness, typical of a printing or proofing process, of an primary ink (P) is set up by

$$R_{\lambda,P} = \frac{1 - R_{\lambda,paper}(a_{\lambda,P} - b_{\lambda,P}\coth(b_{\lambda,P}S_{\lambda,P}X_P))}{a_{\lambda,P} - R_{\lambda,paper} + b_{\lambda,P}\coth(b_{\lambda,P}S_{\lambda,P}X_P)}, \quad (5)$$

where $R_{\lambda,P}$ is the spectral reflectance factor of a primary ink printed on top of a specific paper with spectral reflectance factor, $R_{\lambda,paper}$, and X_P is the effective thickness. X_P again can be solved by a numerical method. Once K_λ, S_λ, and X for each ink are estimated, the prediction of ink overprints is simply a recursive calculation using Eq. (2). That is, taking a three-ink-layer overprint as an example, given that all sets of characteristic parameters for all ink layers were estimated, the prediction of the topmost spectral reflectance factor requires the knowledge about the spectral reflectance factor of the second layer which are predicted based on the *a priori* knowledge about the known spectral reflectance factor of the bottom layer.

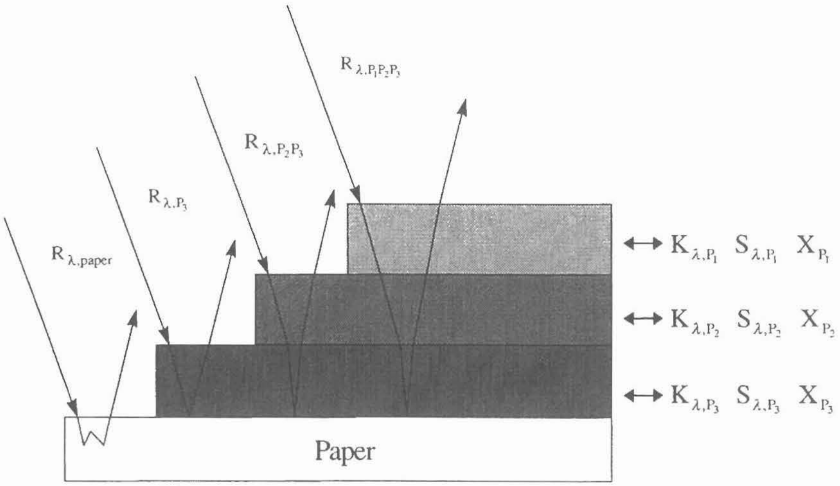

Figure 3: The diagram of a three-ink-layer overprint.

Figure 3 is shown to conceptualize this process where R_{λ,P_3} is the spectral reflectance factor of primary three printed on paper with $R_{\lambda,paper}$, R_{λ,P_2P_3} is the spectral reflectance factor of primary two printed on top of the primary three, and $R_{\lambda,P_1P_2P_3}$ is the spectral reflectance factor of primary one printed on the topmost layer. Analytically, the estimation of spectral reflectance factor for all three ink layers can be described by Eq. (6) for the bottom layer, Eq. (7) for the inter layer, and Eq. (8) for the topmost layer, respectively:

$$R_{\lambda,P_3} = \frac{1 - R_{\lambda,paper}(a_{\lambda,P_3} - b_{\lambda,P_3}\coth(b_{\lambda,P_3}S_{\lambda,P_3}X_{P_3}))}{a_{\lambda,P_3} - R_{\lambda,paper} + b_{\lambda,P_3}\coth(b_{\lambda,P_3}S_{\lambda,P_3}X_{P_3})}, \tag{6}$$

$$R_{\lambda,P_2P_3} = \frac{1 - R_{\lambda,P_3}(a_{\lambda,P_2} - b_{\lambda,P_2}\coth(b_{\lambda,P_2}S_{\lambda,P_2}X_{P_2}))}{a_{\lambda,P_2} - R_{\lambda,P_3} + b_{\lambda,P_2}\coth(b_{\lambda,P_2}S_{\lambda,P_2}X_{P_2})}, \tag{7}$$

$$R_{\lambda,P_1P_2P_3} = \frac{1 - R_{\lambda,P_2P_3}(a_{\lambda,P_1} - b_{\lambda,P_1}\coth(b_{\lambda,P_1}S_{\lambda,P_1}X_{P_1}))}{a_{\lambda,P_1} - R_{\lambda,P_2P_3} + b_{\lambda,P_1}\coth(b_{\lambda,P_1}S_{\lambda,P_1}X_{P_1})}. \tag{8}$$

The accuracy of this process is defined using the CIE94 color difference equation calculated under standard illuminant D50 and the 1931 standard observer (CIE, 1995). The spectral accuracy is quantified both by root-mean-square (RMS) error in units of reflectance factor and the CIE94 color difference equation calculated under standard illuminant A and the 1931 standard observer as the metamerism index (M. I.) after parameric correction (Fairman, 1987).

Experimental

For the convenience of verifying the technical approach described above, the DuPont Water Proof system was used to print the six primaries, which are cyan, magenta, yellow, red, green, and blue, on six pieces of contrast paper shown in Figure 4. Twenty-five overprints, shown in Figure 5, were generated using at most three-primary combinations. Among the 25 overprints, 14 are two-color overprints and 11 are three-color overprints. The printing order corresponds to the color order putting cyan at the bottom most layer, magenta on cyan, yellow on magenta, red on yellow, green on red, and blue on the top most layer. These samples were measured using a Gretag Spectrolino by averaging five measurements for each color. Due to the different refractive indices among air, ink, and support, the Saunderson correction was employed to correct for refractive-index discontinuity at each interface (Saunderson, 1942; Allen, 1987).

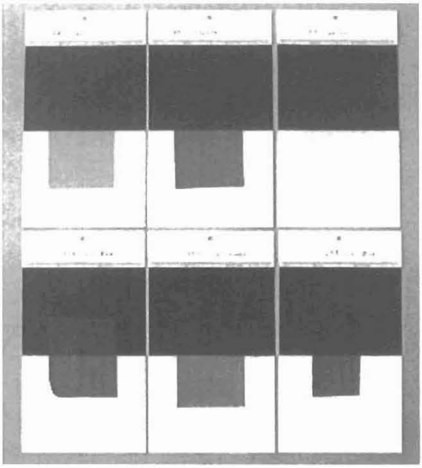

Figure 4: The six primaries printed on contrast paper.

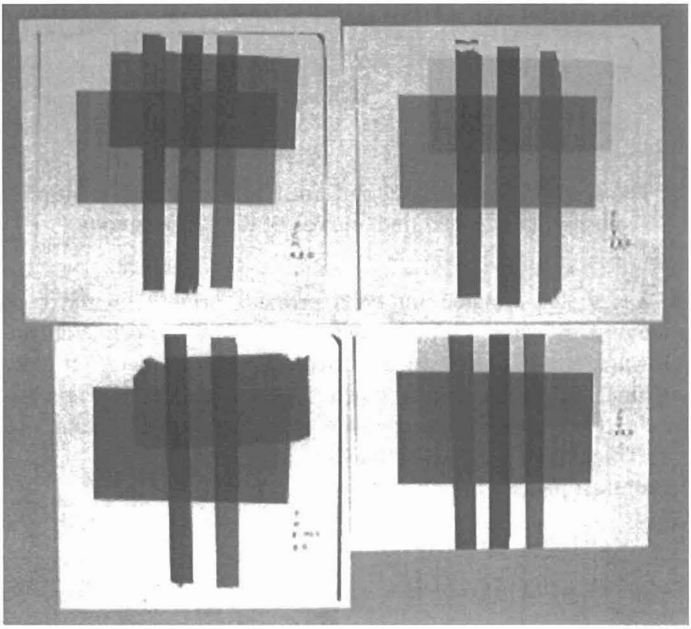

Figure 5: Twenty-five overprints printed on coated paper.

Results

Equations (3) and (4) were used to set up the system of nonlinear equations and solved for the two unknowns by assuming the thickness for each ink is unity. The estimated K_λ and S_λ of the six primaries are plotted in Figure 6.

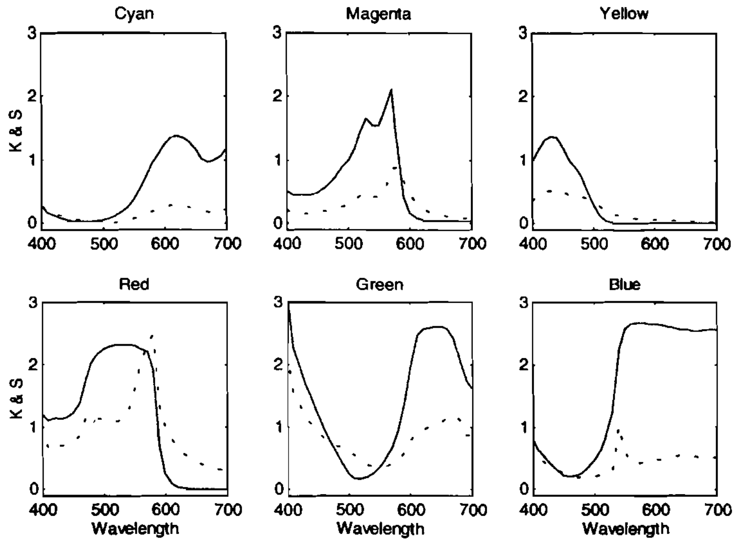

Figure 6: The spectral absorption (solid line) and scattering (multiplied by ten times and dashed line) curves of the six primaries.

Thickness was then estimated for each primary printed on the coated paper shown in Figure 5, using Eq. (2). Due to the high colorimetic and spectral accuracy for predicting primaries, the difference spectra between measured and predicted primaries printed on the coated paper are plotted in Figure 7 and their colorimetric and spectral accuracy as well as their statistical thickness are shown in Table I. Thickness is a ratio related to the ink thickness of each primary printed on contrast paper.

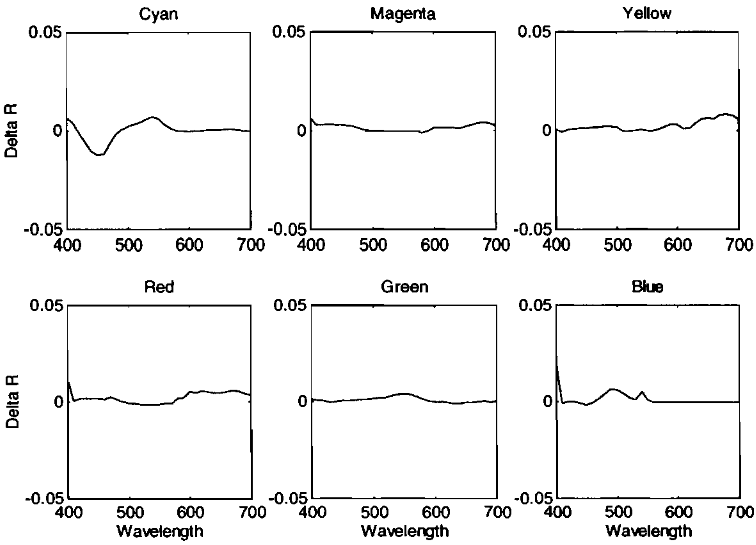

Figure 7: The difference spectra between measured and predicted primaries.

Table I: The colorimetric accuracy, spectral accuracy, and the statistical thickness for the six primaries.

	Cyan	Magenta	Yellow	Red	Green	Blue
ΔE^*_{94}	0.8	0.3	0.1	0.4	0.2	0.8
Metamerism Index	0.0	0.0	0.0	0.0	0.0	0.2
RMS Error	0.005	0.002	0.004	0.004	0.001	0.004
Thickness	0.956	0.958	0.975	1.026	0.962	0.987

According to Table I, the prediction of each primary is of high spectral accuracy as indicated by the near zero metamerism index and the low RMS error. Thus, the first verification ensures the success using of Kubelka-Munk theory to predict the translucent material backed by an opaque support. With the knowledge of optical constants, K_λ and S_λ, and the effective thickness of ink deposited by a typical printing process, the estimation of spectral reflectance factor can be accomplished whenever the paper support is changed under the assumption that there is no interaction between ink and paper (coated paper is preferred). Since the accuracy of the first prediction is high, the interaction between ink and paper is considered insignificant. Second, the prediction of overprints is based on the assumption that no chemical or physical interaction at interfaces of each ink layer. In our experiment, the statistical colorimetric and spectral performance of predictions for the 25 overprints, shown in Table II, is considered high judged by the low average and standard deviation (SDV) of metamerism indices whose histogram is shown in Figure 8. It indicates that almost all the overprints are predicted with high accuracy since most of estimated metamerism indices of 25 overprints are concentrated around 0.3 units of metamerism index.

Table II: The colorimetric and spectral accuracy of the 25 overprints.

	ΔE^*_{94}	Metamerism index
Mean	0.9	0.3
SDV	0.5	0.3
Maximum	2.1	1.2
Minimum	0.2	0.0
RMS Error	0.004	

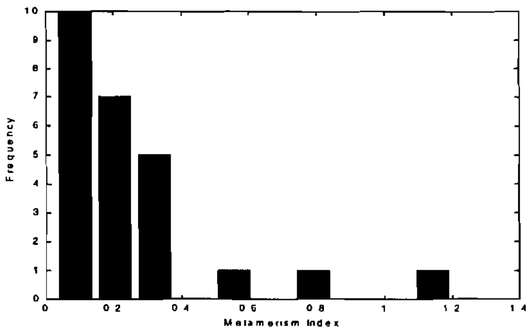

Figure 8: Histogram of the metamerism indices for prediction of the 25 overprints.

Figures (9) and (10) are shown as examples of good predictions and the examples of predictions with relatively low accuracy in terms of their metamerism indices. However, the spectral predictions of these "relative low accuracy" are considered acceptable judged by their low colorimetric and spectral error. The appearance of predicted spectral curves is tracing along the corresponding measured spectral curves indicated by the difference spectra.

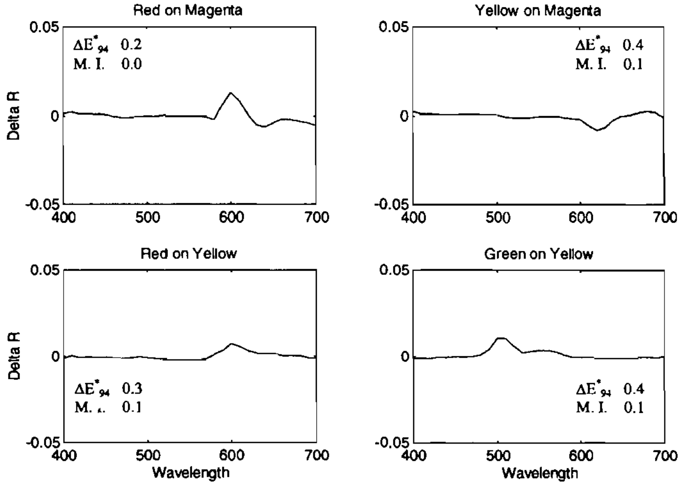

Figure 9: The difference spectra of four overprints best predicted with high accuracy.

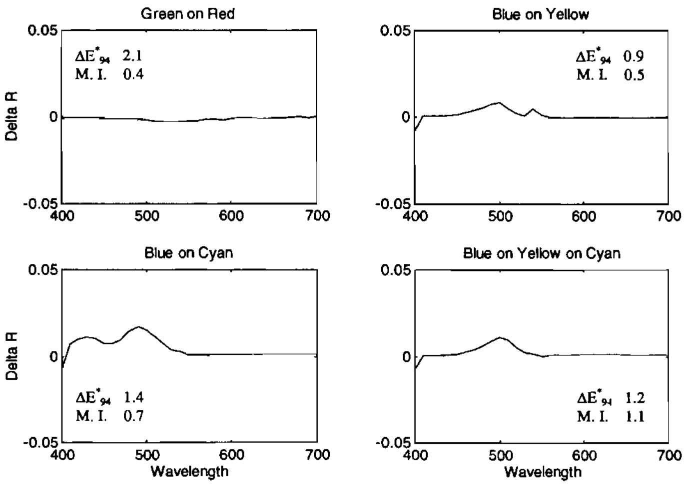

Figure 10: The difference spectra of four overprints predicted with relatively low accuracy.

Discussion

The errors contributed to the prediction using the technical approach described herein can be attributed from three types of error. The first is the uniformity of the paper support to be printed. Second is the homogeneity of ink thickness delivered by a printing process. The last is the accuracy of the determination of the two optical constants, K_λ and S_λ. Recall that K_λ and S_λ are solved numerically. In the process of solving for K_λ and S_λ, the constraints of the positivity of these two optical constants should be superimposed in numerical estimation to account for the absorption and scattering properties of real material, i.e., the negativity of K_λ and S_λ is not realizable. Another realistic consideration for the numerical estimation is that the term a_λ in Eq. (2) is a function of the ratio of absorption to scattering coefficient, $(K/S)_\lambda$, at a sampled wavelength. When estimated S_λ is zero, the numerical error of division by zero happens. The process required setting the lower boundary of S_λ to be nonzero to prevent the numerical error of the division by zero in addition to that Eq. (2) can be confounded by the numerical result of zero divided by zero. This limitation causes the over-prediction for the reflectance factor when both the estimated K_λ and S_λ are low and close to zero. The strongest evidence is the prediction for cyan ink printed on the coated paper.

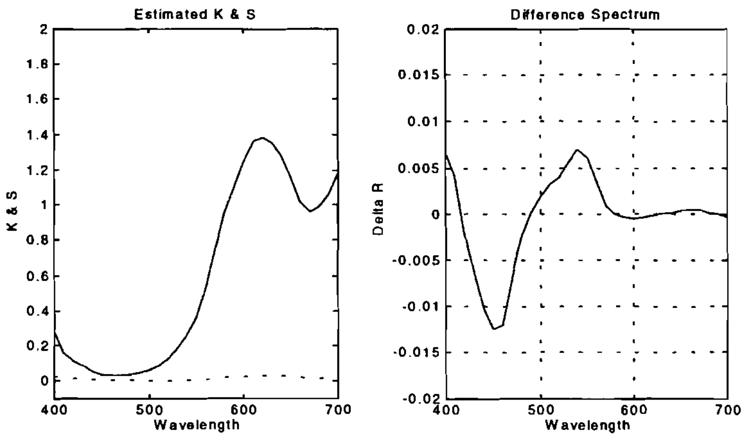

Figure 11: The estimated two optical constants for cyan ink and corresponding difference spectrum in units of reflectance factor.

Figure 11 indicates that estimated low absorption (solid line) and scattering (dashed line) for cyan ink happens from 440 nm to 470 nm. Its corresponding prediction of spectral reflectance factor is over-predicted (the difference spectrum is obtained by the predicted subtracted from the measured) at the same spectral region due to the numerical limitation.

Future Efforts

This analysis was performed by a proofing device with a small number of ink overprints. However, the ultimate goal for this process is its application to an offset printing press for multiple-ink printing minimizing metamerism. Hence, the verification for the approach in predicting overprints printed by offset press is a necessary future effort.

Conclusions

An algorithm for predicting overprints by a proofing device was exercised and verified. The errors contributing to this verification process are the uniformity of a coated paper to be printed, homogeneity of the printed ink thickness, and numerical limitations when estimating the two optical constants. Nevertheless, high colorimetric and spectral accuracy was achieved by this approach using Kubelka-Munk turbid media theory.

Acknowledgments

The authors wish to express their thanks to E. I. du Pont de Nemours and Company, Inc. for their financial support of this research.

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